University of Bahrain

Department of Electrical and Electronics Engineering

EENG372

Communication Systems I

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Topic 2:

Frequency Modulation (FM)

This Topic will cover

- Angle Modulation
  - Phase Modulation (FM)
  - Frequency Modulation (FM)
- FM Generation
- FM Demodulation
- FM Receivers

Frequency Modulation: Generation

FM Generation

Q: What is an FM modulator?

We will study two types:

1. Direct Method - Voltage Controlled Oscillator (VCO))
2. Indirect method (Armstrong Modulator)

Direct FM Generation

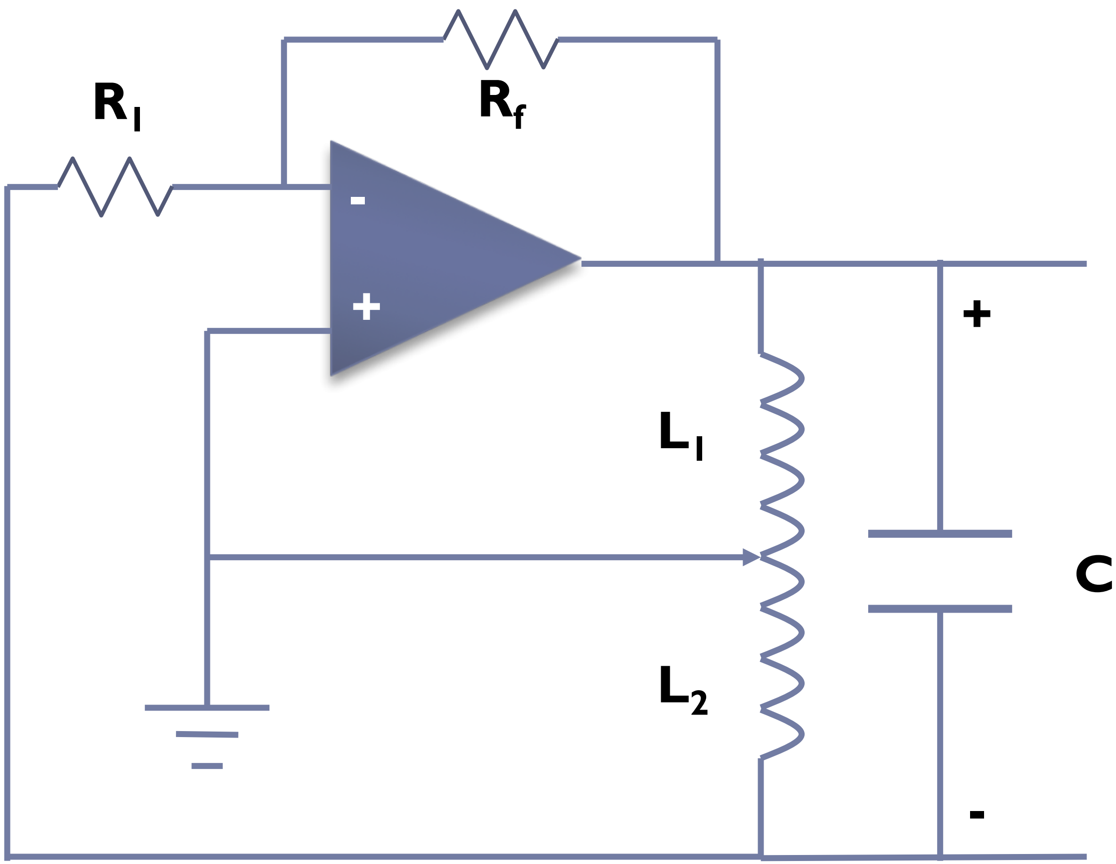
Q: What is a VCO?

It is a circuit that oscillates (produces a frequency) that varies with an applied voltage.

It is an oscillator than generates a carrier frequency and the instantaneous frequency of this carrier can be varied with the amplitude of the message signal.

