University of Bahrain
Department of Electrical and Electronics Engineering

EENG372: Communication Systems I

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Topic 0:

Signal Analysis

This topic will cover

- Signal Classification
- Fourier representation of signals
- Spectrum, Bandwidth, Channel and Frequency allocation
- Power and Decibel

Signals

Q: What is a Signal?

A signal is a function representing information or data.

$$x(t)$$

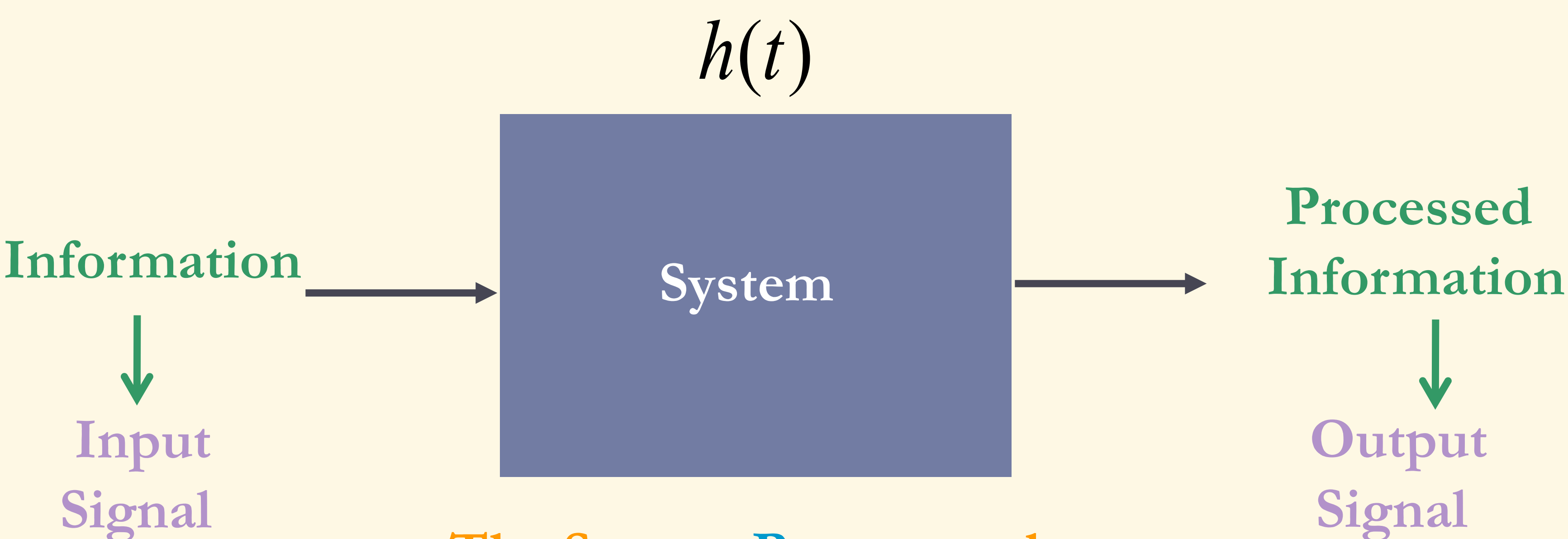
Examples:

- Speech or music
- Medical signals, such as EEG or ECG.

Systems

Q: What is a System?

A system is an entity that can process information to produce another 'form' of information



The System Processes the Input signal to produce an output signal

Size of Signals

Q: Can a signal that is varying with time be measured by one number that indicates its "size"?

For $x(t)$

We can find the energy

$$E_g = \int_{-\infty}^{\infty} |x(t)|^2 \, dt$$

Or the average power

$$P_g = \lim_{t \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 \, dt$$

Signal Classification

Types of Signals:

Continuous Time and Discrete Time

A signal is either:

Continuous–time (CT)

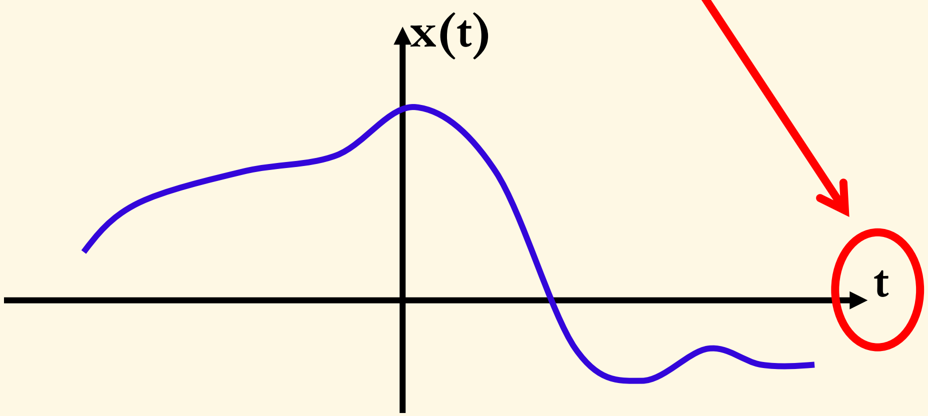
It is defined for all time **t**

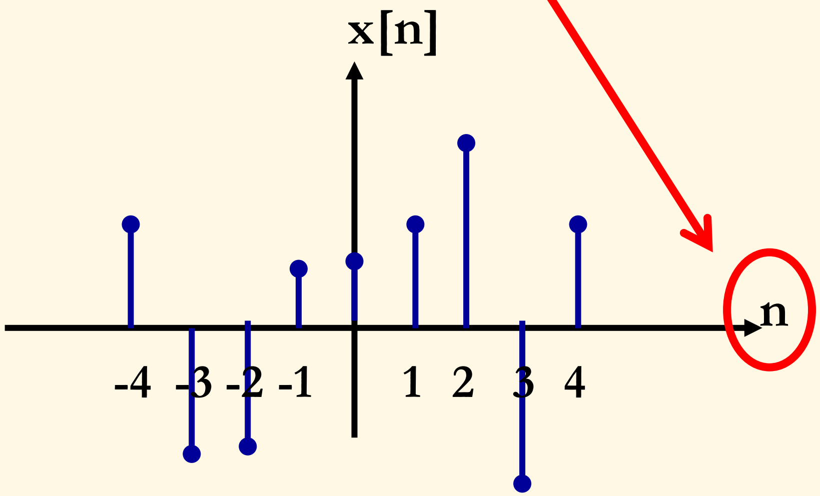
Discrete–time (DT)

It is defined at discrete instants in time **n**

This is related to the independent variable

Example:





Analogue and Digital

A signal is either:

Analogue

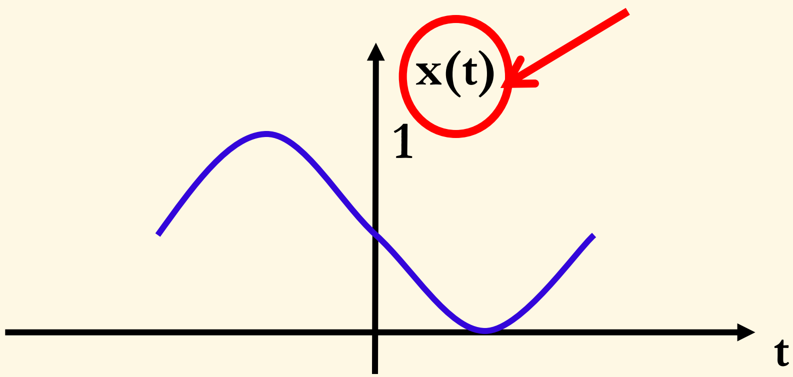
amplitude varies continuously between two values.

Digital

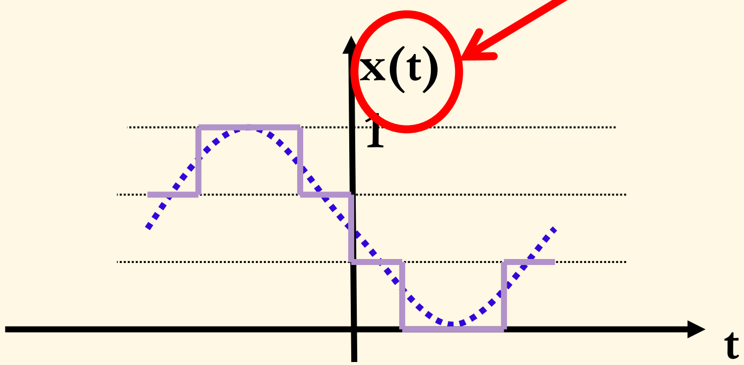
amplitude can only have a finite number of values between two values

Example:

This is related to the dependent variable



the amplitude varies between 0 and 1



amplitude can take only 4 values between 0 and 1

Periodic or Aperiodic

A signal is either:

Periodic

It repeats itself every T time

There is a Time T such that

$x(t) = x(t + T)$

where T is called the period of the signal

Non Periodic

Aperiodic

It never repeats

There is NO Time T such that

$x(t) = x(t + T)$

Periodicity in CT

In continuous time a periodic signal is one that repeats itself every T seconds.

- Mathematically it satisfies the condition: $x(t) = x(t+T)$
  for all t where T is a positive constant.
- Note that if $T = T_0$ satisfies the above equation then so does $T_1 = 2T_0, T_2 = 3T_0, \ldots\ldots.$
- The smallest constant T that satisfies the condition is called the fundamental period of the periodic signal. In this case T_0.
- The reciprocal $f = (1/T_0)$ is the fundamental frequency and it defines the frequency at which $x(t)$ repeats itself.
- The angular frequency in *radians/ second* is defined as $\omega = 2\pi f$
- A signal that does not satisfy the condition is called a *non-periodic* or an *aperiodic* signal.

Periodicity in DT

Q: Can a DT signal be periodic? Give an example

Periodicity in CT

Exercise: Find the period, and angular frequency

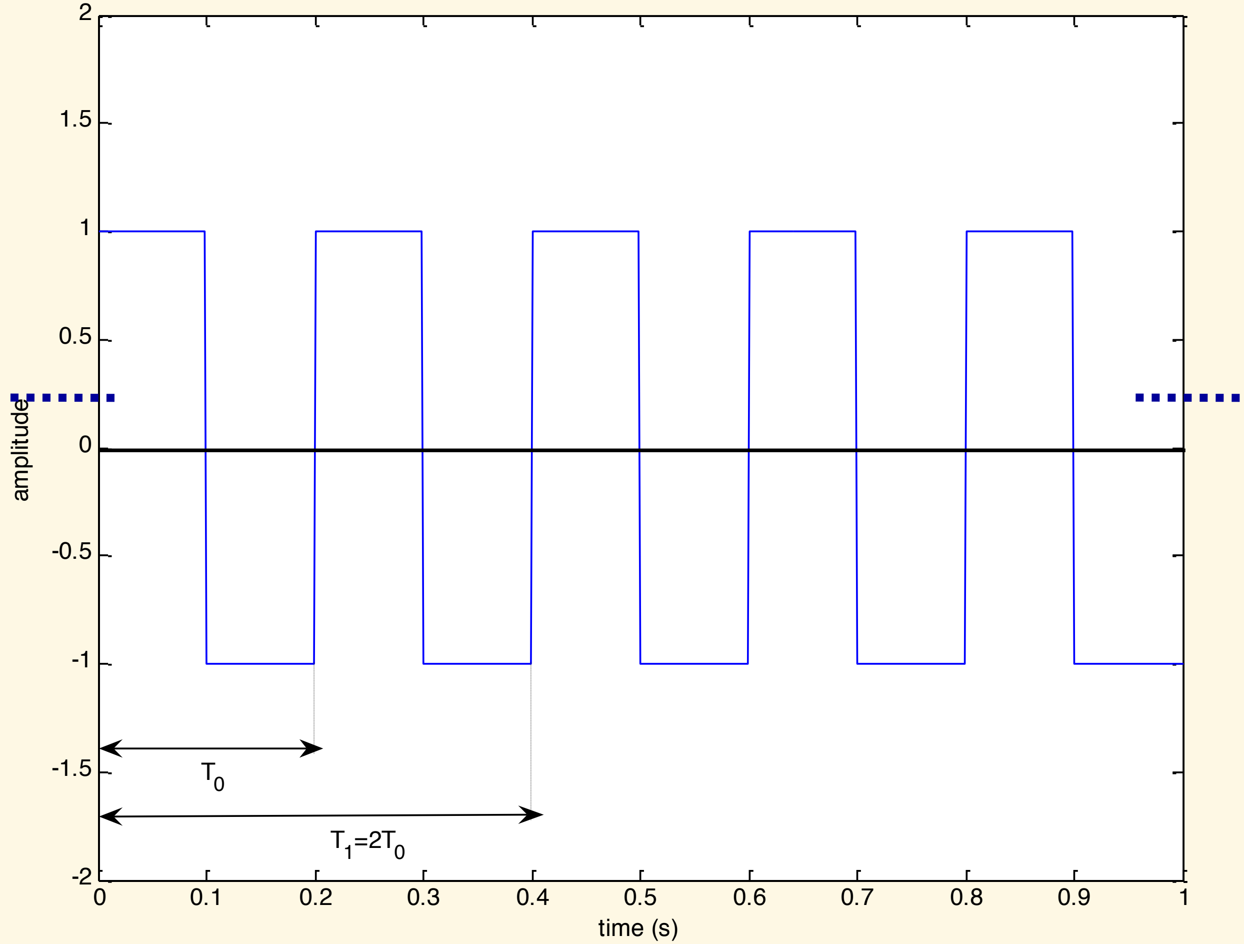
This is a CT periodic signal.

T_o = .2 seconds.

Multiples of T_0: .4,.6,.8 … are also periods of the signal.

f = 5 Hz

ω = 31.42 radians/sec.



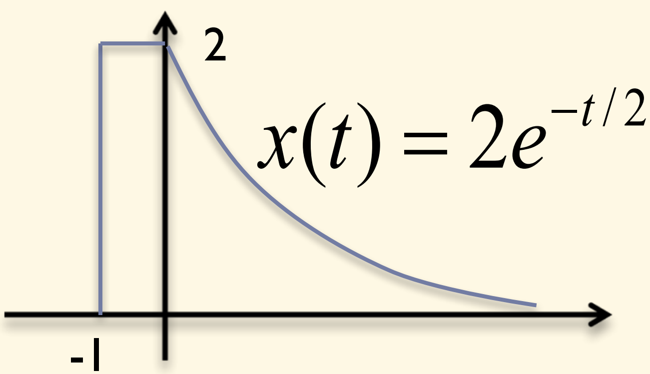
Energy Signal and Power Signal

A signal is either:

Energy Signal

Has finite energy

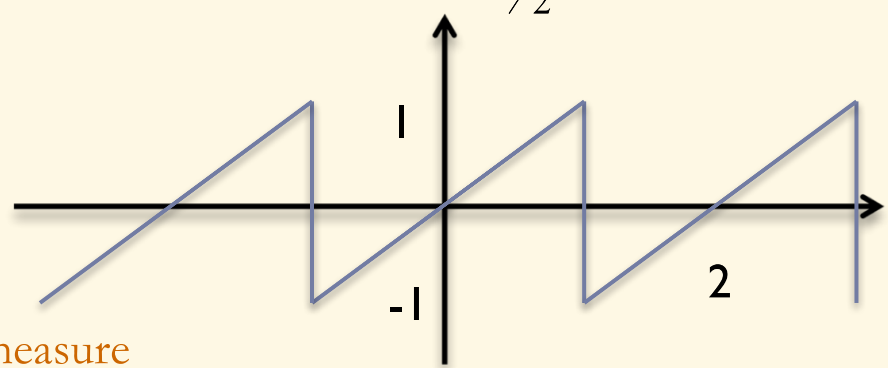
$$\int_{-\infty}^{\infty} |x(t)|^2 \, dt < \infty$$



Power Signal

Has finite power (mean square value)

$$0 < \lim_{t\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 \, dt < \infty$$



A measure

Energy Signal and Power Signal

Exercise: Determine the suitable measure of "size" for the signals on the previous slide.

Deterministic Signal and Random Signal

A signal is either:

Deterministic Signal

physical description completely known

mathematical or graphical

Random Signal

probabilistic description known

mean or mean squared

Noise

Message Signal

Fourier Series

Sinusoidal Signals

Q: What is Sinusoidal Signal?

A cosine (or sine) function which is defined as:

Amplitude

time

Phase Shift

$$x(t) = A\cos(\omega t + \phi)$$

Angular Frequency

Cosine or sine

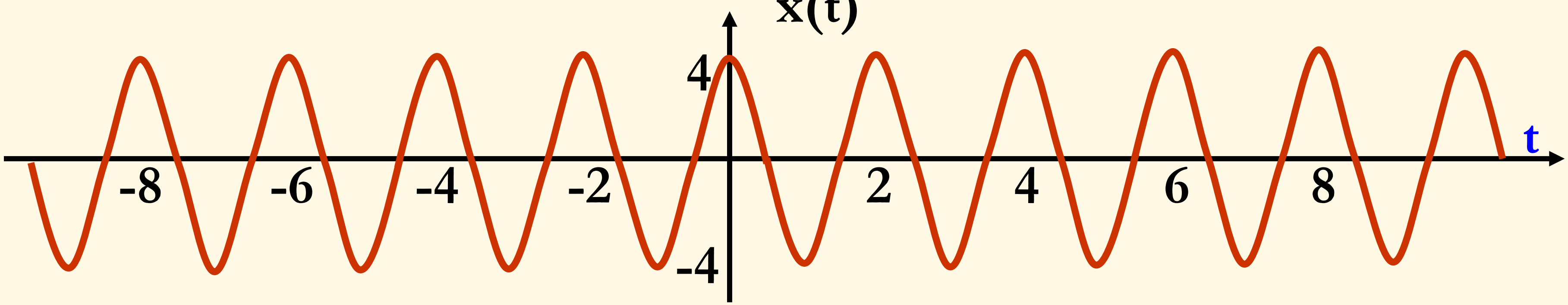
$$f = \frac{1}{T}$$

$$\omega = 2\pi f$$

Sinusoidal Signals

Exercise: What is T_0, f, ω and A?

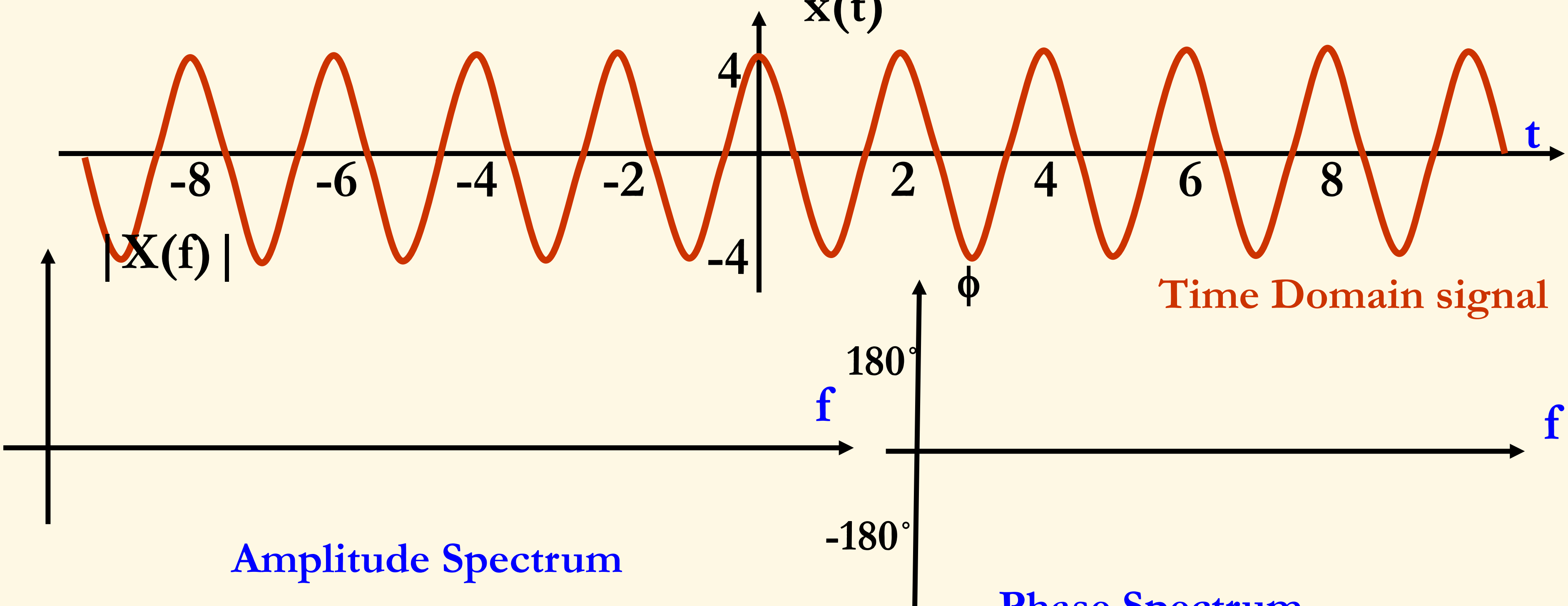
$$x(t) = \ldots\cos(\ldots\ldots t + \ldots) \qquad = \frac{}{2}\left[e^{j(\ldots\ldots t)} + e^{-j(\ldots\ldots t)}\right]$$



Amplitude Spectrum

Q: What is the amplitude spectrum of x(t) signal?

$$x(t) = \ldots\cos(\ldots\ldots t + \ldots) \qquad = \frac{\;}{2}\left[e^{j(\ldots\ldots t)} + e^{-j(\ldots\ldots t)}\right]$$



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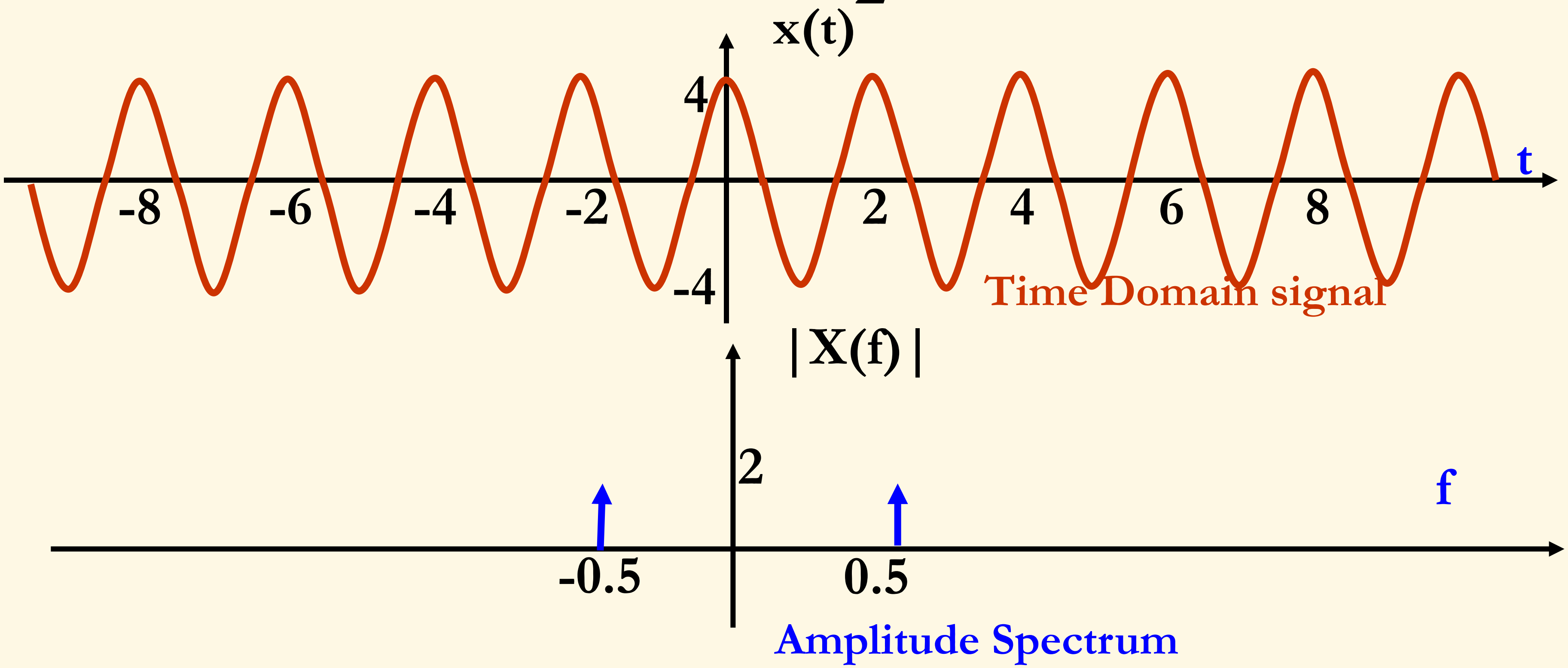

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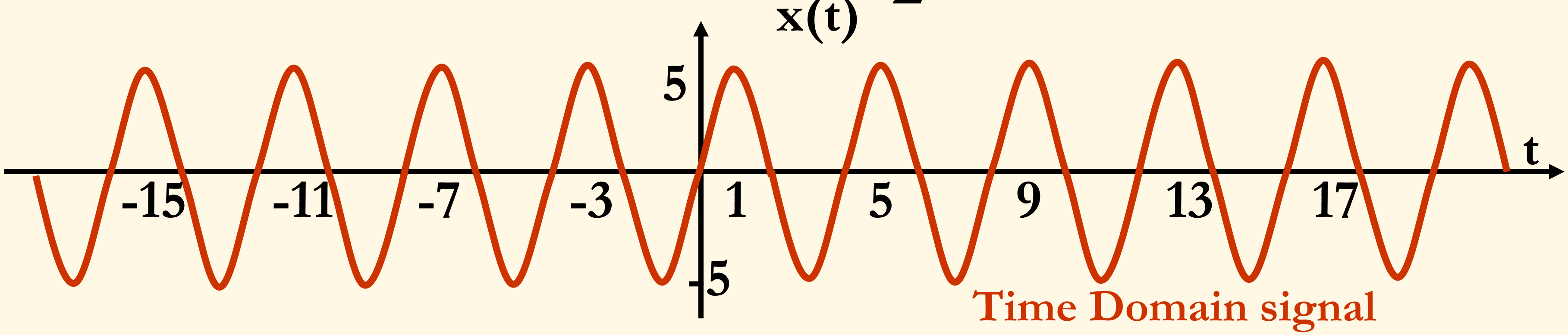

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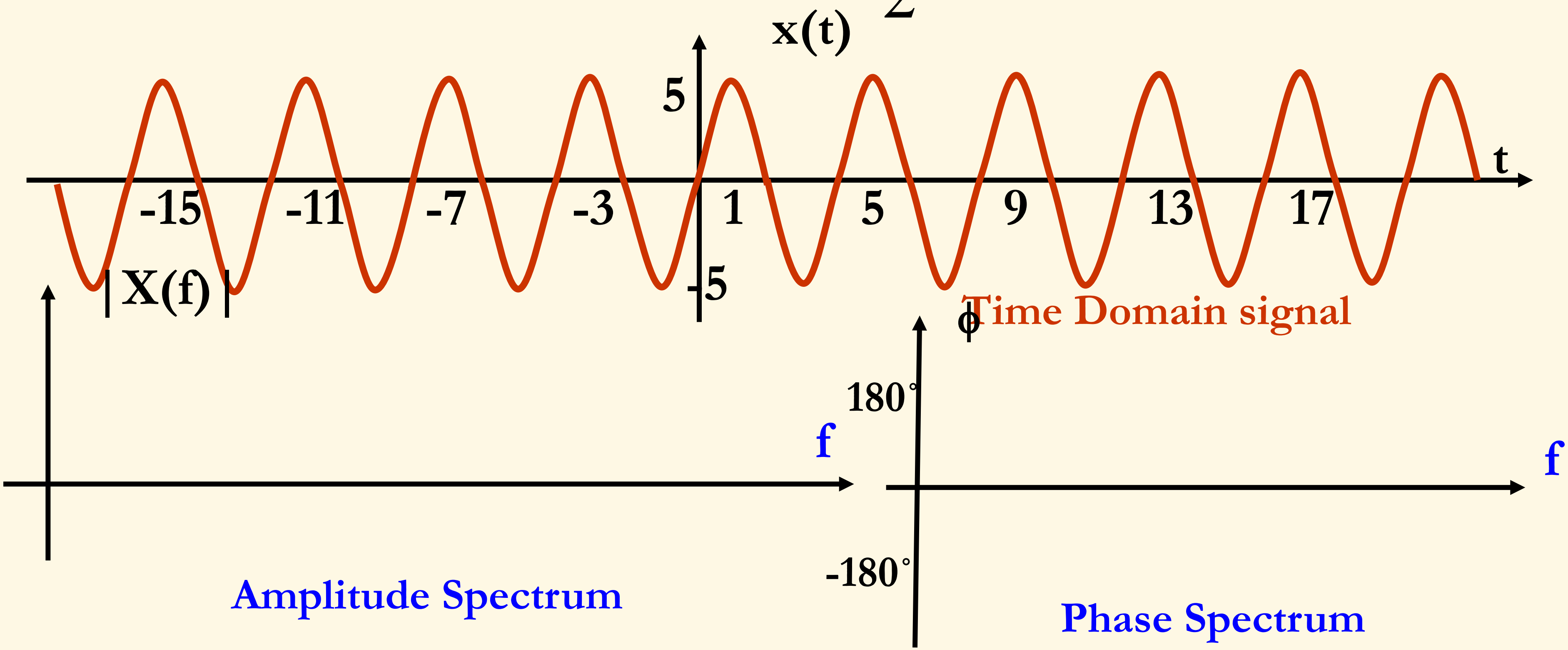

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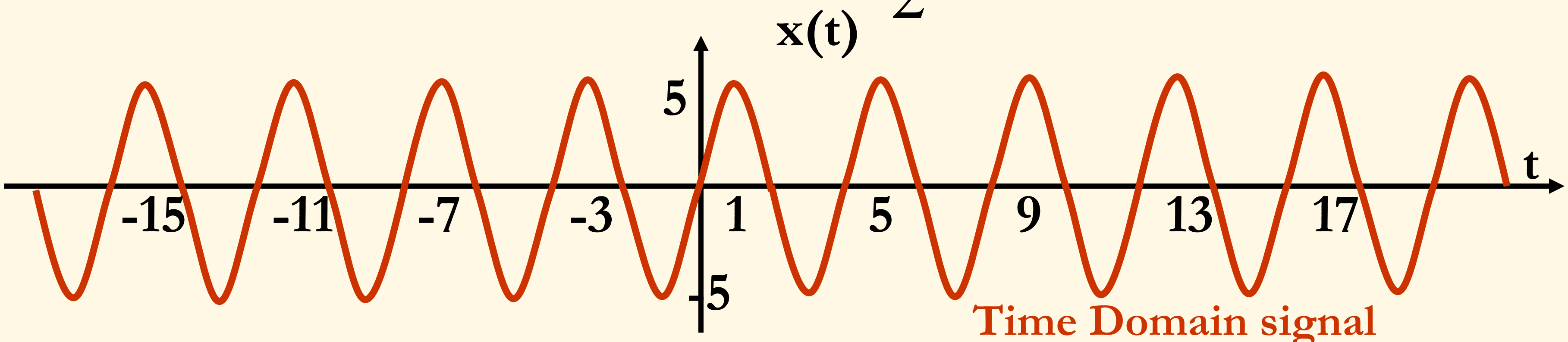

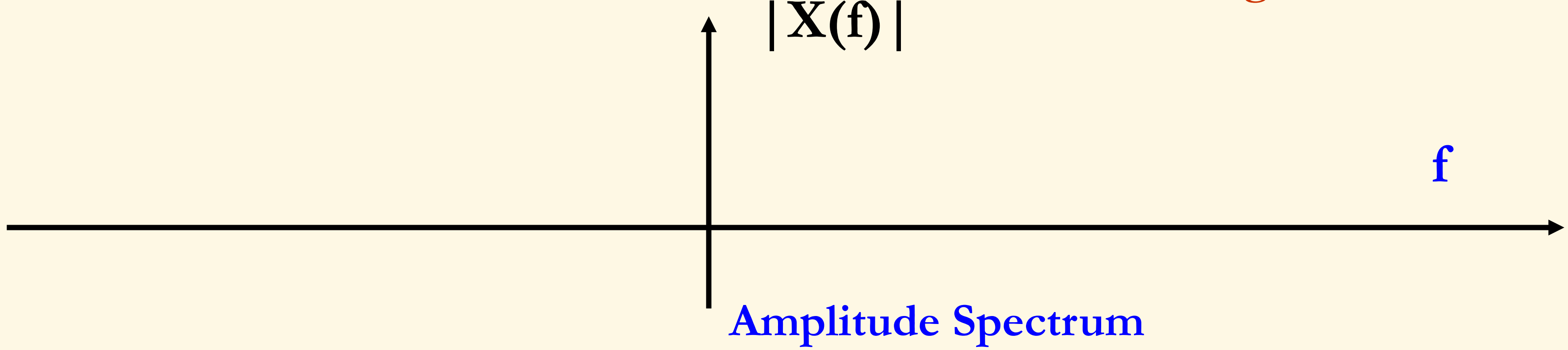

Sinusoidal Signals

Exercise: What is T_0, f, ω and A?

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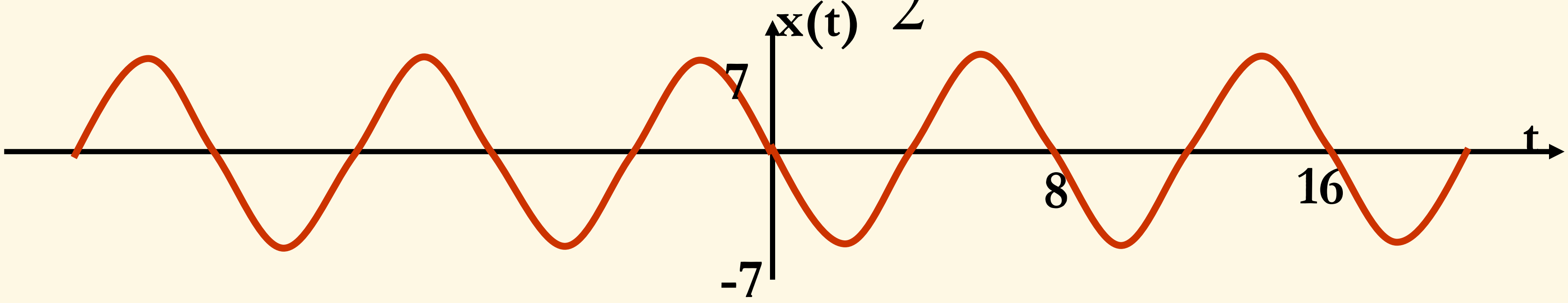

Time Domain signal

Amplitude Spectrum

Q: What is the amplitude spectrum of x(t) signal?

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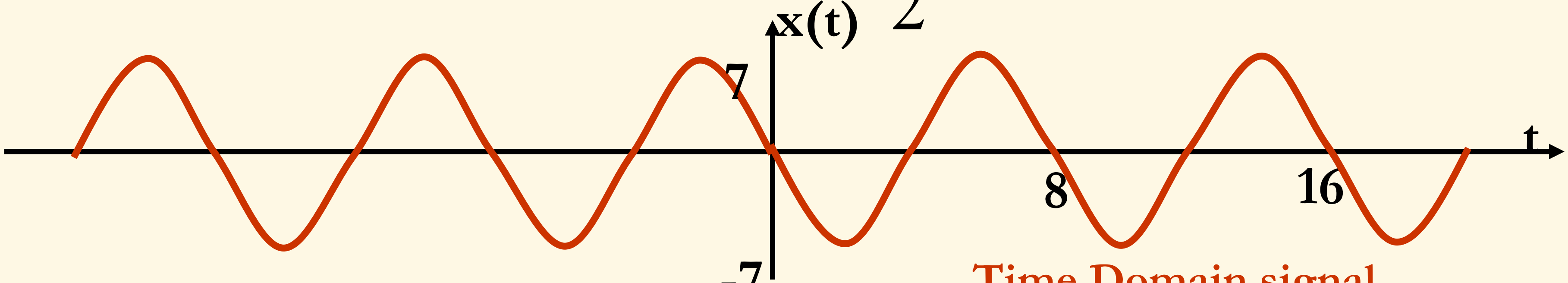
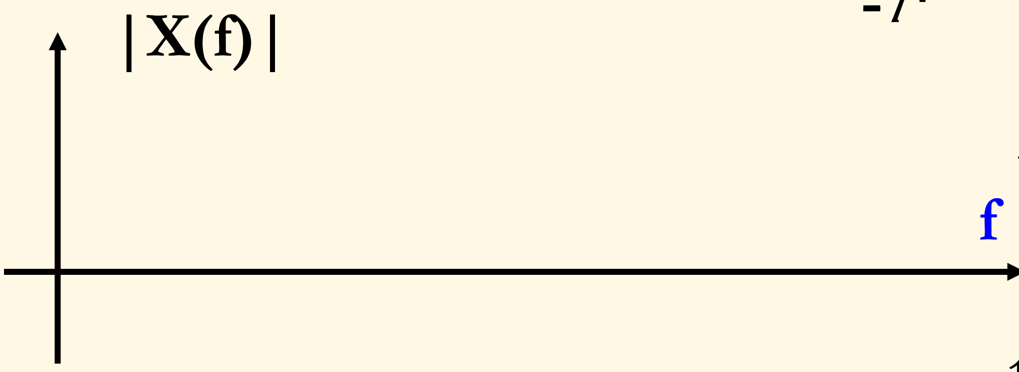
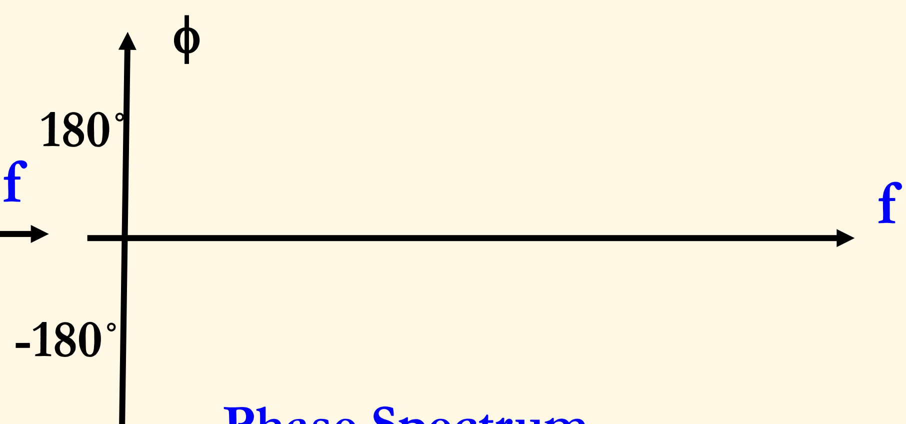


Time Domain signal



Amplitude Spectrum

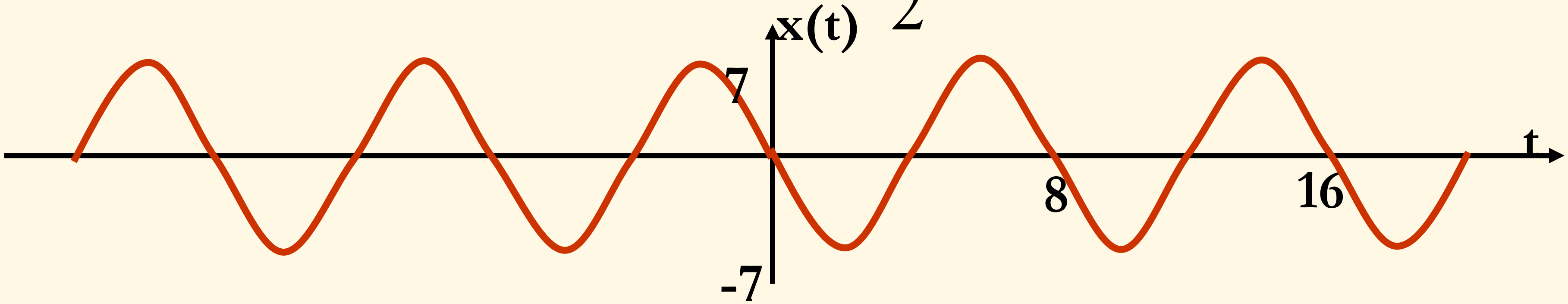


Phase Spectrum

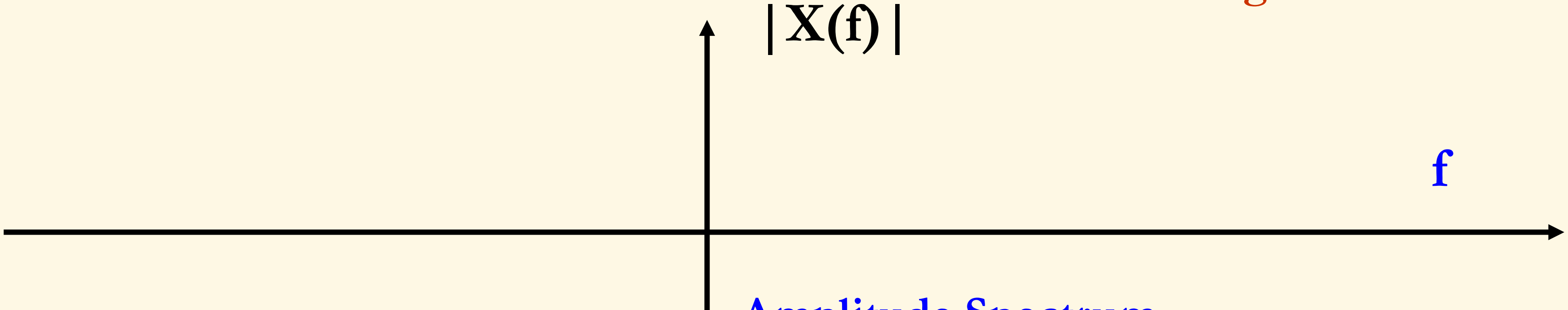
Amplitude Spectrum

Q: What is the amplitude spectrum of x(t) signal?

$$x(t) = \ldots\cos(\ldots\ldots t + \ldots) \qquad = \frac{\ }{2}\left[e^{j(\ldots\ldots t)} + e^{-j(\ldots\ldots t)}\right]$$



Time Domain signal



Amplitude Spectrum

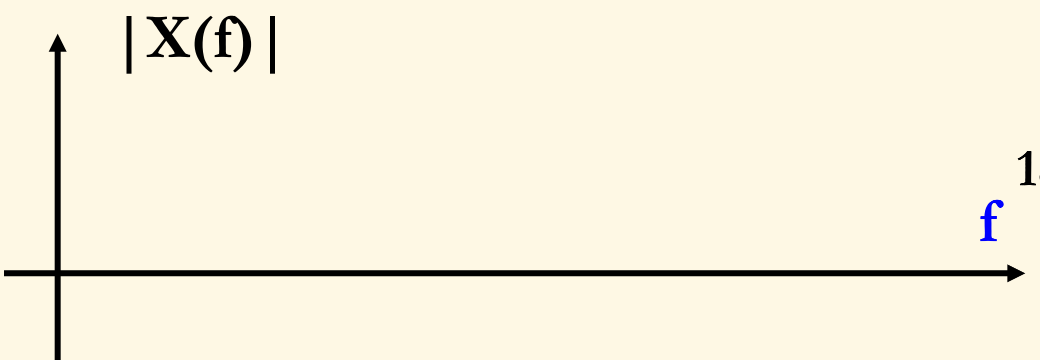
Amplitude Spectrum

Exercise: **What is the spectrum of:**

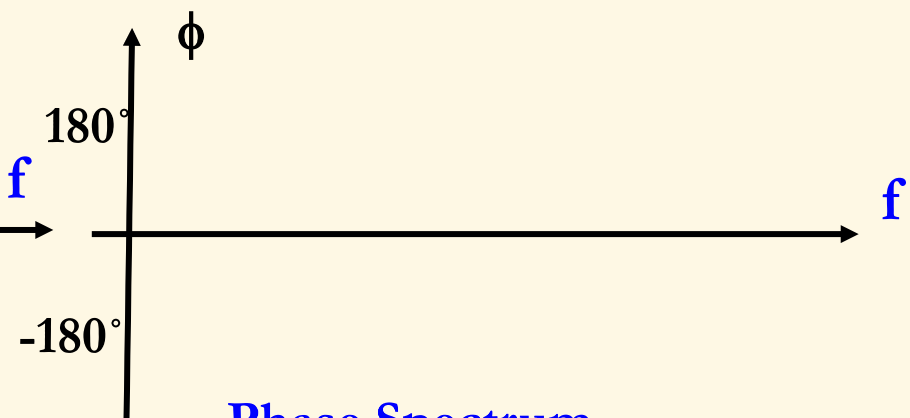
$$x(t) = 10\cos(6\pi\, t) + 7\sin(8\pi\, t) - 3\sin(16\pi\, t)$$

$$x(t) = 10\cos(6\pi\, t) + 7\cos(8\pi\, t - \pi/2) + 3\cos(16\pi\, t + \pi/2)$$

$$x(t) = 5\left[e^{j(\ldots\ldots t)} + e^{-j(\ldots\ldots t)}\right] + 3.5\left[e^{j(\ldots\ldots t)} + e^{-j(\ldots\ldots t)}\right] + 1.5\left[e^{j(\ldots\ldots t)} + e^{-j(\ldots\ldots t)}\right]$$



Amplitude Spectrum

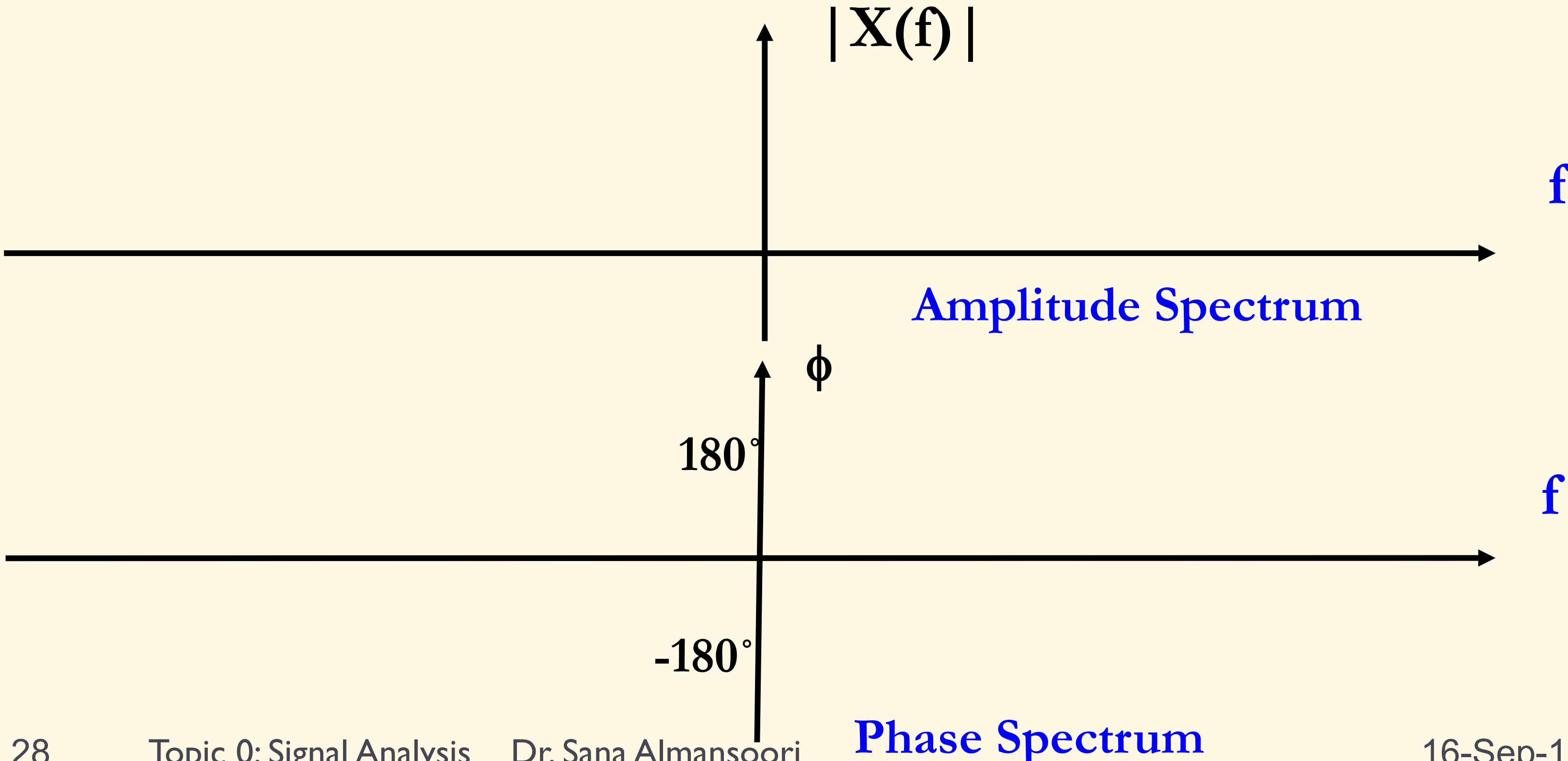


Phase Spectrum

Amplitude Spectrum

Exercise: What is the spectrum of:

$$x(t) = 5\left[e^{j(6\pi t)} + e^{-j(6\pi t)}\right] + 3.5\left[e^{j(8\pi t - \pi/2)} + e^{-j(8\pi t - \pi/2)}\right] + 1.5\left[e^{j(16\pi t + \pi/2)} + e^{-j(16\pi t + \pi/2)}\right]$$



Fourier Analysis

Q: What if a signal is not a sinusoidal function?

According to Fourier, any periodic signal x(t) can be decomposed into a sum (integral) of cosine functions.

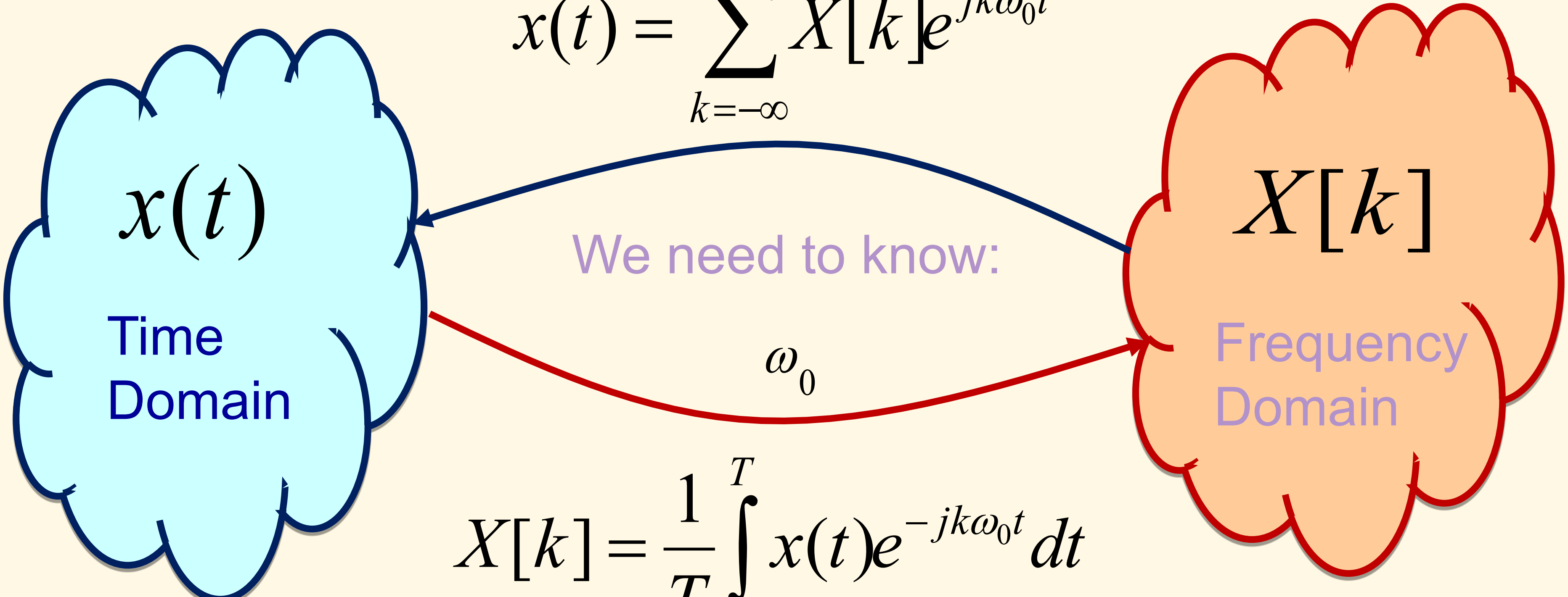
This is called the Fourier Series.

Fourier Series

The Fourier Series Pair representation for a continuous time signal x(t) is:

- This is referred to as the exponential form of the Fourier Series pair.

$$x(t) = \sum_{k=-\infty}^{\infty} X[k] e^{jk\omega_0 t}$$



$$X[k] = \frac{1}{T}\int_{0}^{T} x(t) e^{-jk\omega_0 t}\, dt$$

Fourier Series

Q: Can Fourier Series be expressed as cosines instead of exponentials?

Yes..

$$x(t) = A_0 + \sum_{n=1}^{\infty} A_n \cos(n\omega_0 t) + \sum_{n=1}^{\infty} B_n \sin(n\omega_0 t)$$

OR..

Amplitude

Phase Shift

$$x(t) = C_0 + \sum_{n=1}^{\infty} C_n \cos(n\omega_0 t + \theta_n)$$

Fundamental angular frequency

Fourier Series

where:

$$A_0 = \frac{1}{T_0} \int_{t_1}^{t_1+T_0} x(t)\,dt$$

$$A_n = \frac{2}{T_0} \int_{t_1}^{t_1+T_0} x(t)\cos(n\omega_0\, t)\,dt \qquad B_n = \frac{2}{T_0} \int_{t_1}^{t_1+T_0} x(t)\sin(n\omega_0\, t)\,dt$$

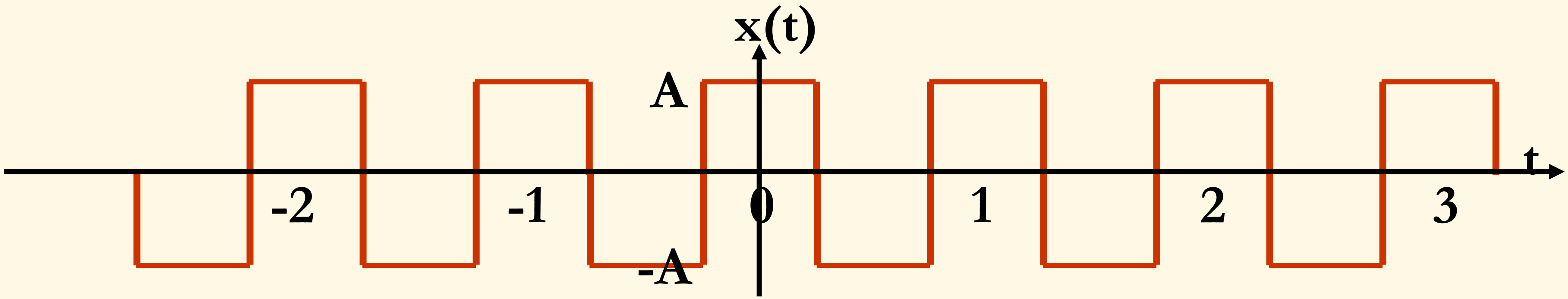
$$C_0 = A_0$$

$$C_n = \sqrt{A_n^2 + B_n^2} \qquad \theta_n = \tan^{-1}\left(\frac{-B_n}{A_n}\right)$$

Fourier Analysis

Q: How do we apply Fourier Analysis?

Consider the following square wave:

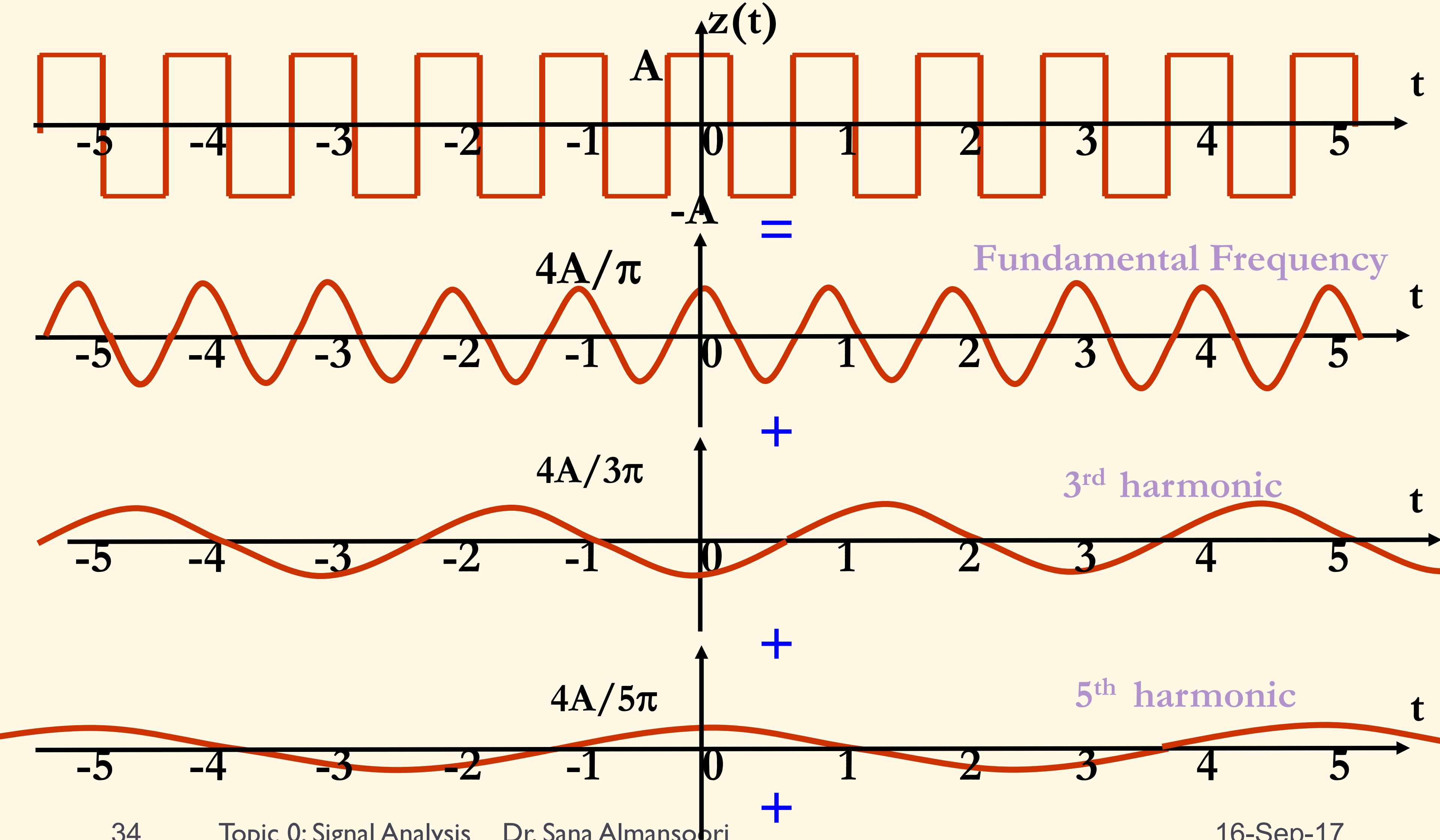


According to Fourier, it is equal to:

$$x(t) = \frac{4A}{\pi}\left(\cos(\omega_0\, t) - \frac{1}{3}\cos(3\omega_0\, t) + \frac{1}{5}\cos(5\omega_0\, t) - .....\right)$$

Q: What does this mean?

Fourier Analysis

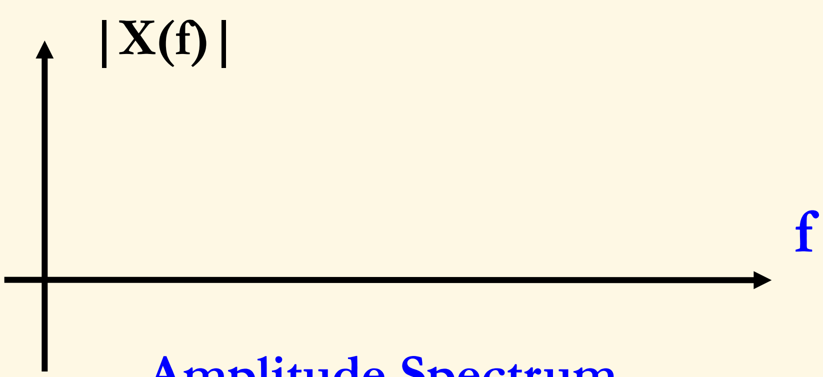


Amplitude and Phase Spectrum

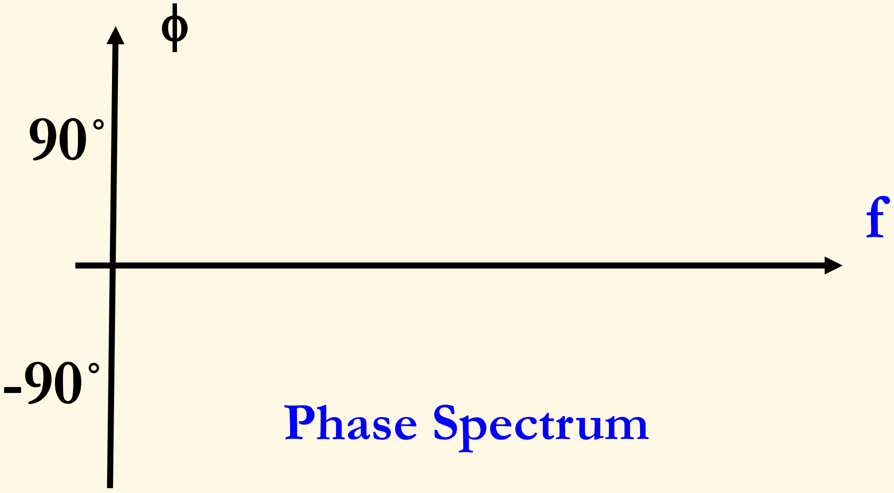
Exercise: What is the Spectrum of the square wave?

$$x(t) = \frac{4A}{\pi}\left(\cos(\omega_0\, t) - \frac{1}{3}\cos(3\omega_0\, t) + \frac{1}{5}\cos(5\omega_0\, t) - \frac{1}{7}\cos(7\omega_0\, t).....\right)$$

f_0 **Is the fundamental frequency of the square wave**



Amplitude Spectrum

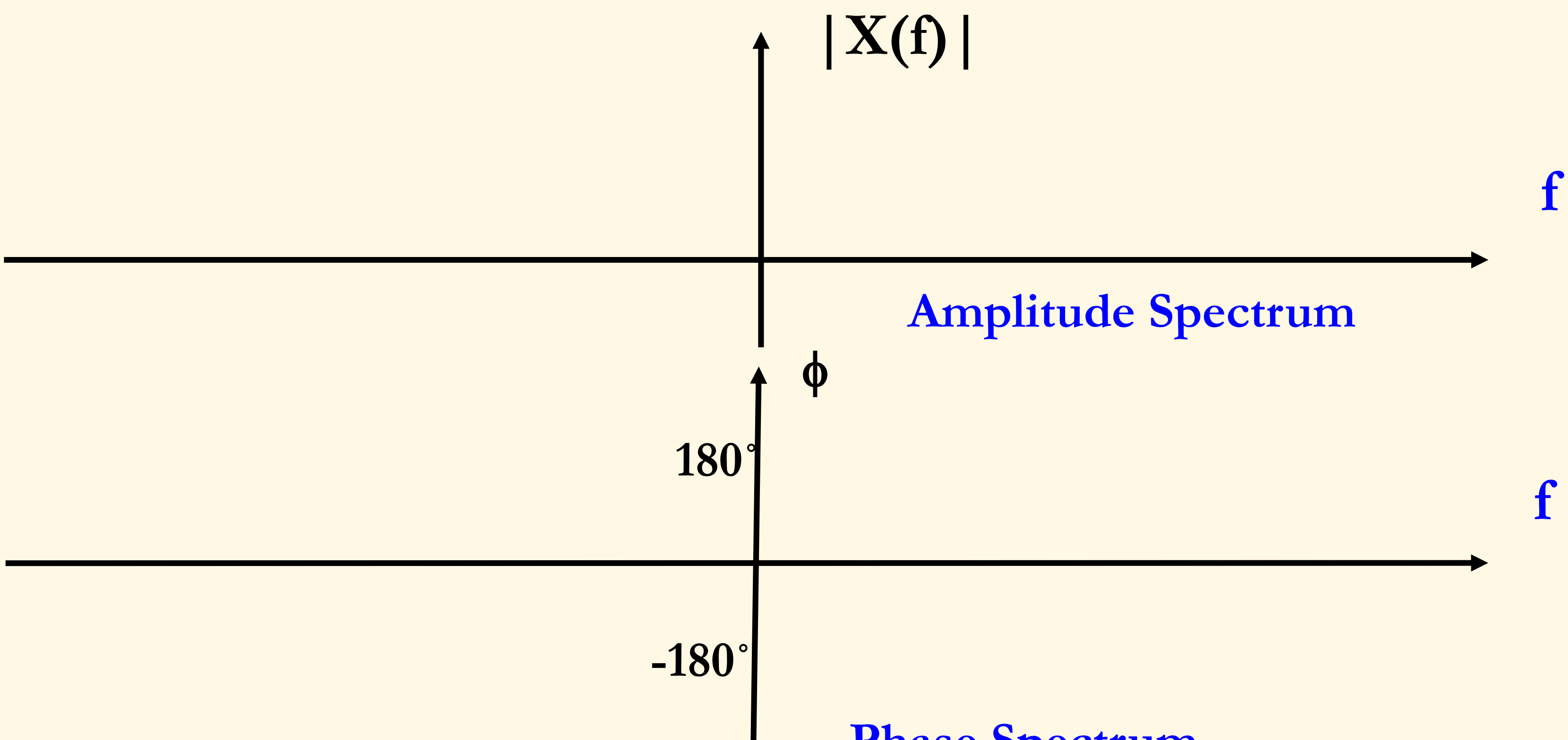


Phase Spectrum

Amplitude and Phase Spectrum

Exercise: What is the Spectrum of the square wave?

$$x(t) = \frac{4A}{\pi}\left(\cos(\omega_0\, t) - \frac{1}{3}\cos(3\omega_0\, t) + \frac{1}{5}\cos(5\omega_0\, t) - \frac{1}{7}\cos(7\omega_0\, t).....\right)$$



Amplitude and Phase Spectrum

Q: What is the Amplitude Spectrum of a signal?

The Amplitude spectrum shows the relative amplitude of the different frequency (cosine) components contained within a signal (function).

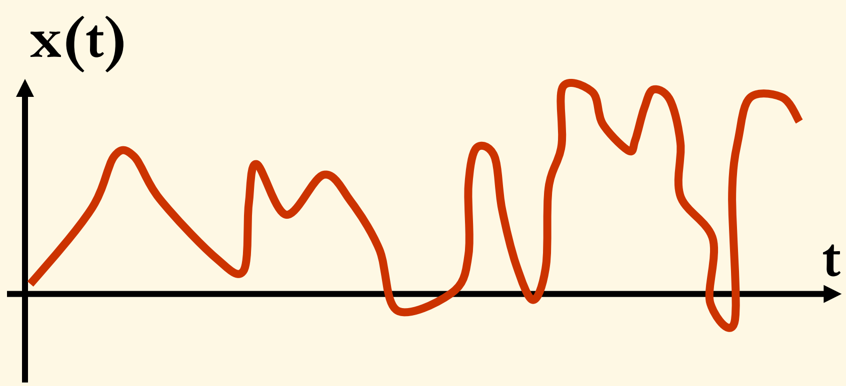
Q: What is the Phase Spectrum of a signal?

The Phase spectrum shows the Phase of the different frequency (cosine) components contained within a signal (function).

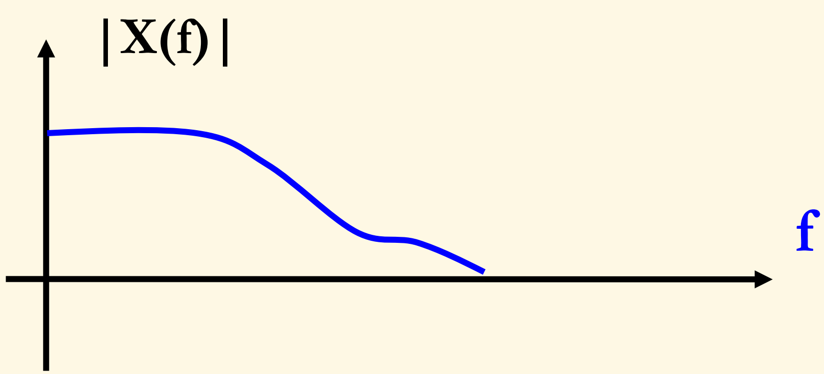
Fourier Analysis

Q: Can we use Fourier Analysis if x(t) is <u>not</u> periodic?

Yes, and the spectrum will be continuous function of f.



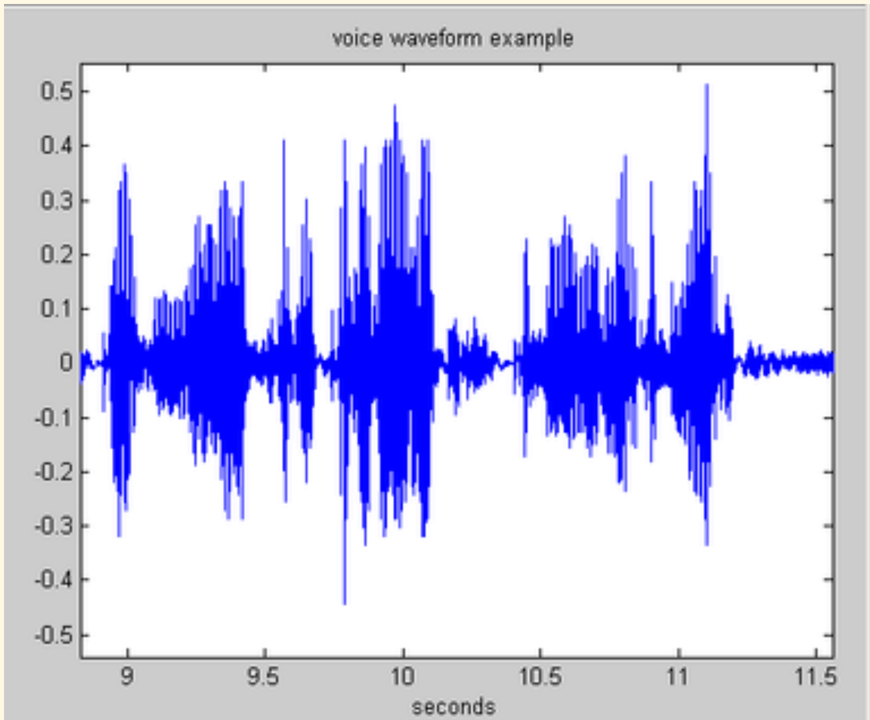
Time Domain signal

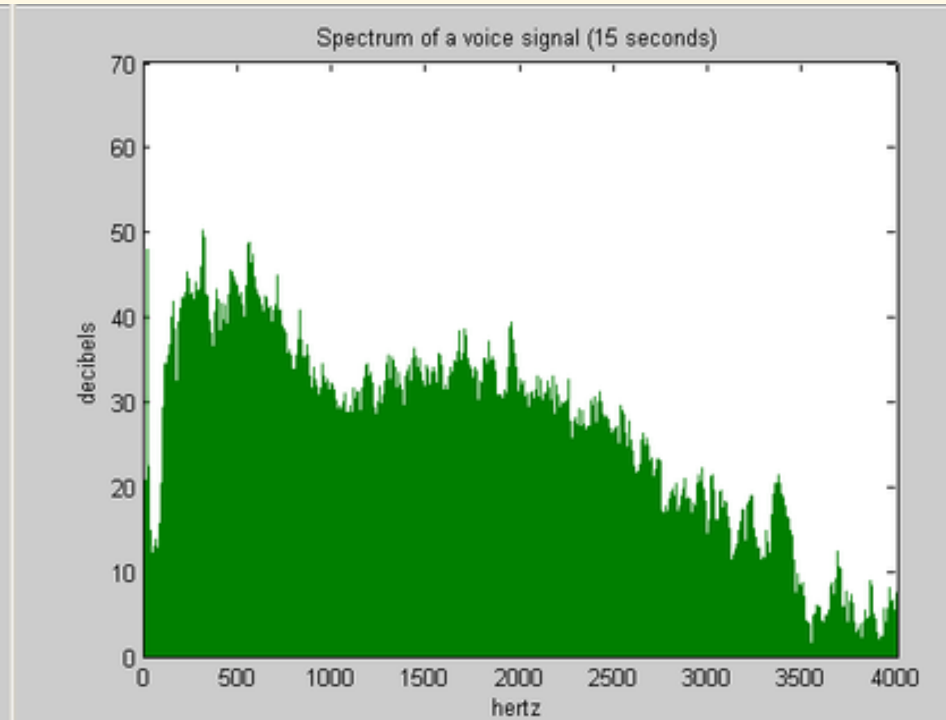


Amplitude Spectrum

Voice Signal and Spectrum

Wikapedia





Time Domain signal

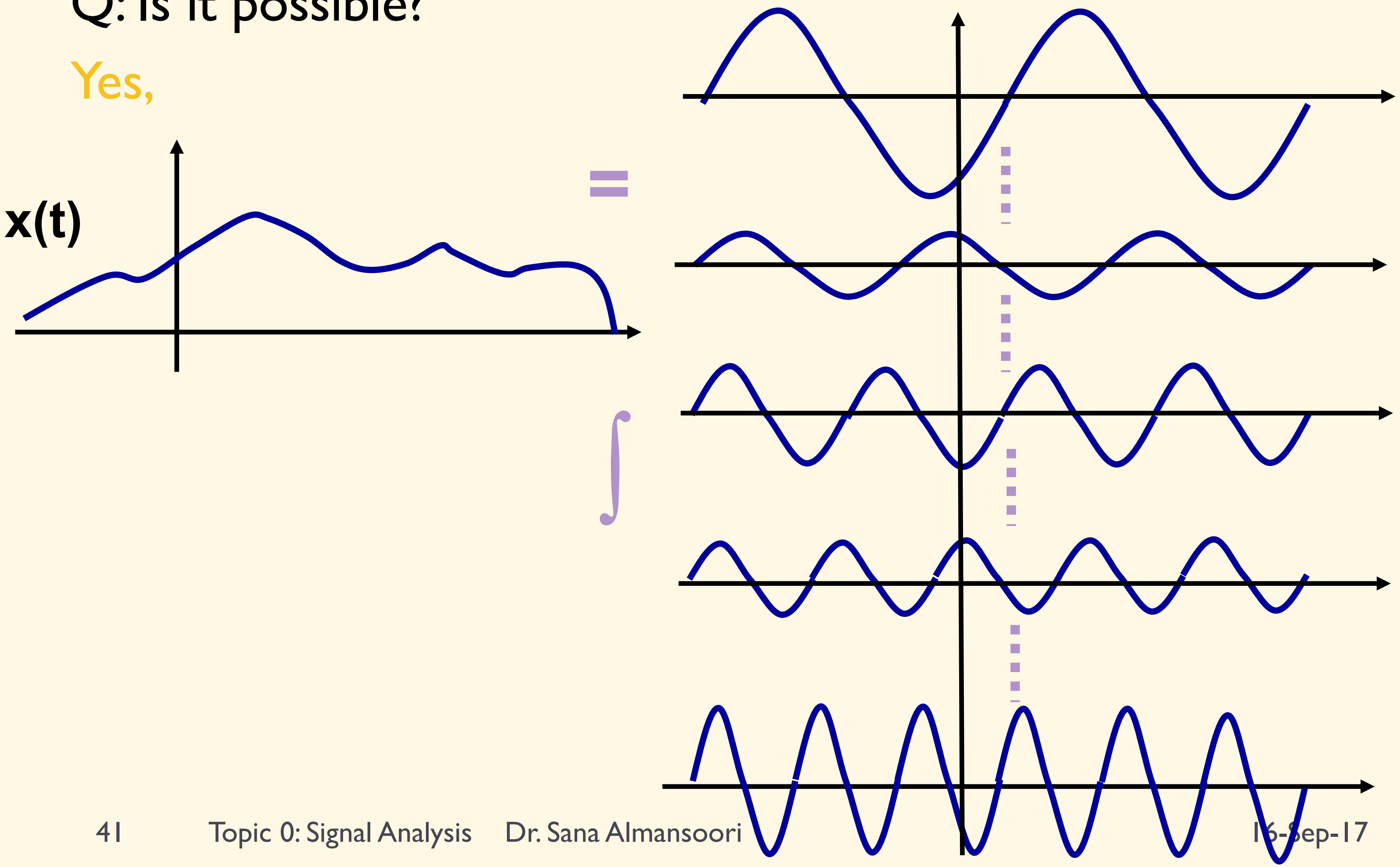
Amplitude Spectrum

Fourier Transform

Fourier Representation of Non Periodic Signals

Q: Is it possible?

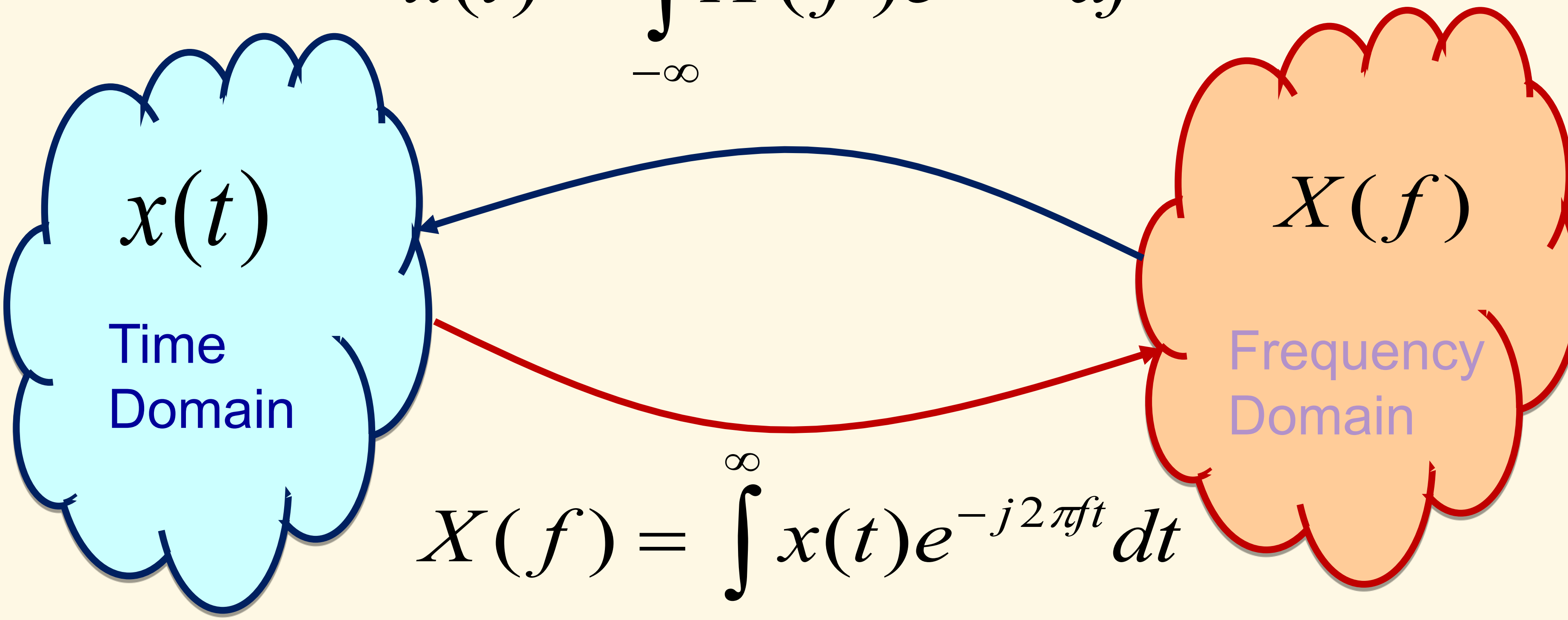
Yes,



Fourier Transform

The Fourier Transform Pair representation for a continuous time signal x(t) is:

$$x(t) = \int_{-\infty}^{\infty} X(f) e^{j2\pi ft} df$$



$$X(f) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi ft} dt$$

Fourier Transform

Exercise: Find the Fourier representation of the rectangular pulse defined as:

Answer:

$$x(t) = \begin{cases} 1, & -T_0 < t < T_0 \\ 0, & |t| > T_0 \end{cases}$$

$$X(f) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi f t} dt = \int_{-T_0}^{T_0} e^{-j2\pi f t} dt = \left. \frac{e^{-j2\pi f t}}{-j2\pi f} \right|_{-T_0}^{T_0}$$

f = 0

$$X(f) = \int_{-T_0}^{T_0} 1 dt$$

$$X(f) = t \Big|_{-T_0}^{T_0} = 2T_0$$

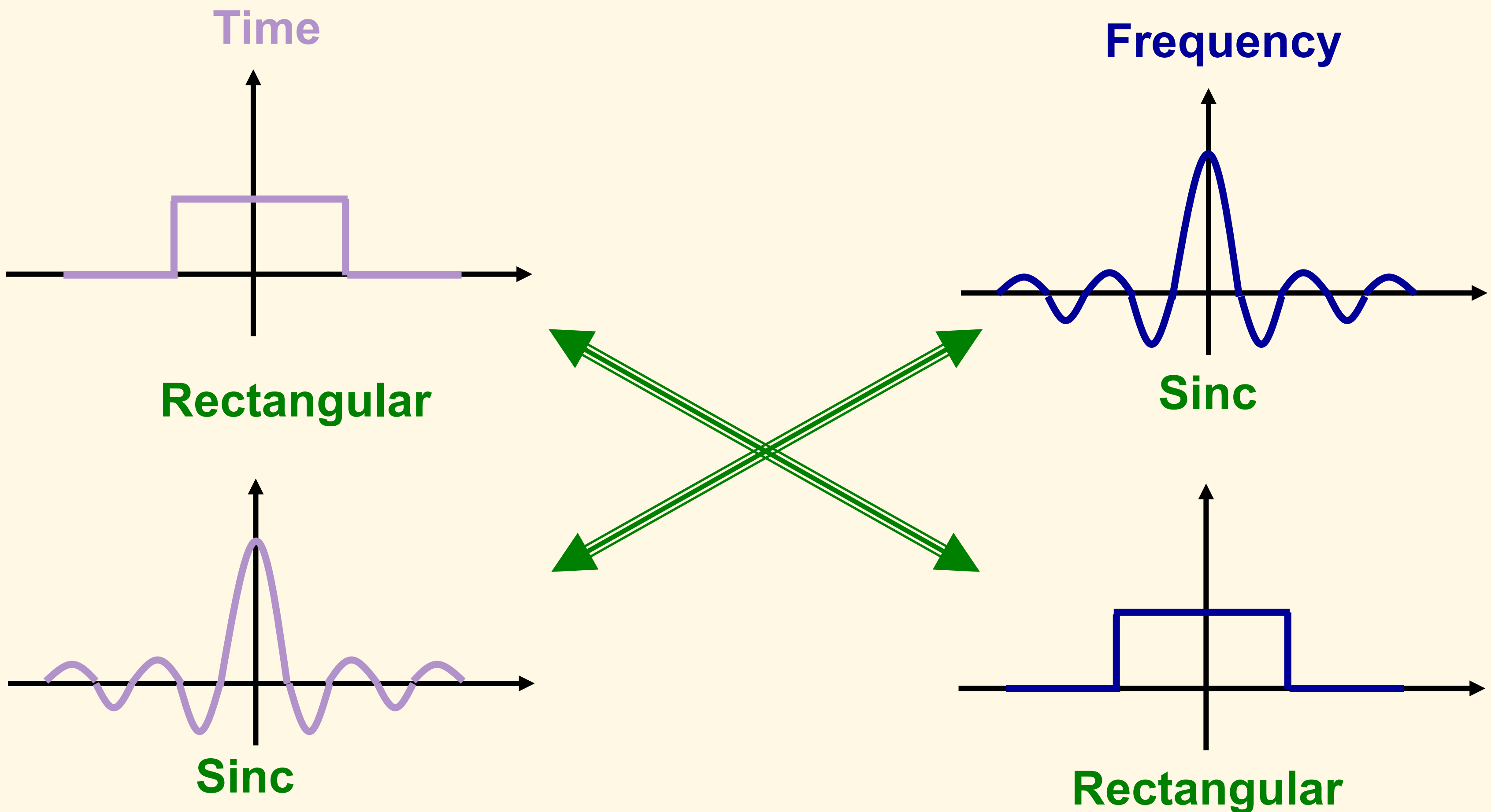
f ≠ 0

$$X(f) = \frac{e^{-j2\pi f T_0} - e^{j2\pi f T_0}}{-j2\pi f} = \frac{e^{j2\pi f T_0} - e^{-j2\pi f T_0}}{j2\pi f}$$

$$X(f) = \frac{1}{\pi f} \sin(2\pi f T_0) = \frac{\sin(2\pi f T_0)}{\pi f} = 2T_0 \sin c(2 f T_0)$$

Duality

Q: What is the duality property of the FT?



Fourier Transform

Exercise: Find the Fourier Transform of unit impulse δ(t). $x(t) = \delta(t)$

Answer:

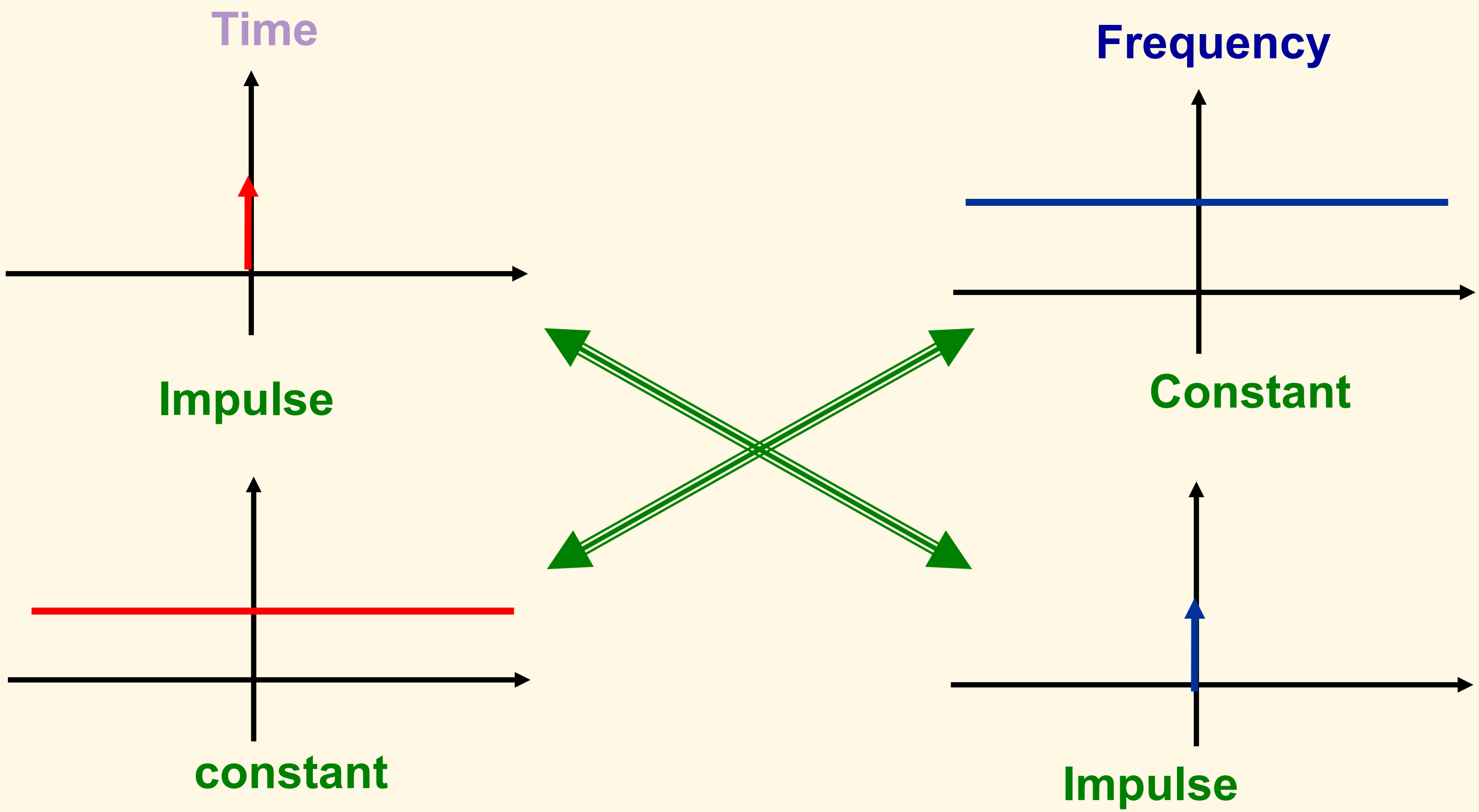
$$X(f) = \int_{-\infty}^{\infty} \delta(t) e^{-j2\pi ft} dt$$

$$X(f) = \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$x(t) = \delta(t) \quad \xleftrightarrow{\text{FT}} \quad X(f) = 1$$

Duality

Q: What is the duality property of the FT?



Fourier Transform

Exercise: Find the Fourier Transform of a shifted unit impulse $x(t) = \delta(t - t_0)$

Answer:

$$X(f) = \int_{-\infty}^{\infty} \delta(t - t_0) e^{-j2\pi ft} dt$$

$$X(f) = \int_{-\infty}^{\infty} \delta(t - t_0) e^{-j2\pi ft_0} dt$$

$$X(f) = e^{-j2\pi ft_0} \int_{-\infty}^{\infty} \delta(t - t_0) dt = e^{-j2\pi ft_0}$$

$$x(t) = \delta(t - t_0) \xleftrightarrow{\text{FT}} X(\omega) = e^{-j2\pi ft_0}$$

Fourier Transform

Exercise: Find the Inverse Fourier Transform of a shifted impulse spectrum $X(f) = \delta(f - f_0)$

Answer:

$$x(t) = \int_{-\infty}^{\infty} \delta(f - f_0) e^{j2\pi ft} df$$

$$x(t) = \int_{-\infty}^{\infty} \delta(f - f_0) e^{j2\pi f_0 t} df$$

$$x(t) = e^{j2\pi f_0 t} \int_{-\infty}^{\infty} \delta(f - f_0) df = e^{j2\pi f_0 t}$$

$$x(t) = e^{j2\pi f_0 t} \quad \overset{FT}{\longleftrightarrow} \quad X(f) = \delta(f - f_0)$$

Fourier Transform

Exercise: Find the Fourier Transform of a cosine function $x(t) = \cos(2\pi f_0 t)$

Answer:

$$x(t) = \cos(2\pi f_0 t) = \frac{e^{j2\pi f_0 t} + e^{-j2\pi f_0 t}}{2} = \frac{1}{2}\left(e^{j2\pi f_0 t} + e^{-j2\pi f_0 t}\right)$$

Using the previous slide:

$$X(f) = \frac{1}{2}\left[\delta(f - f_0) + \delta(f + f_0)\right]$$

Draw the spectrum of the cosine function.

Fourier Transform

Exercise: Find the Fourier Transform of a sine function

$$x(t) = \sin(2\pi f_0 t)$$

Answer:

$$x(t) = \sin(2\pi f_0 t) = \frac{e^{j2\pi f_0 t} - e^{-j2\pi f_0 t}}{2j} = \frac{1}{2j}\left(e^{j2\pi f_0 t} - e^{-j2\pi f_0 t}\right)$$

Using the previous slide:

$$X(f) = \frac{1}{2j}\left[\delta(f - f_0) - \delta(f + f_0)\right]$$

Draw the spectrum of the sine function.

Properties of Fourier Transform

Convolution

Q:When we convolve signals in the time domain, what happens in the frequency domain?

$$y(t) = h(t) * x(t) = \int_{-\infty}^{\infty} h(\tau) x(t-\tau) d\tau$$

$$y(t) = \int_{-\infty}^{\infty} h(\tau) \left[\int_{-\infty}^{\infty} X(f) e^{j2\pi f(t-\tau)} df \right] d\tau$$

$$y(t) = \int_{-\infty}^{\infty} H(f) X(f) e^{+j2\pi ft} df = \int_{-\infty}^{\infty} Y(f) e^{+j2\pi ft} df$$

$$\therefore y(t) = x(t) * h(t) \Rightarrow Y(f) = X(f) H(f)$$

Convolution

Exercise: Find y(t)=x(t)*h(t) if:

$$x(t) = \frac{1}{\pi t}\sin(\pi t) = \sin c(t) \qquad h(t) = \frac{1}{\pi t}\sin(2\pi t) = 2\sin c(2t)$$

Answer:

We know that:

$$X(f) = \begin{cases} 1, & |f| < 1/2 \\ 0, & |f| > 1/2 \end{cases} \qquad H(f) = \begin{cases} 1, & |f| < 1 \\ 0, & |f| > 1 \end{cases}$$

Using the convolution property:

$$Y(f) = X(f)H(f) = \begin{cases} 1, & |f| < 1/2 \\ 0, & |f| > 1/2 \end{cases}$$

Doing the Inverse FT :

$$\Rightarrow y(t) = \frac{1}{\pi t}\sin(\pi t)$$

Convolution

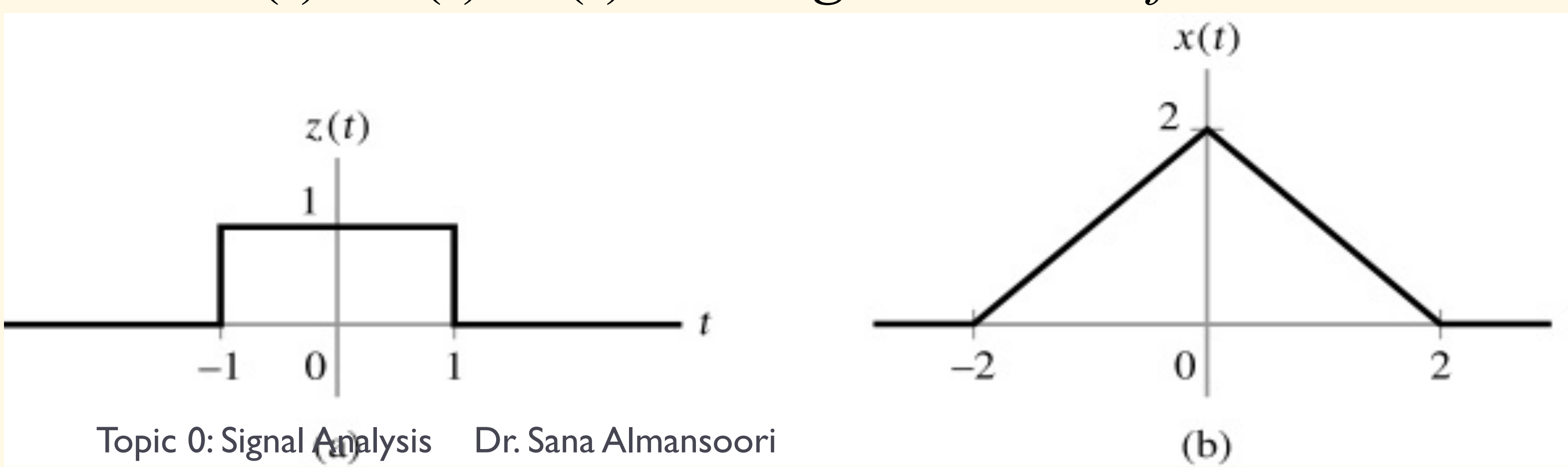
Exercise: Find x(t) if: $X(f) = 4\sin c^2(2f)$

Answer:

$$X(f) = 2\sin c(2f) \bullet 2\sin c(2f) = Z(f) \bullet Z(f)$$

$$Z(f) = 2\sin c(2f) \qquad z(t) = \begin{cases} 1, & |t| < 1 \\ 0, & |t| > 1 \end{cases}$$

$$\Rightarrow x(t) = z(t) * z(t) = triangular \quad waveform$$



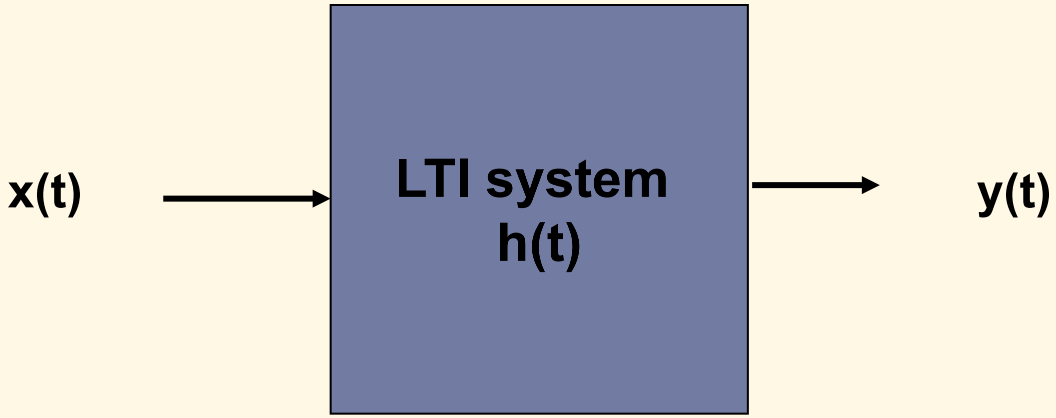
Convolution

Exercise: Find the output of the following system y(t) if

$$x(t) = \cos(2\pi t) + 2\cos(6\pi t)$$

and

$$h(t) = 2\sin c(4t)$$

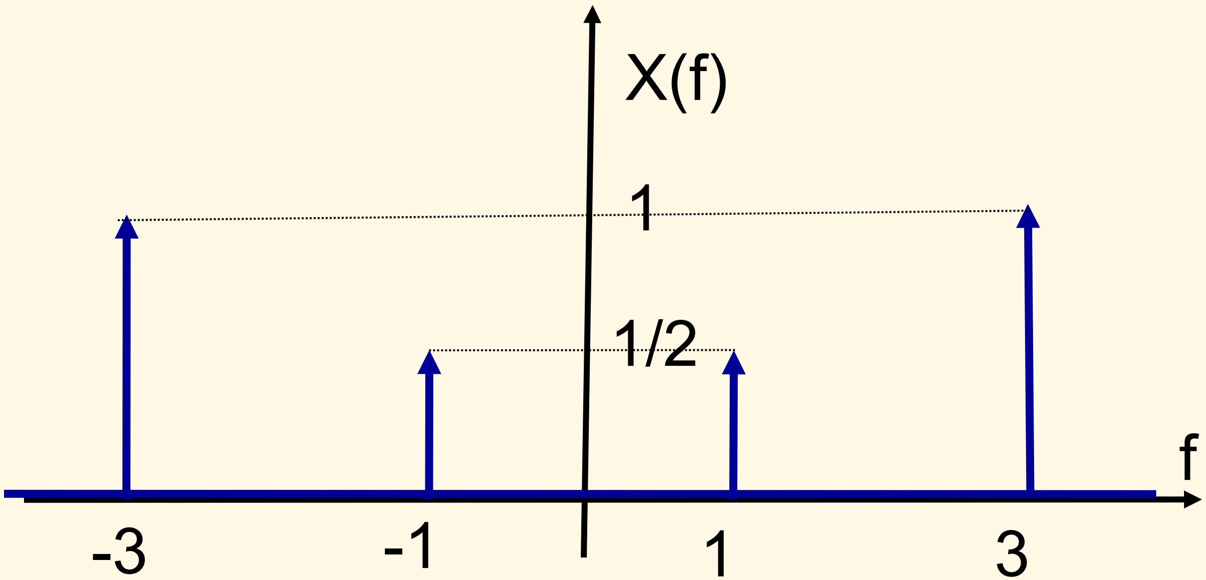


Convolution

Answer:

$$x(t) = \cos(2\pi t) + 2\cos(6\pi t)$$

$$X(f) = \frac{1}{2}\delta(f-1) + \frac{1}{2}\delta(f+1) + \delta(f-3) + \delta(f+3)$$

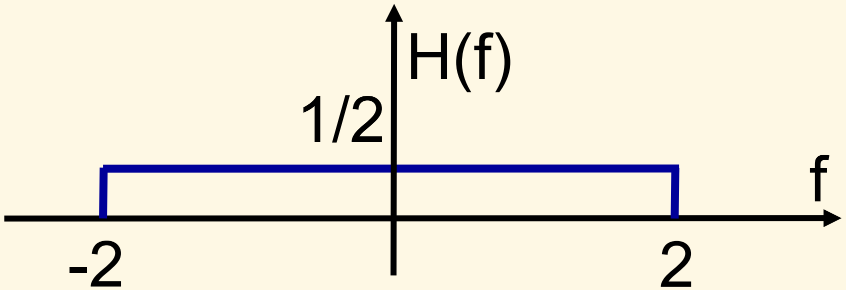


Convolution

Answer:

$$h(t) = 2\sin c(4t)$$

$$H(f) = \begin{cases} 1/2 & |f| < 2 \\ 0 & |f| > 2 \end{cases}$$

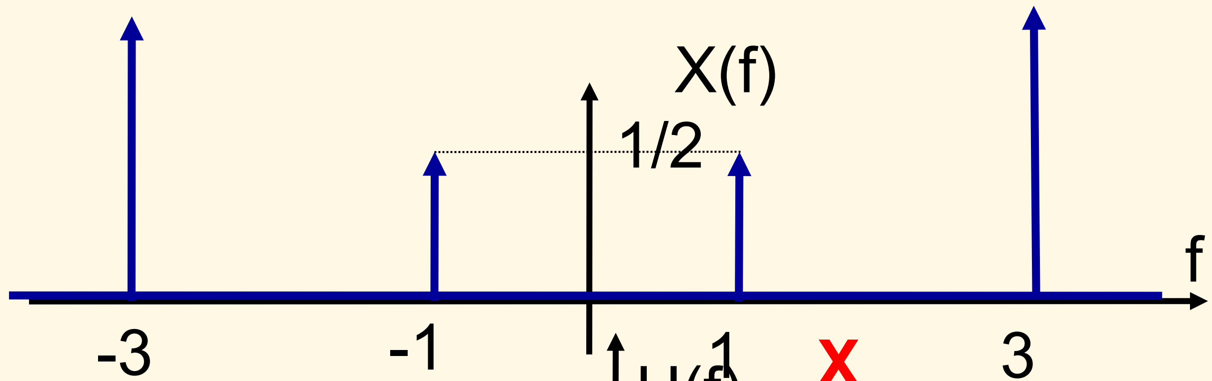


Convolution

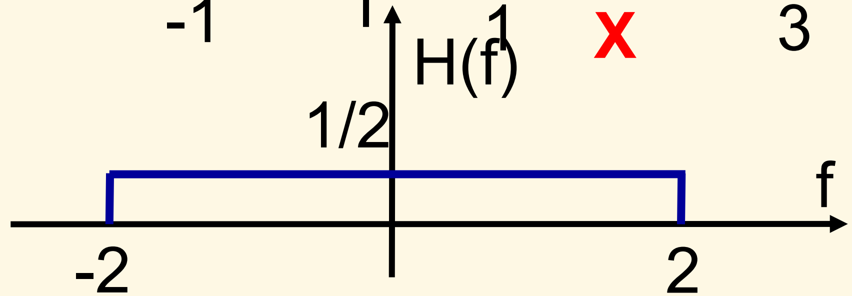
Answer:

$$Y(f) = X(f)H(f)$$

$$Y(f) = \frac{1}{4}\delta(f-1) + \frac{1}{4}\delta(f+1)$$



x



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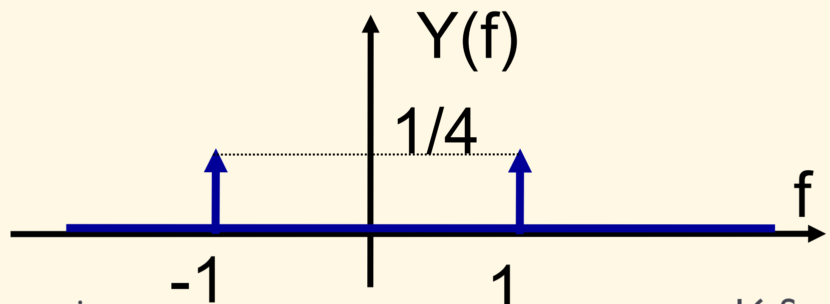


To get the output y(t):

$$y(t) = IFT\{Y(f)\}$$

$$\Rightarrow y(t) = \frac{1}{2}\cos(2\pi t)$$

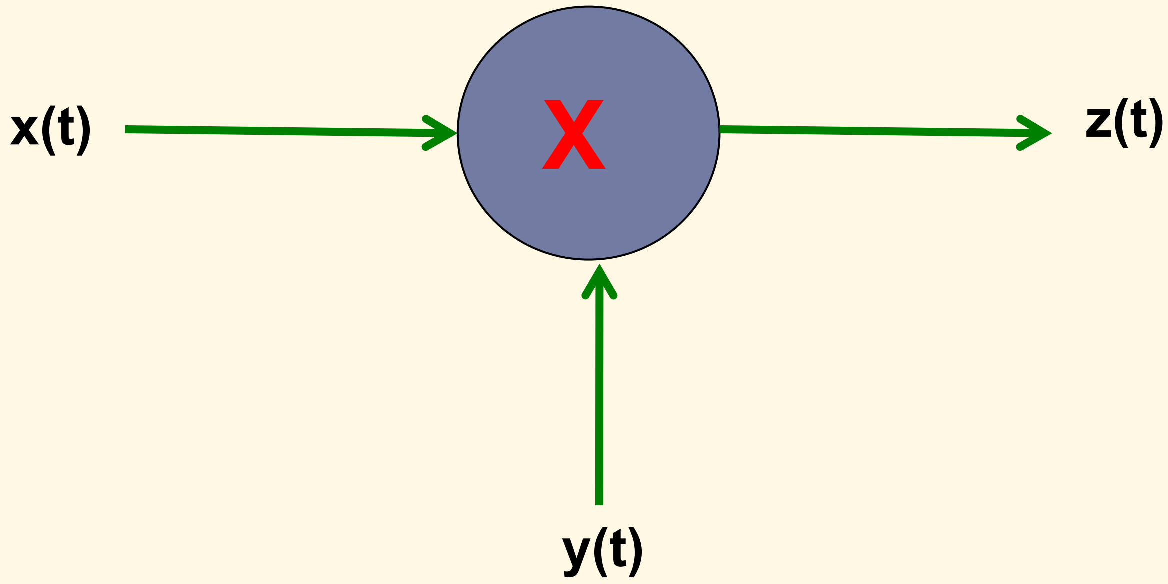
Multiplication

Q: When we multiply two signals in the time domain, what happens in the frequency domain?

$$If \quad y(t) = x(t)h(t) \quad then \quad Y(f) = X(f) * H(f)$$

Multiplication

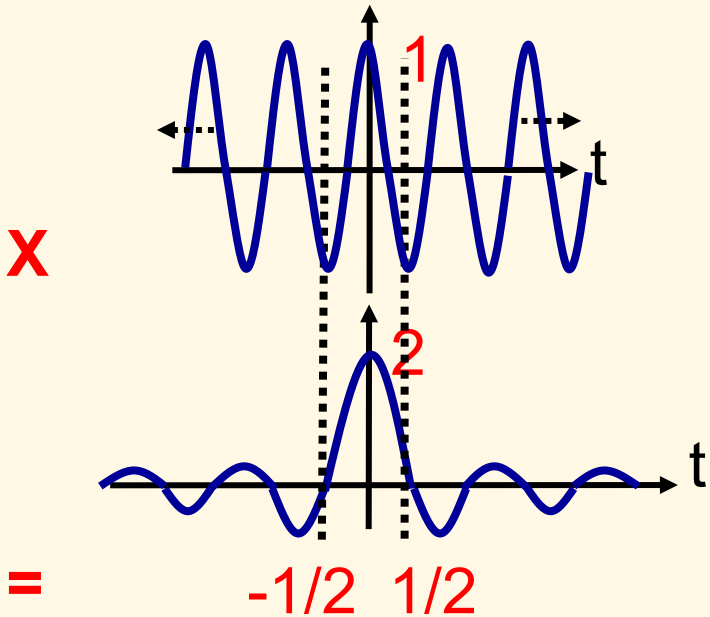
Exercise: Find z(t) where $x(t) = 2\sin c(2t)$ and $y(t) = \cos(2\pi t)$



Multiplication

Answer:Let us first draw the functions

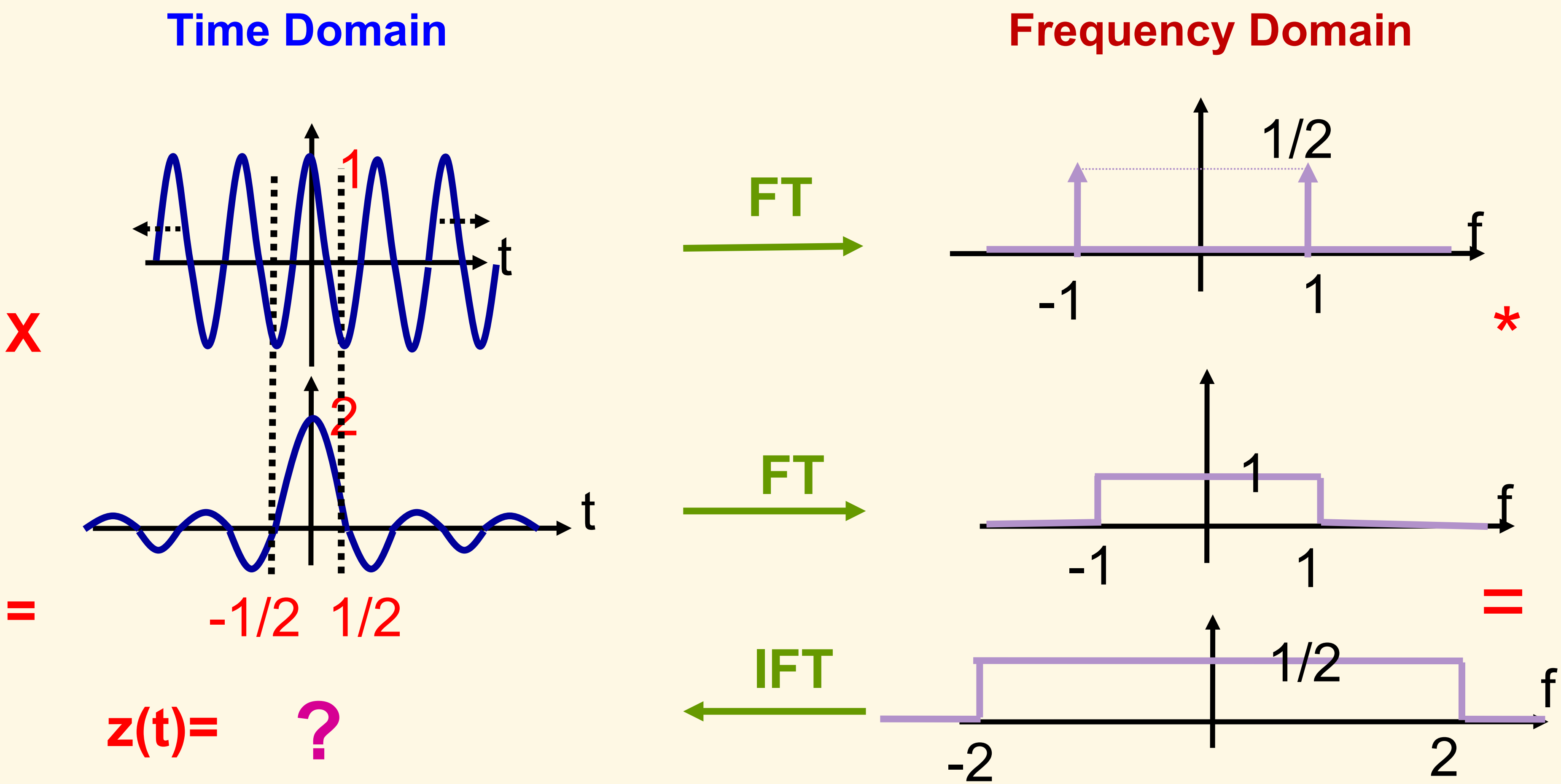
Time Domain



z(t)= ?

Multiplication

Answer: Let us first draw the functions



Cosine Modulation

Q: When we multiply a signal x(t) in the time domain with a cosine, what happens in the frequency domain?

If

$$y(t) = \cos(2\pi f_0 t) \bullet x(t)$$

Then

$$Y(f) = \frac{1}{2}\left[X(f - f_0) + X(f + f_0)\right]$$

Q: Can you verify this in the previous example?

Cosine Modulation

Proof:

$$y(t) = \cos(2\pi f_0 t)x(t) = \frac{1}{2}\left(e^{-j2\pi f_0 t} + e^{+j2\pi f_0 t}\right)x(t) = \frac{1}{2}e^{-j2\pi f_0 t}x(t) + \frac{1}{2}e^{+j2\pi f_0 t}x(t)$$

Using the frequency shift property:

If,

$$x(t) \xleftrightarrow{FT} X(f)$$

Then,

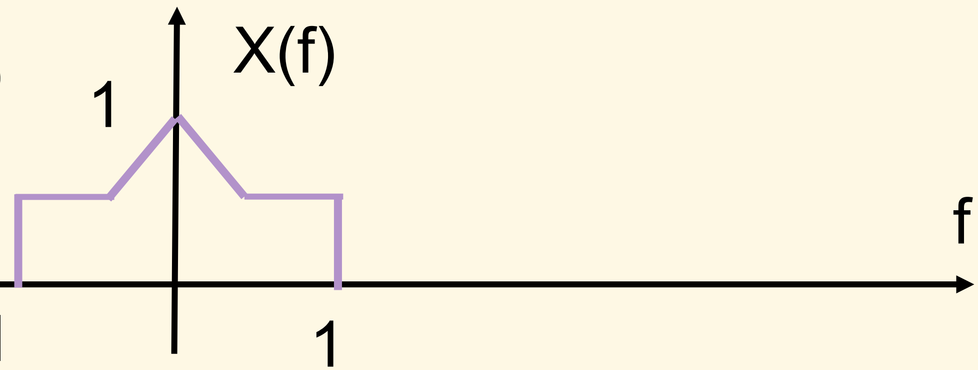
$$Y(f) = \frac{1}{2}X(f - f_0) + \frac{1}{2}X(f + f_0) = \frac{1}{2}\left(X(f - f_0) + X(f + f_0)\right)$$

This is called the cosine modulation property.

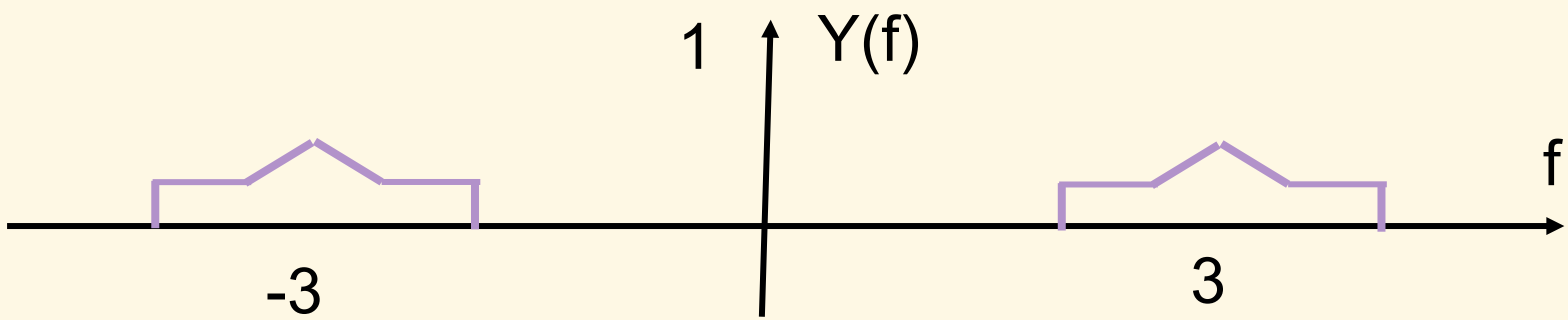
Cosine Modulation

Exercise: If x(t) has the spectrum shown in the figure below, find (by plotting) Y(f) given:

$$y(t) = \cos(6\pi t) \bullet x(t)$$



Answer: Since f_0=3, then Y(f) =



Bandwidth

Bandwidth

Q: What is the Bandwidth of a communication *system*?

It is the range of frequencies it can transmit.

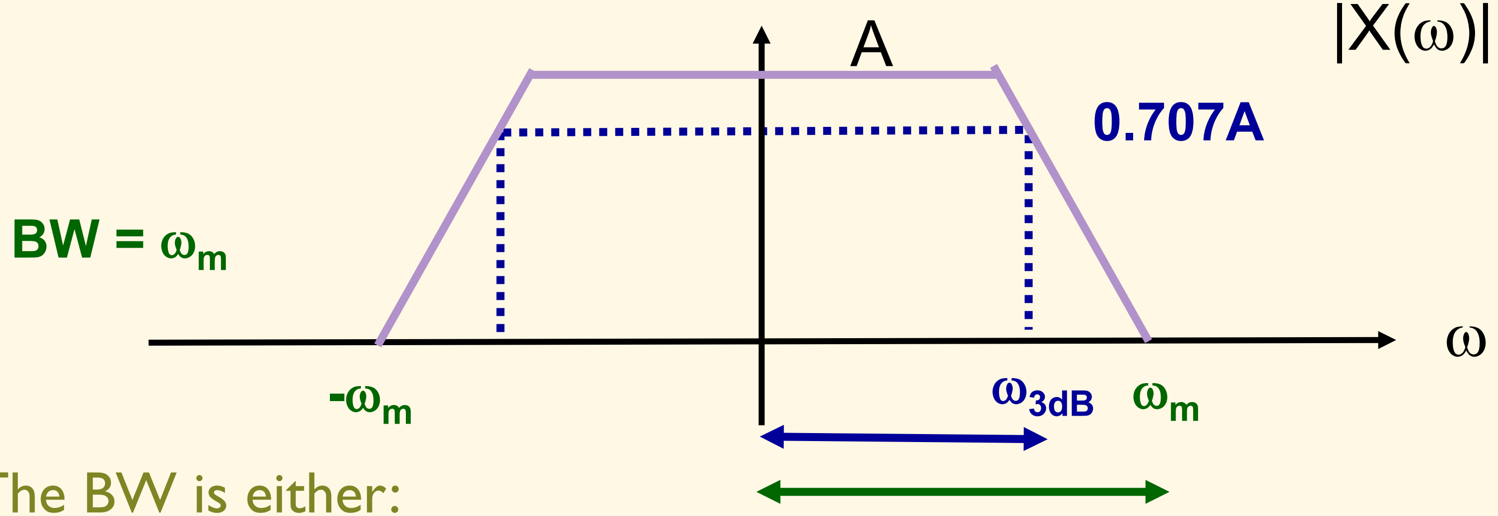
Q: What is the Bandwidth of a *Signal*?

It is the range of frequencies the are contained in a signal.

Bandwidth

Q: What is the BW of a signal or system with finite Maximum frequency?

For a signal with a maximum frequency in the Fourier Presentation,



The BW is either:

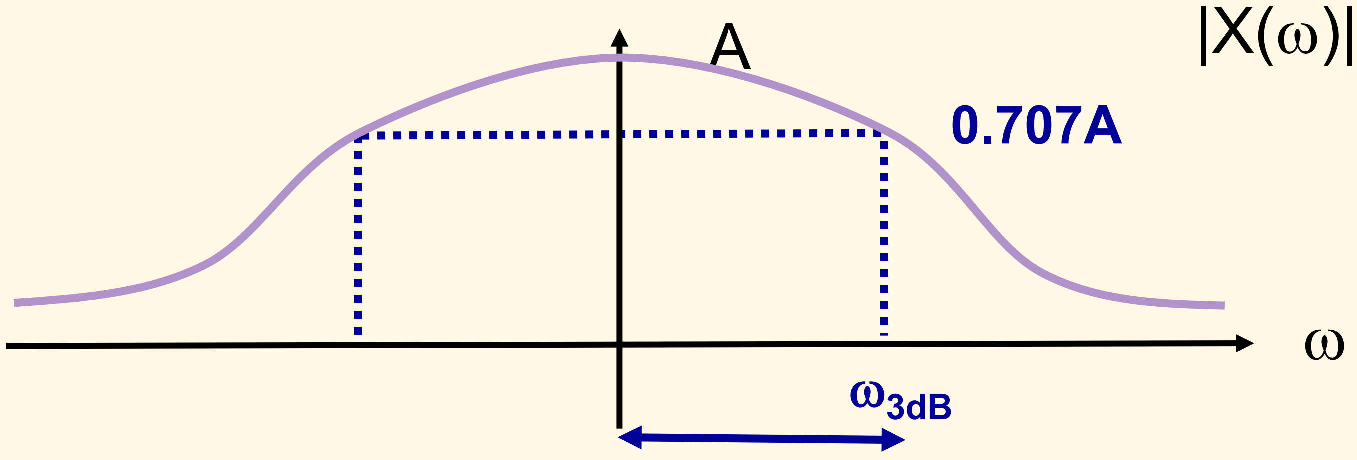
1. the maximum frequency ω_m
2. the frequency at which the power is ½ of the maximum. That is $1/\sqrt{2}$ the amplitude. This is referred to as the 3dB BW.

Bandwidth

Q: What is the BW of a signal or system with infinite Maximum frequency

It is the 3dB BW

3dB BW = ω_{3dB}

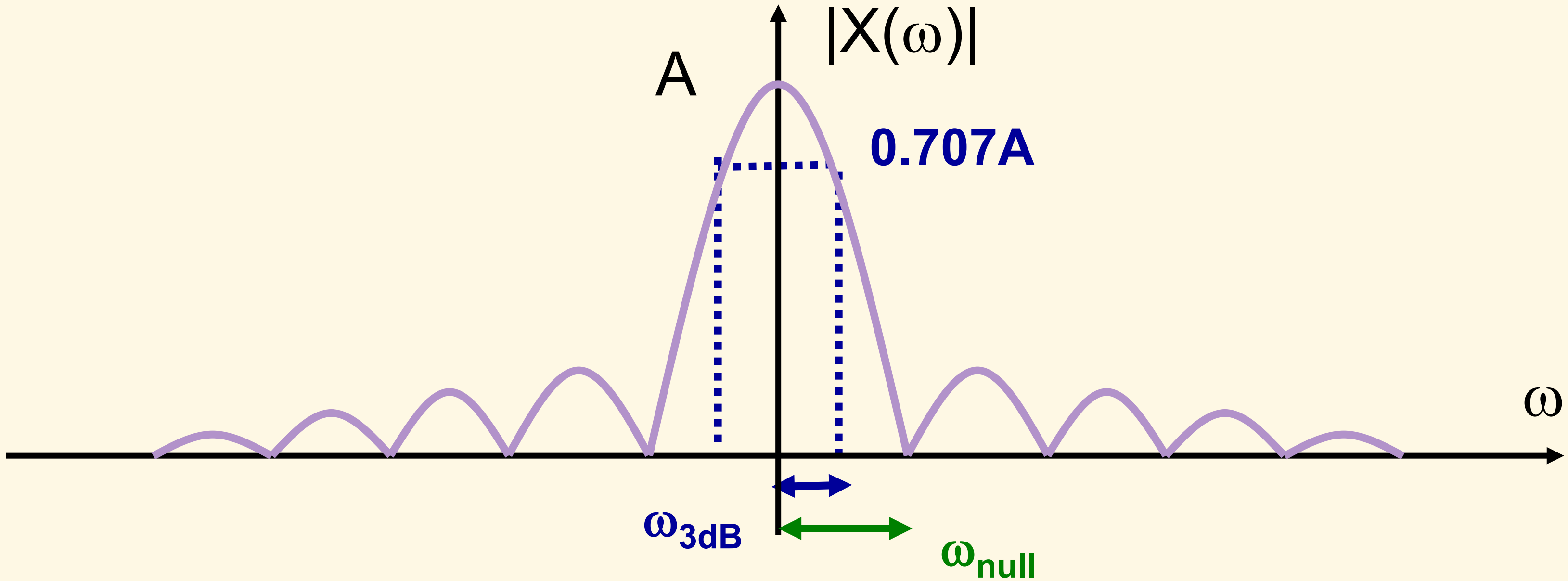


Bandwidth

Q: What is the BW of a sinc function?

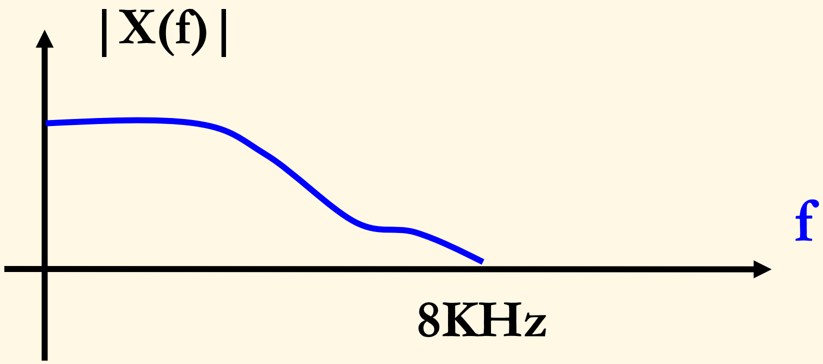
There are several approximations for the BW:

1. the range of frequencies from DC to the first null.
2. the range of frequencies from the DC to the frequency which has ½ of the power at the origin. 1/√2 the amplitude. This is referred to as the 3dB BW.

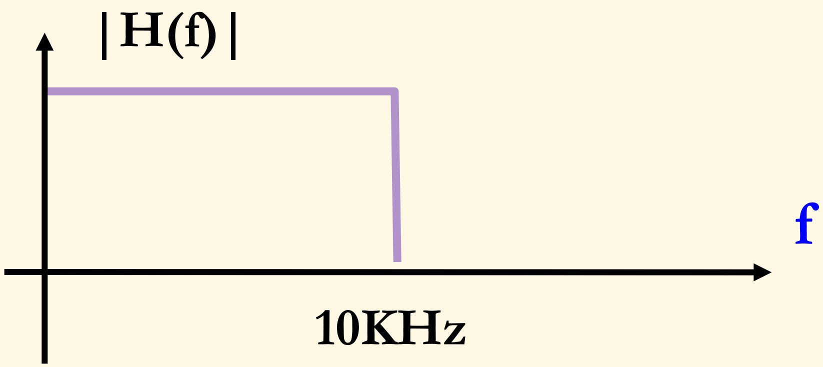


Bandwidth

Example: Is it possible to transmit signal x(t) through system h(t)?



Amplitude Spectrum



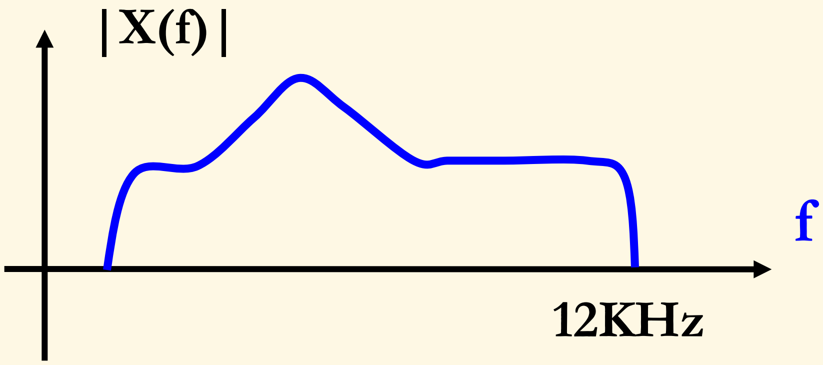
Amplitude Spectrum

Signal BW=………

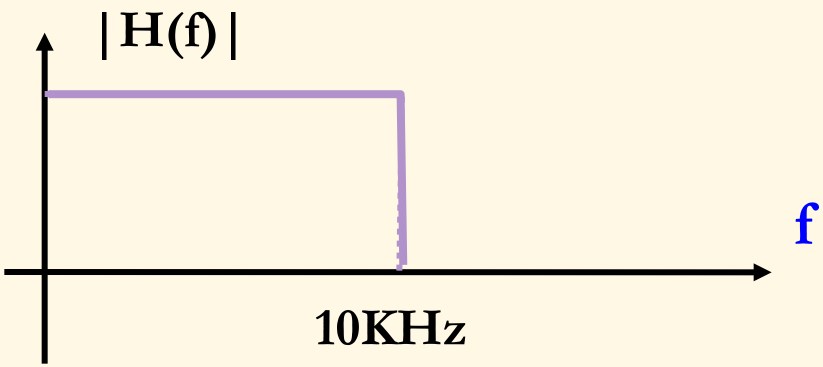
System BW=…….

Bandwidth

Example: Is it possible to transmit signal x(t) through system h(t)?



Amplitude Spectrum



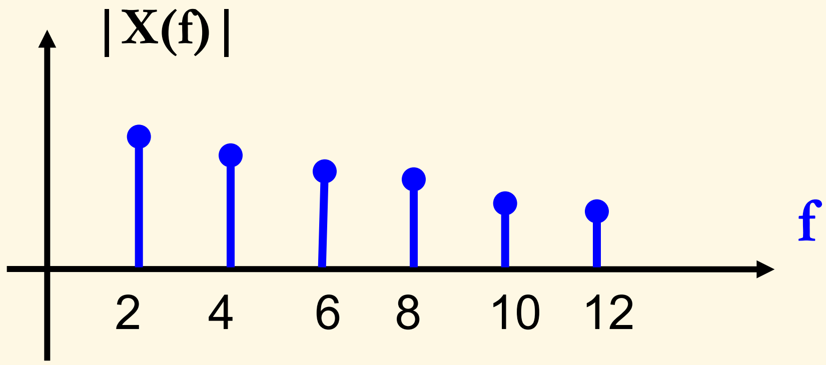
Amplitude Spectrum

Signal BW=………

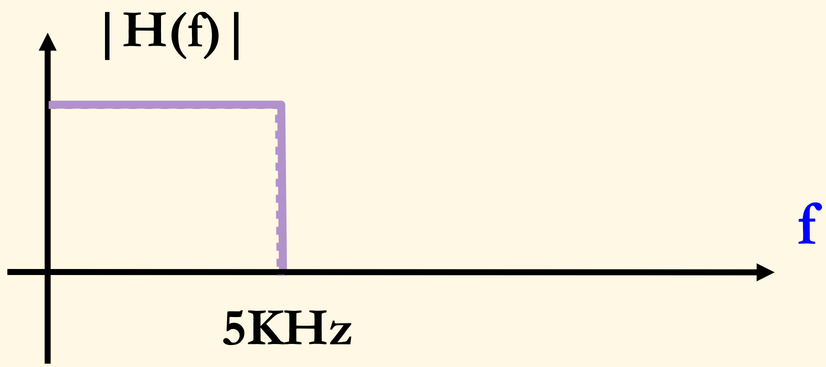
System BW=……..

Bandwidth

Example: Is it possible to transmit signal x(t) through system h(t)?



Amplitude Spectrum



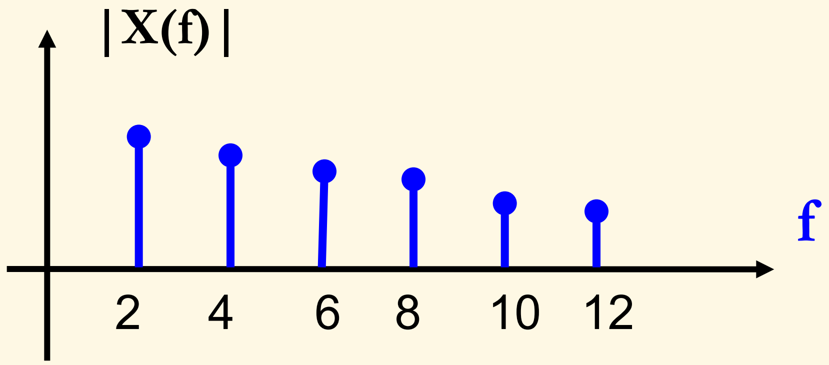
Amplitude Spectrum

Signal BW=………

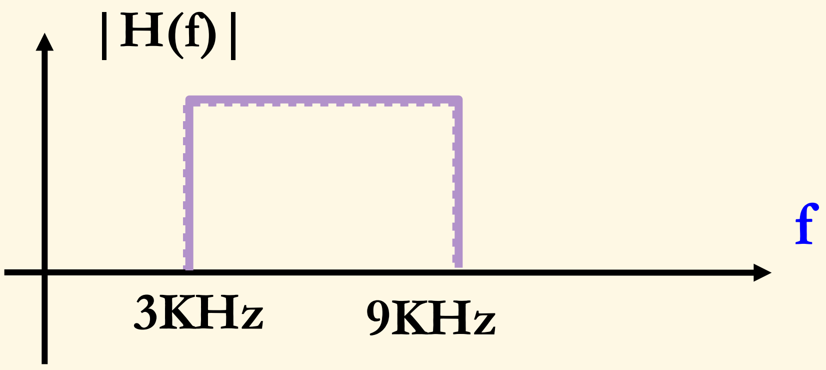
System BW=……..

Bandwidth

Example: Is it possible to transmit signal x(t) through system h(t)?



Amplitude Spectrum



Amplitude Spectrum

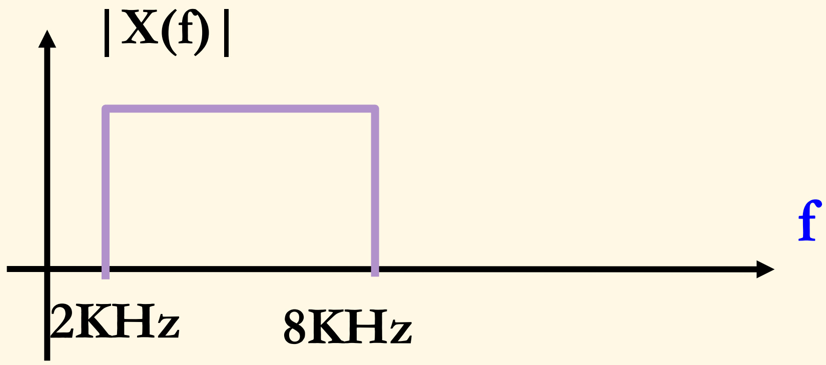
Signal BW=………

System BW=$f_1 - f_2$=…….

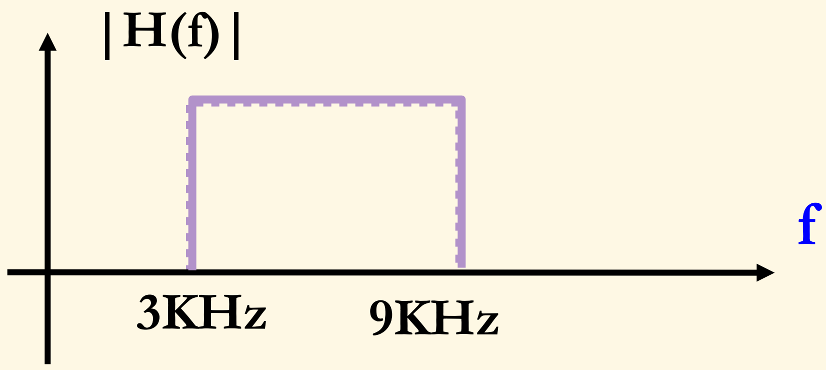
f_c = ……….

Bandwidth

Example: Is it possible to transmit signal x(t) through system h(t)?



Amplitude Spectrum



Amplitude Spectrum

Signal BW= $f_1 - f_2$ = ………

System BW= $f_1 - f_2$=……..

f_c = ………..

f_c = ………..

Energy and Power

Energy in the Frequency domain Parseval Theorem

Q: What is the total energy in a signal?

$$E_x = \int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\infty}^{\infty} |X(f)|^2 df$$

This means that the energy of the signal in the time domain is equal to the energy in the frequency domain

We can calculate the energy in a signal in any of the two domains depending on which is simpler.

Parseval Theorem

Exercise: Find the energy of the signal

$$x(t) = 4\sin c(4t)$$

Answer: In the time domain:

$$E_x = \int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\infty}^{\infty} |4\sin c(4t)|^2 dt$$

This is very difficult to evaluate, try it in the frequency domain

$$X(f) = \begin{cases} 1, & |f| \le 2 \\ 0, & |f| > 2 \end{cases}$$

$$E_x = \int_{-\infty}^{\infty} |X(f)|^2 df = \int_{-2}^{2} |1|^2 df = \int_{-2}^{2} 1 df = 4$$

Parseval Theorem

Q: What is the average power in a periodic signal?

We look at the average power since the signal x(t) is periodic

For the Exponential Form, the average Power is:

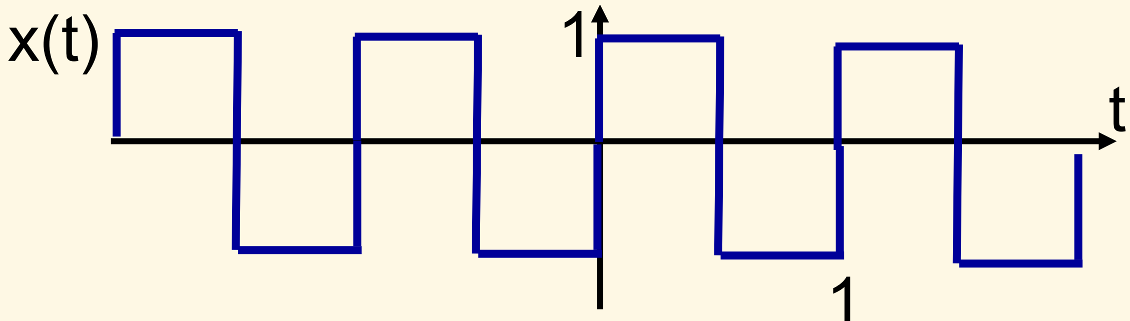
$$P_{av} = \frac{1}{T}\int_0^T |x(t)|^2 dt \qquad = \sum_{k=-\infty}^{\infty} |X[k]|^2$$

For the Trigonometric Form:

$$P_{av} = \frac{1}{T}\int_0^T |x(t)|^2 dt \quad = |X[0]|^2 + \sum_{k=1}^{\infty}\left(\frac{1}{2}|A[k]|^2 + \frac{1}{2}|B[k]|^2\right)$$

Parseval Theorem

Exercise: Find the average power of the following signal.



Answer:

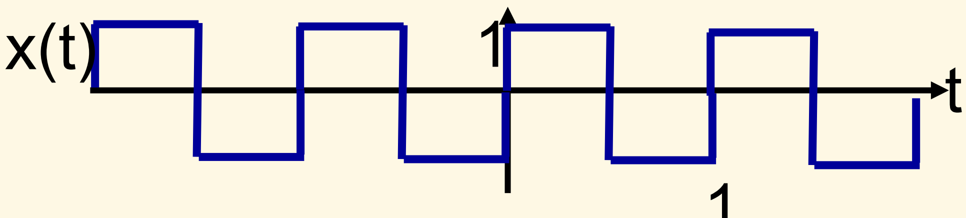
$$P_{av} = \frac{1}{T}\int_0^T |x(t)|^2 dt = \sum_{k=-\infty}^{\infty} |X[k]|^2$$

This is easier to do in the time domain

$$P_{av} = \frac{1}{T}\int_0^T |x(t)|^2 dt = \frac{1}{1}\left(\int_0^{0.5} |1|^2 dt + \int_{0.5}^{1} |-1|^2 dt\right) = \ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

Parseval Theorem

Exercise: Find the average power in the first 3 non zero FS components of the following signal



For a square function:

$$|X[k]| = \begin{cases} \frac{2A}{\pi k}, & k \quad odd \\ 0, & k \quad even \end{cases}$$

Answer:

Exponential Form

$$P_{av} = \frac{1}{T}\int_0^T |x(t)|^2 dt = \sum_{k=-\infty}^{\infty} |X[k]|^2$$

In the first 3 non zero components of the FS:

$$P_{av\_1st \quad 3 \quad \neq 0 \quad comp} = \sum_{k=-5}^{5} |X[k]|^2$$

$$= |X[-5]|^2 + |X[-3]|^2 + |X[-1]|^2 + |X[1]|^2 + |X[3]|^2 + |X[5]|^2$$

Parseval Theorem

Exercise: Find the average power in the first 3 non zero FS components of the following signal

For a square function:

$$X[0]=0, \quad B[k]=0,$$

$$|A[k]| = \begin{cases} \dfrac{4A}{\pi k}, & k \quad odd \\ 0, & k \quad even \end{cases}$$

Answer:

Using trigonometric form

$$P_{av} = \frac{1}{T}\int_0^T |x(t)|^2 dt = |X[0]|^2 + \sum_{k=1}^{\infty}\left(\frac{1}{2}|A[k]|^2 + \frac{1}{2}|B[k]|^2\right)$$

In the first 3 non zero components of the FS:

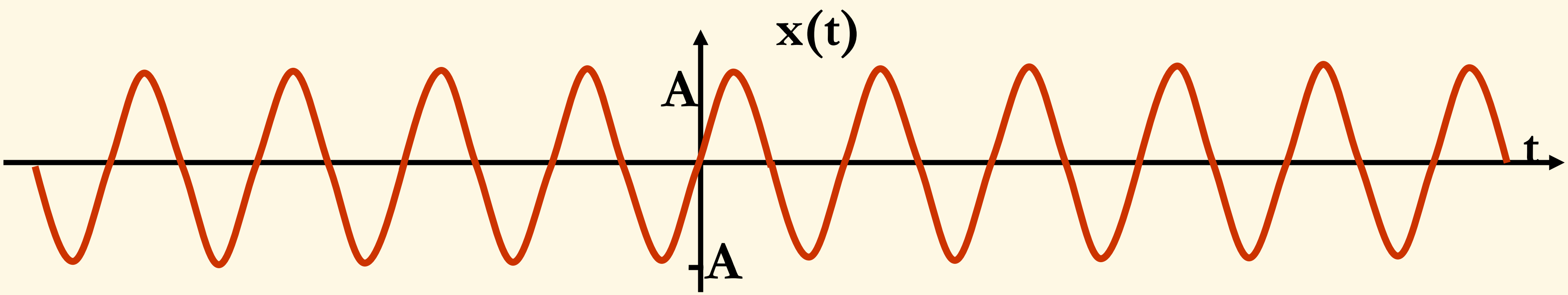
$$P_{av\_1st \quad 3 \quad \neq 0 \quad comp} = \sum_{k=1}^{5} \frac{1}{2}|A[k]|^2$$

$$= \frac{1}{2}\left(|A[1]|^2 + |A[3]|^2 + |A[5]|^2\right) \qquad = \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

Average of a Sinusoidal signal

Q: What is the Average (X_{av}) of a sinusoidal signal?

The Average is the DC part of the signal and is equal to:

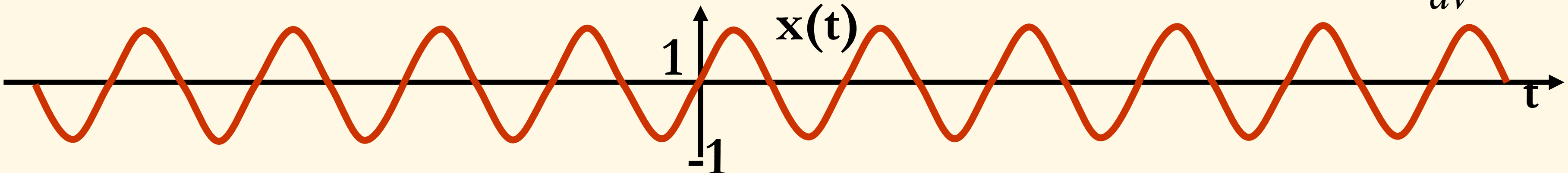


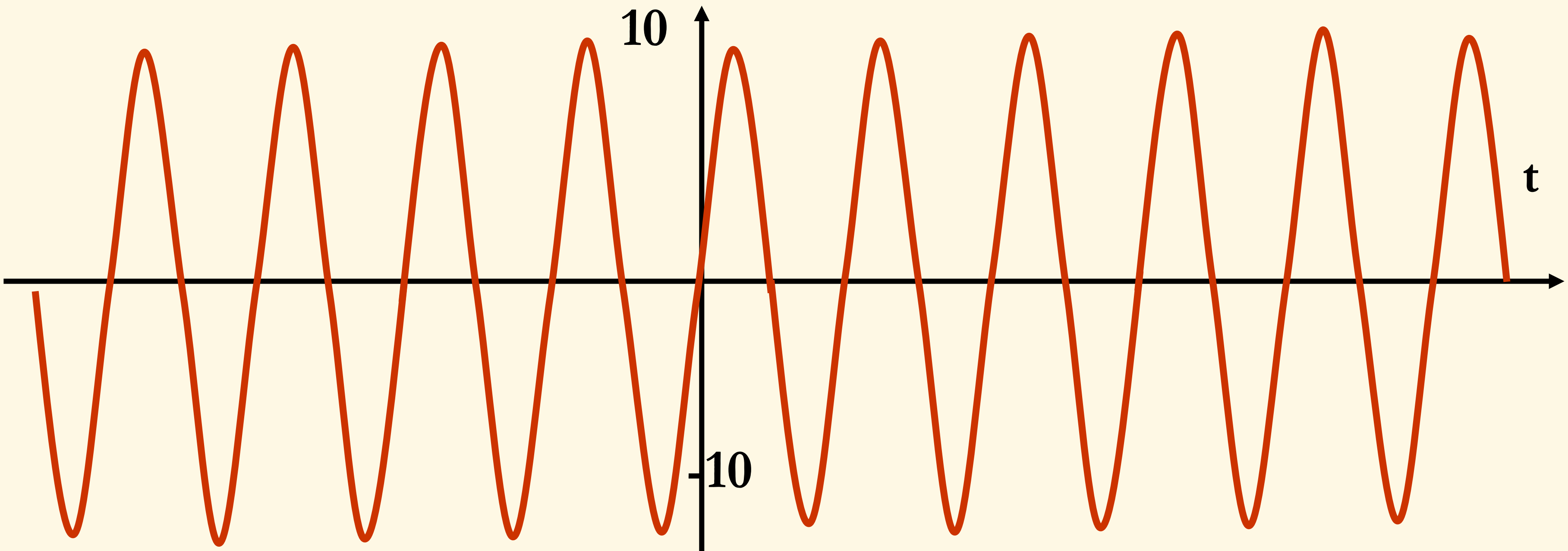
$$X_{av} = \frac{1}{T}\int_T x(t)dt = 0$$

Average of a Sinusoidal signal

Q: Does the Average (X_{av}) of a sinusoidal signal give an indication of the size of the signal?

$$X_{av} = 0$$

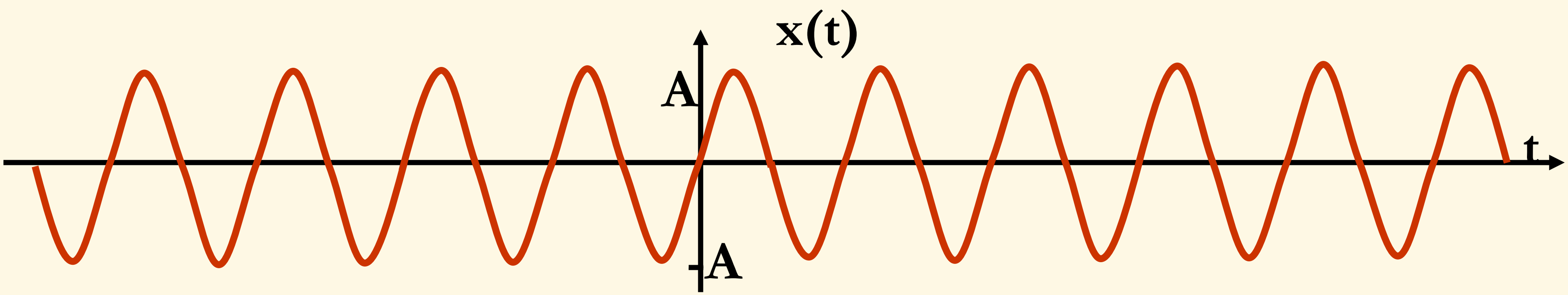




RMS of a Sinusoidal signal

Q: What is the Root mean square (X_{RMS}) of a sinusoidal signal?

The RMS is:

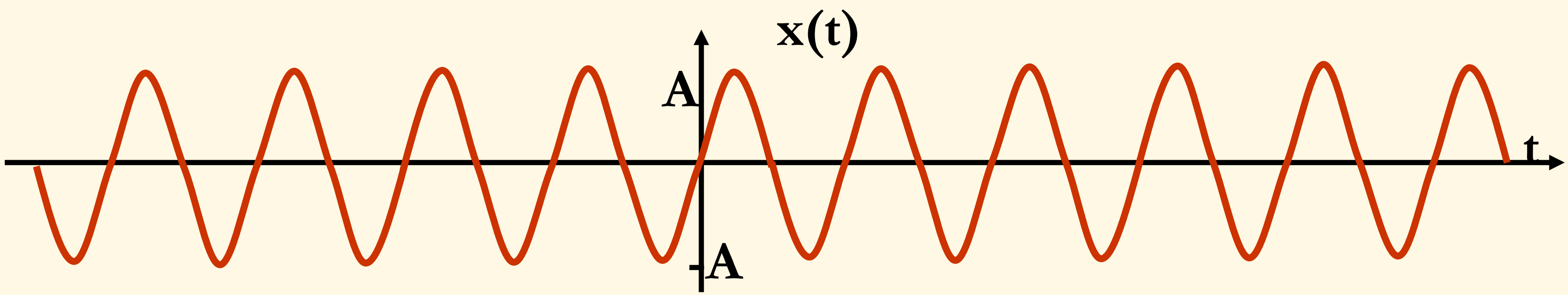


$$X_{RMS} = \sqrt{\frac{1}{T}\int_{T} |x(t)|^2 dt} = \frac{A}{\sqrt{2}} = 0.707A$$

Average Power of a Sinusoidal signal

Q: What is the total Average Power (P_a) of a sinusoidal signal?

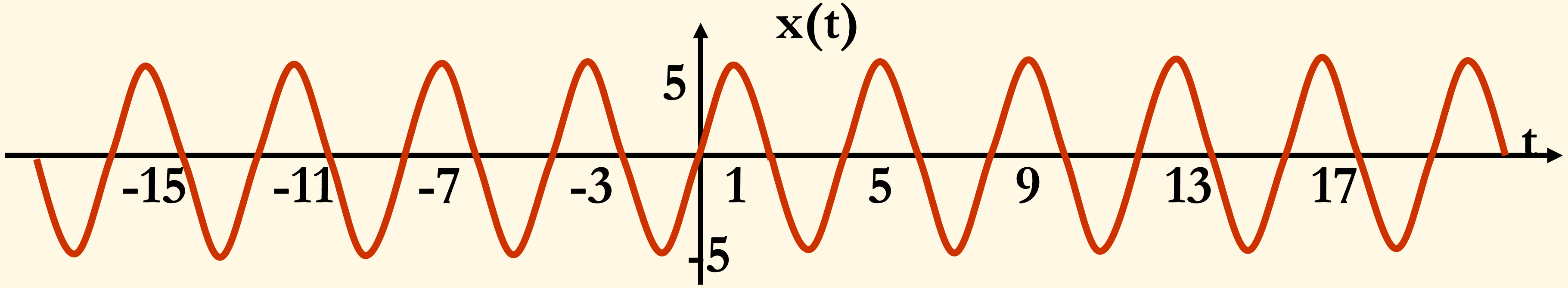
The average power is:



$$P_{av} = \frac{1}{T}\int_{T} |x(t)|^2 dt = \left(X_{RMS}\right)^2 = \frac{A^2}{2} = \left(\frac{A}{\sqrt{2}}\right)^2$$

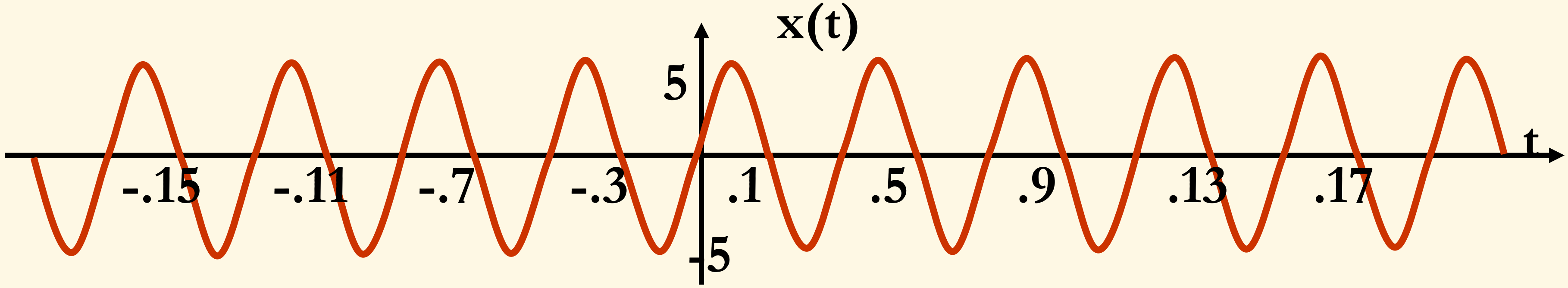
Sinusoidal signals

Exercise: What is the X_{AV}, X_{RMS} and P_{av} of the following signal?



Sinusoidal signals

Exercise: What is the X_{AV}, X_{RMS} and P_{av} of the following signal?

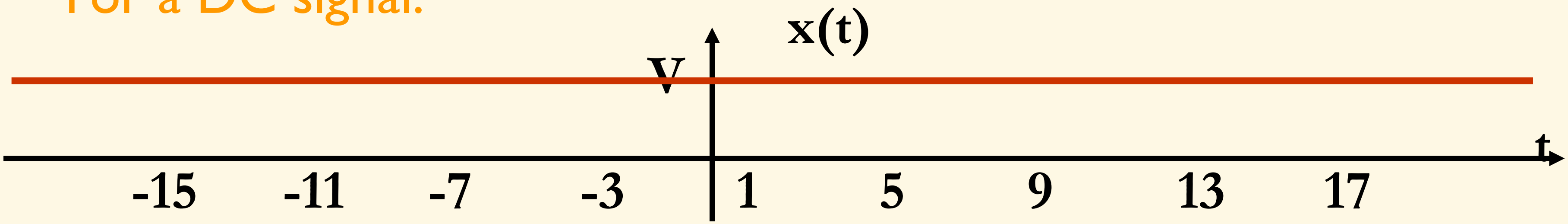


It is the same because the period and the frequency do not effect the average power

DC signal

Exercise: What is the X_{AV}, X_{RMS} and P_{av} of the following signal?

For a DC signal:



$$X_{av} = V$$

$$X_{RMS} = V$$

$$P_{av} = V^2$$

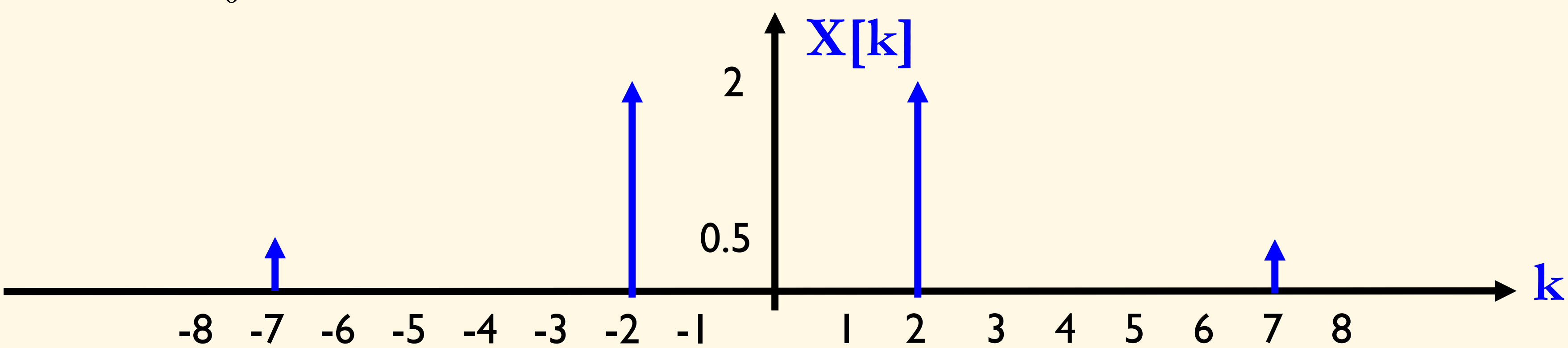
Sum of Sinusoidal signals

Exercise: What is the total Average Power of the sum of two sinusoids?

$$x(t) = \cos(7\pi\, t) - 4\sin(2\pi\, t)$$

It is a periodic signal: $f_o = 0.5$

$$P_{av} = \frac{1}{T}\int_0^T |x(t)|^2 dt = \sum_{k=-\infty}^{\infty} |X[k]|^2$$



$$P_{av} = (0.5)^2 + 0.5)^2 + (2)^2 + (2)^2 = \ldots\ldots\ldots\ldots W$$

Sum of Sinusoidal signals

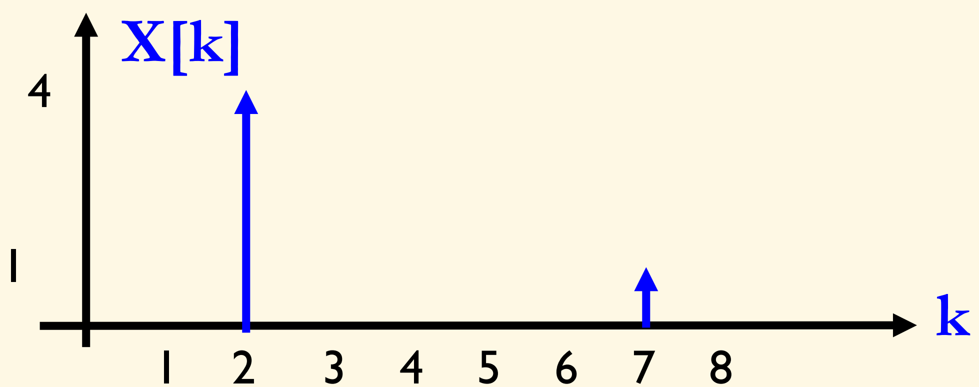
Exercise: What is the total Average Power of the sum of two sinusoids?

$$x(t) = \cos(7\pi\, t) - 4\sin(2\pi\, t)$$

It is a periodic signal:

$$f_o = 0.5$$

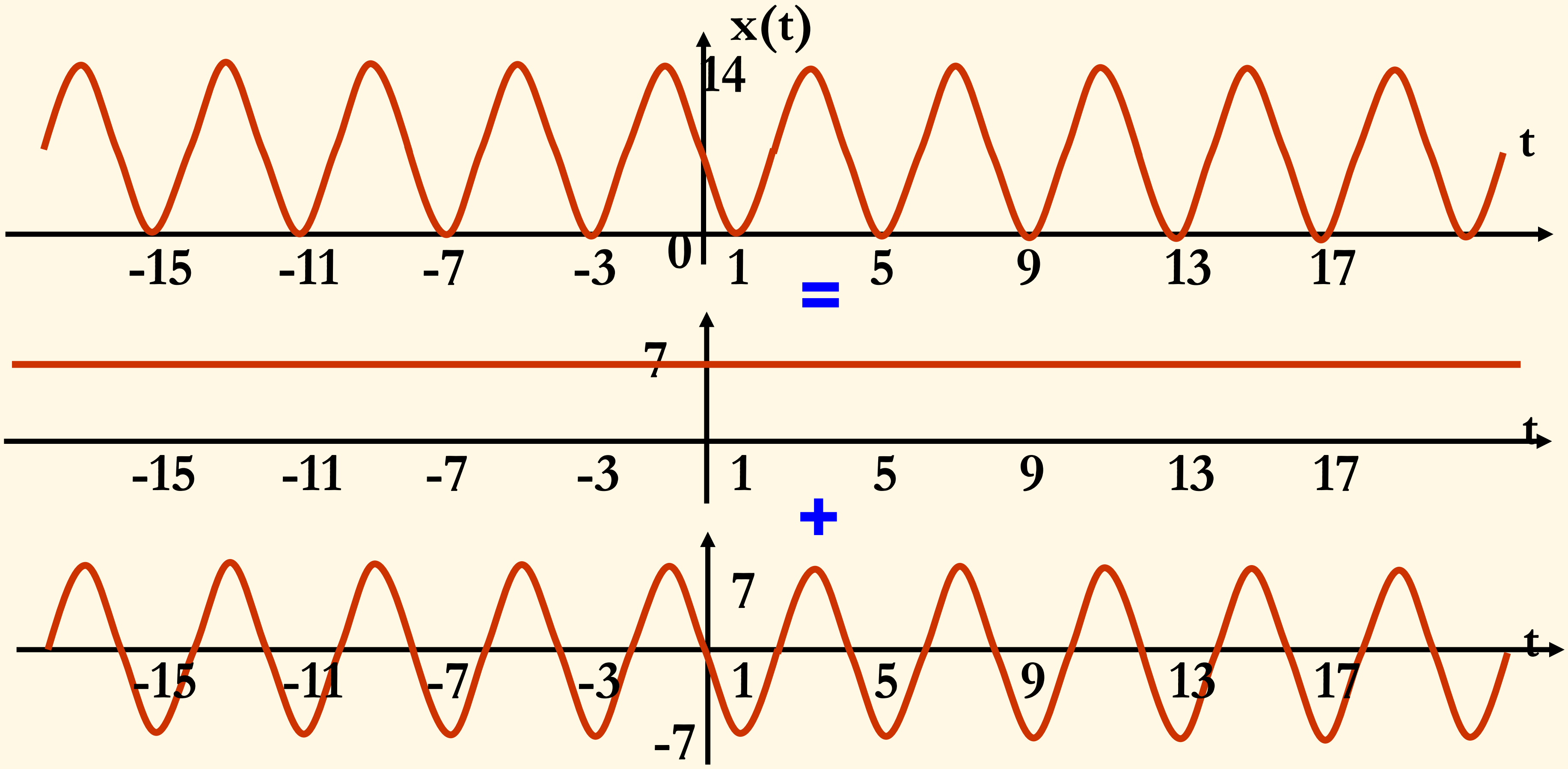
$$P_{av} = \frac{1}{T}\int_0^T |x(t)|^2 dt = |X[0]|^2 + \sum_{k=1}^{\infty}\left(\frac{1}{2}|A[k]|^2 + \frac{1}{2}|B[k]|^2\right)$$



$$P_{av} = \frac{1^2}{2} + \frac{4^2}{2} = \ldots\ldots\ldots\ldots W$$

DC and Sinusoidal signals

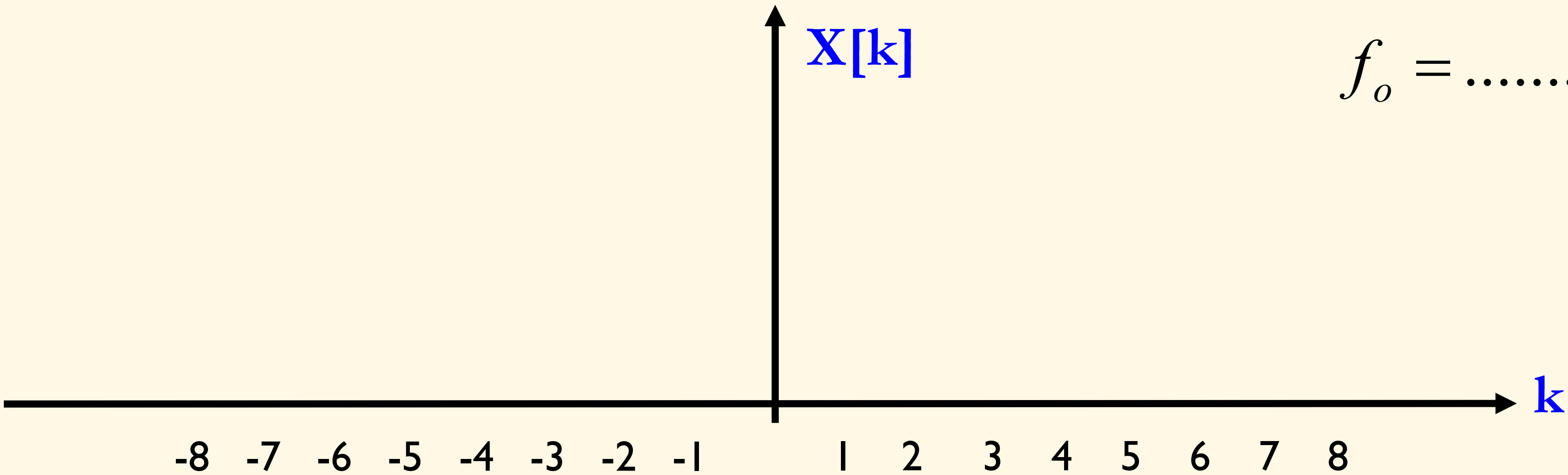
Exercise: What is the X_{AV}, X_{RMS} and P_{av} of the following signal?



Sum of Sinusoidal signals

Exercise: What is the total Average Power of the following signal?

$$x(t) = 5 + 10\cos(6\pi\, t) + 7\sin(8\pi\, t) - 3\sin(16\pi\, t)$$



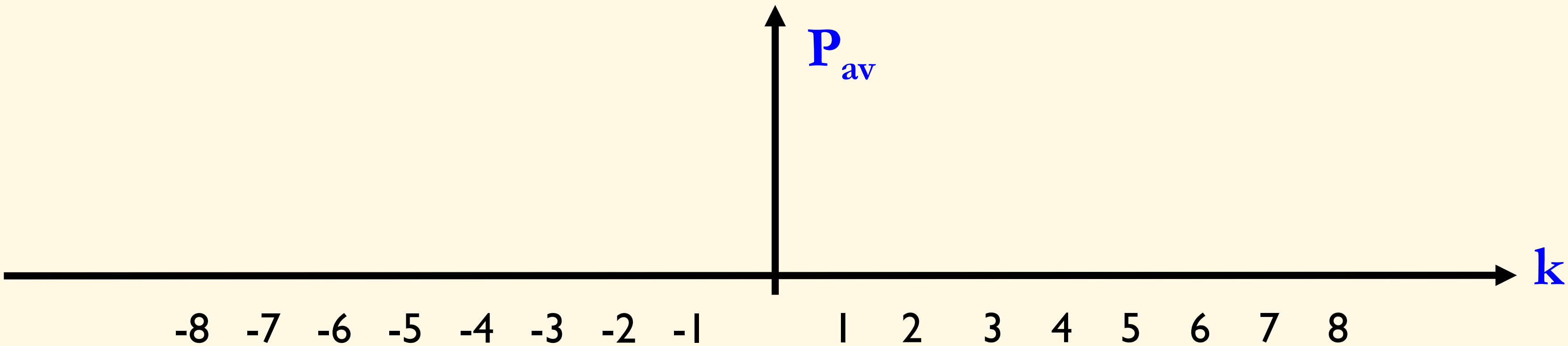
$$f_o = \ldots\ldots$$

$$P_{av} =$$

Power Spectrum

Q: What is the Power Spectrum of a signal?

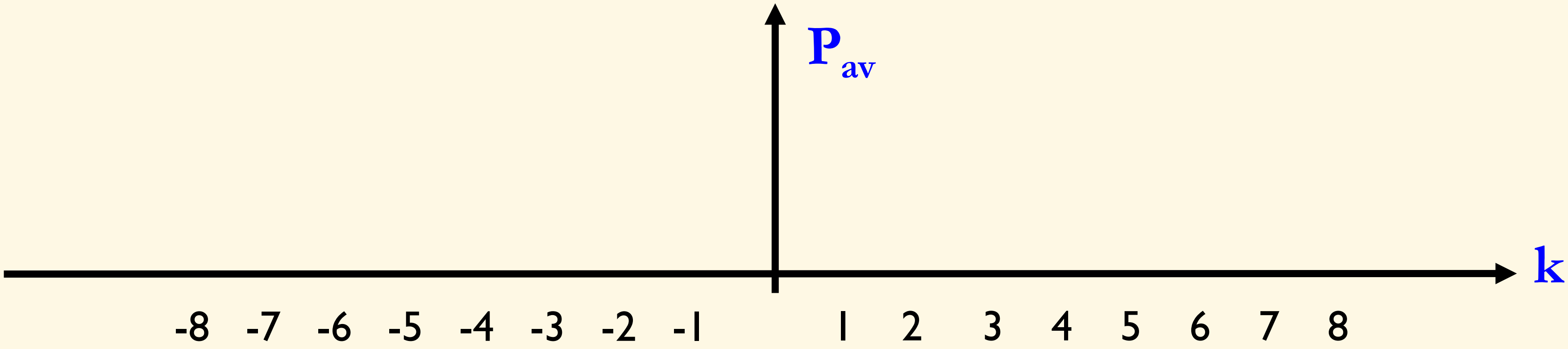
It is the plot of the average power for each sinusoidal component of the signal.



Power Spectrum

Exercise: What is the power spectrum for the following signal?

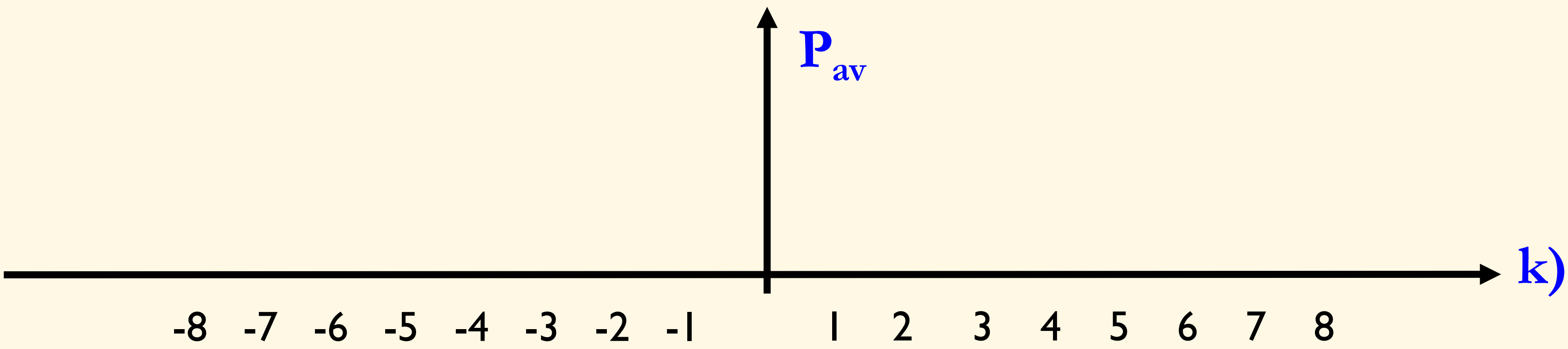
$$x(t) = \cos(7\pi\, t) - 4\sin(2\pi\, t)$$



Power Spectrum

Exercise: What is the power spectrum for the following signal?

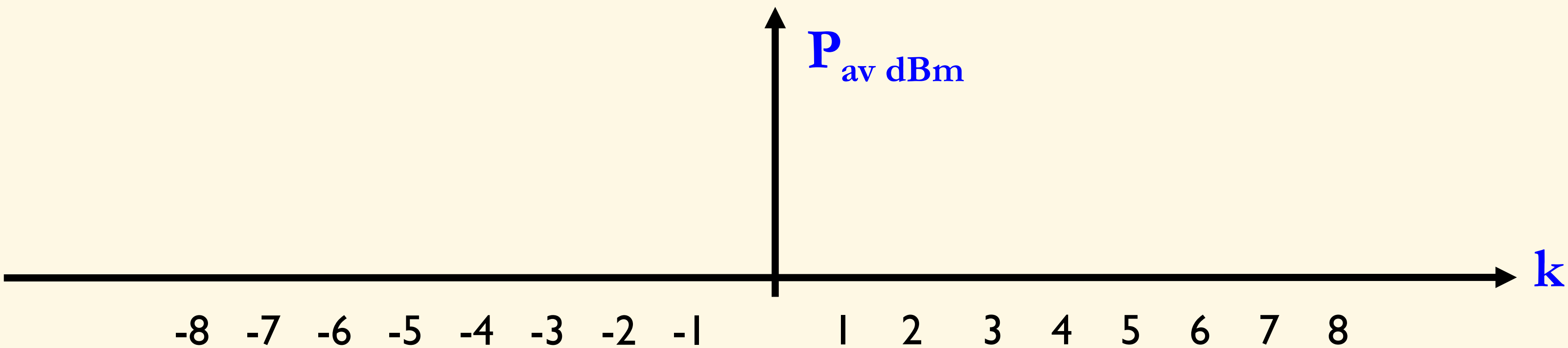
$$x(t) = 5 + 10\cos(6\pi\, t) + 7\sin(8\pi\, t) - 3\sin(16\pi\, t)$$



Power Spectrum in Decibel (dBm)

Q: What is the Power Spectrum in dBm of a signal?

It is the plot of the average power in dBm for each sinusoidal component of the signal.



Power in Decibel (dBm)

Q: What is the Power in dBm?

A logarithmic ratio with a reference power 1mW

It is defined as:

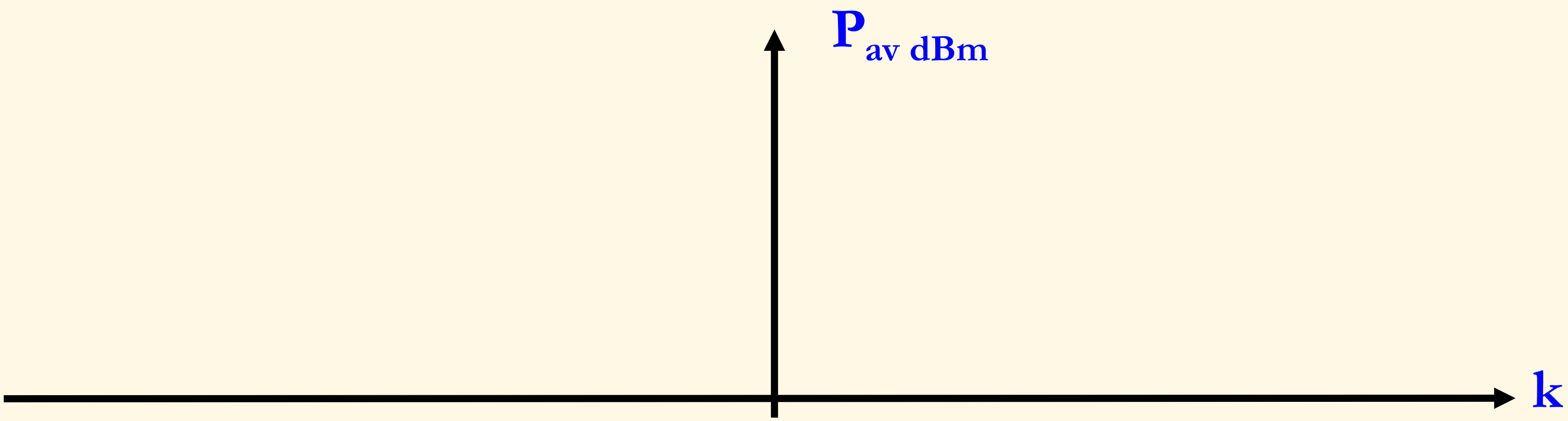
$$P_{av\quad dBm} = 10\log\left(P_{av}(mW)\right)$$

$$P_{av\quad dBm} = 10\log\left(1000 \times P_{av}(W)\right)$$

Power in Decibel (dBm)

Exercise: What is the Power Spectrum in dBm of the following signal?

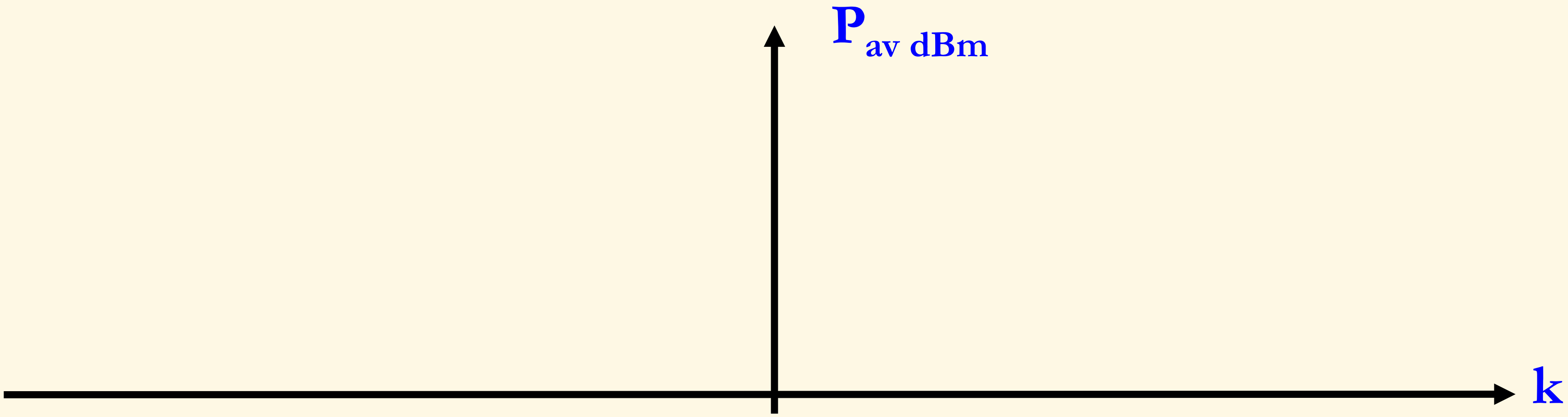
$$x(t) = \cos(7\pi\, t) - 4\sin(2\pi\, t)$$



Power in Decibel (dBm)

Exercise: What is the Power Spectrum in dBm of the following signal?

$$x(t) = 5 + 10\cos(6\pi\, t) + 7\sin(8\pi\, t) - 3\sin(16\pi\, t)$$



Power in dBm

Q: Why represent power in dBm?

Because using the decibel makes the calculation of loss and gain in different components in a communication system easier

Q: How?

Assume we have:

The output power in Watts is equal to:

$$P_{out} = P_{in} \times G$$

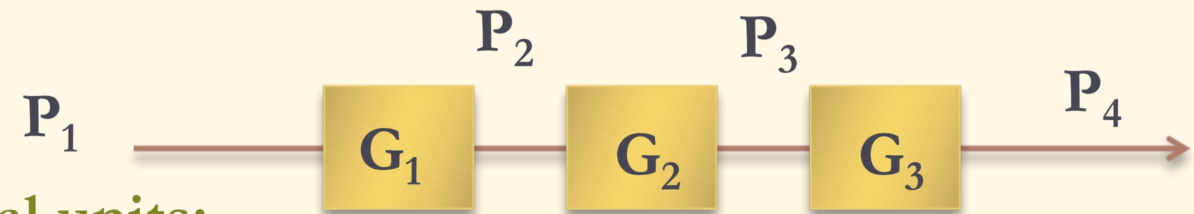
But the output power in dBm is equal to:

$$\left(P_{out}\right)_{dBm} = \left(P_{in}\right)_{dBm} + \left(G\right)_{dB}$$

So we covert a multiplication to an addition

Why dB?

Q: What is the total gain G for this cascade of systems?



In normal units:

$$G = \frac{P_4}{P_1} = \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

In dB:

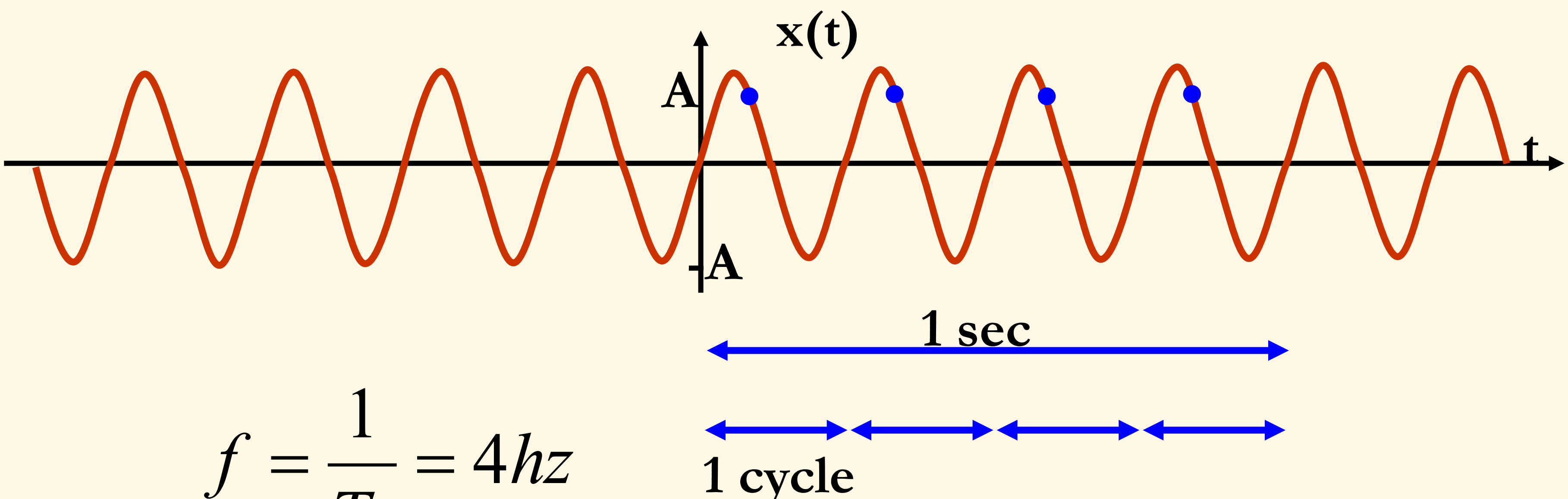
$$G_{dB} = \left(\frac{P_4}{P_1}\right)_{dB} = \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

Waves

Frequency

Q: What is the frequency of a signal?

The frequency of a sinusoidal signal is the number of times a certain point occurs in 1 sec.



$$f = \frac{1}{T_0} = 4hz$$

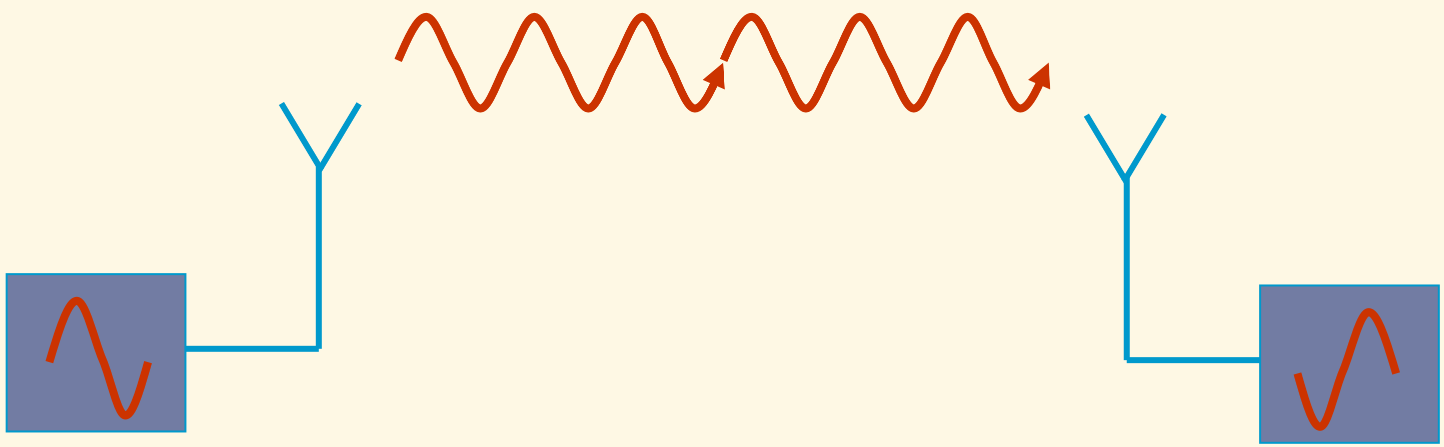
$$T_0 = 0.25\,\sec$$

Electromagnetic Waves

Q: What is the Electromagnetic Waves?

Electromagnetic Waves are electronic signals that radiate in space. They consist of Electrical and magnetic Signals.

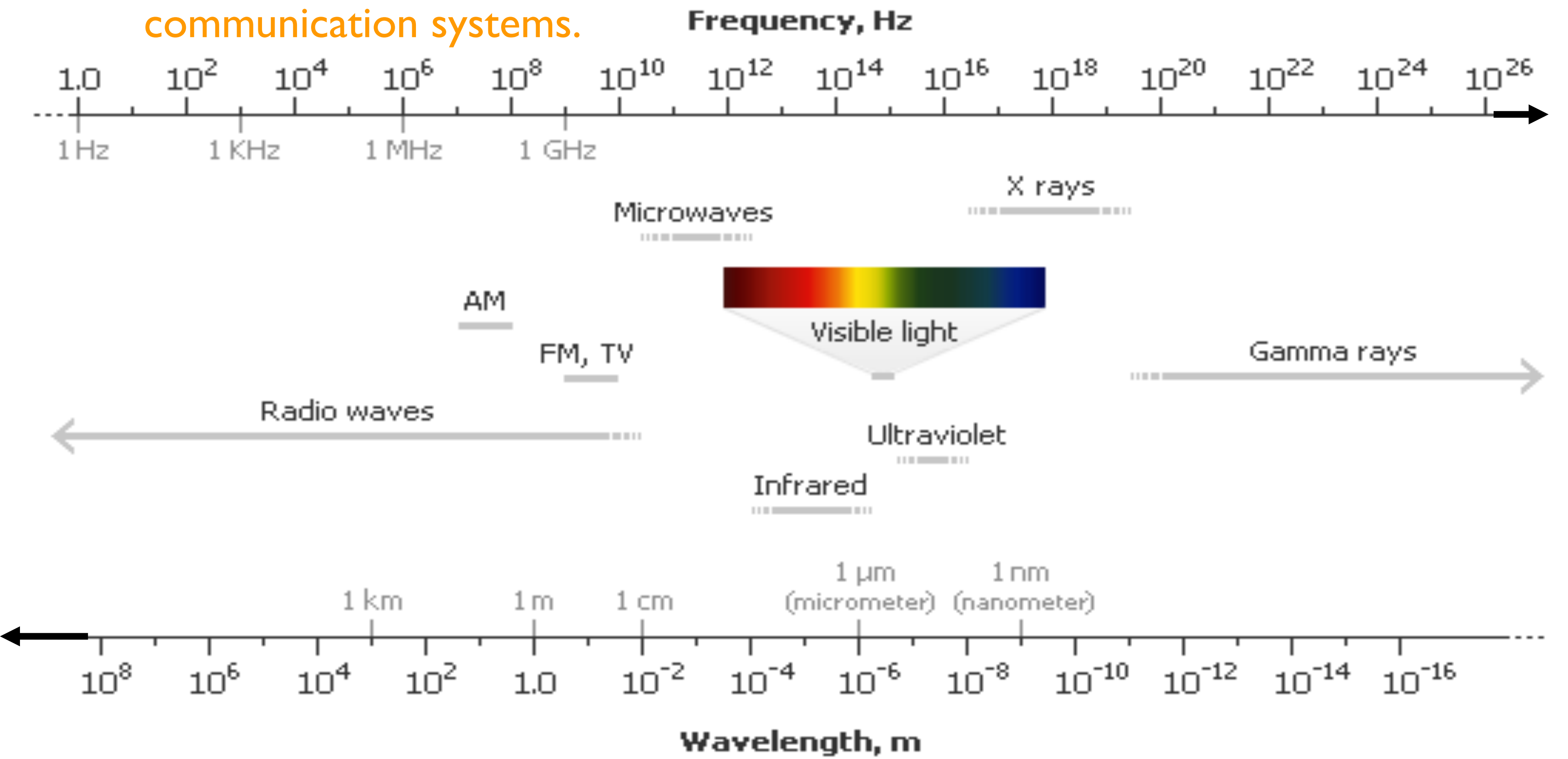
They are also called Radio Frequency (RF) waves.



Electromagnetic Spectrum

Q: What is the Electromagnetic Spectrum?

The Electromagnetic Spectrum is the range of frequencies used in communication systems.



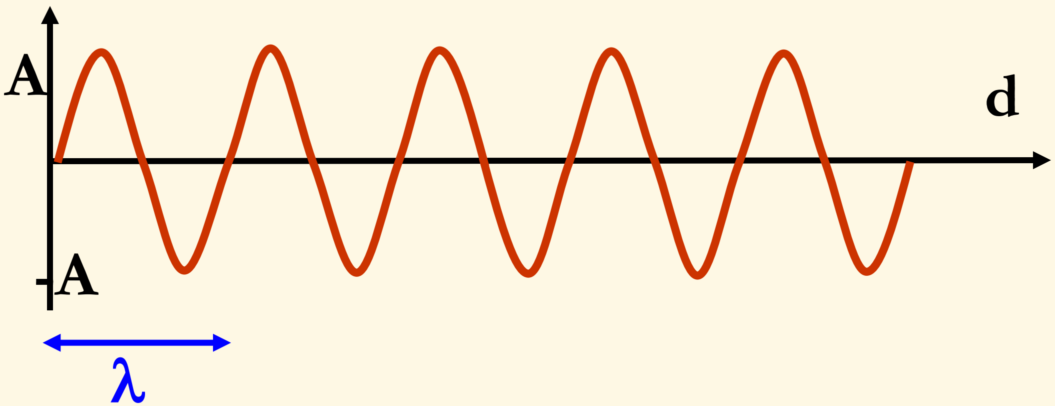
Wavelength

Q: What is the wavelength of a signal?

The wavelength of a traveling sinusoidal signal is the distance the wave travels in one period.

It is equal to:

$$\lambda = \frac{c}{f}$$



where c is the speed of electromagnetic waves,

$c = 300{,}000{,}000$ m/s $= 3 \times 10^{8}$ m/s

Wavelength

Exercise: What is the wavelength of the following signal assuming it is propagating in free space?

$$x(t) = 10\cos(6\pi f\ t)$$