Example: Hartley Oscillator

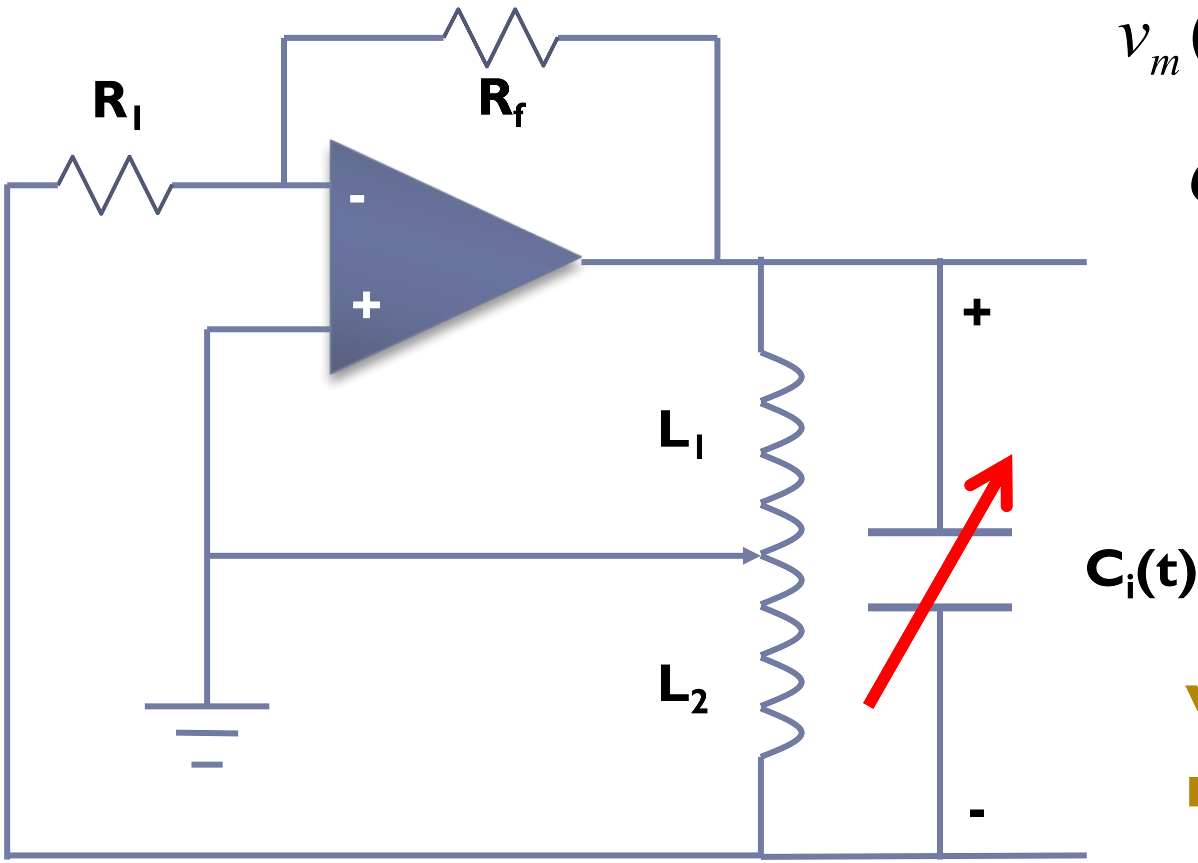
Hartley Oscillator



$$v_0(t) = V_c \cos(2\pi f_c t)$$

$$f_c = \frac{1}{2\pi\sqrt{(L_1 + L_2)C}}$$

Hartley Oscillator



$$v_m(t) = V_m \cos(2\pi f_m t)$$

$$C_i(t) = C_0 + C_f v_m(t)$$

Varies with the message signal

Hartley Oscillator

Q: What is the instantaneous frequency of the Oscillations?

$$f_i(t) = \frac{1}{2\pi\sqrt{(L_1+L_2)C_i(t)}} = \frac{1}{2\pi\sqrt{(L_1+L_2)(C_0+C_f v_m(t))}}$$

$$f_i(t) = \frac{1}{2\pi\sqrt{(L_1+L_2)C_0\left(1+\frac{C_f}{C_0}v_m(t)\right)}} = f_c\frac{1}{\sqrt{(1+\Delta C v_m(t))}}$$

$$f_i(t) = f_c(1+\Delta C v_m(t))^{-\frac{1}{2}} \approx f_c\left(1-\frac{\Delta C}{2}v_m(t)\right) \approx \left(f_c-\frac{\Delta C f_c}{2}v_m(t)\right)$$

$$f_i(t) \approx f_c + k_f v_m(t) \qquad k_f = -\frac{\Delta C}{2}f_c = -\frac{C_f}{2C_0}f_c$$

Hartley Oscillator

Q: What is the output of the Oscillations?

$$v_0(t) = V_c \cos[\theta_i(t)] = V_c \cos\left[2\pi \int_{-\infty}^{t} f_i(\tau)d\tau\right]$$

$$v_0(t) = V_c \cos\left[2\pi(f_c t + k_f \int_{-\infty}^{t} v_m(\tau)d\tau)\right] = v_{FM}(t)$$

Indirect FM Generation

Q: What is an indirect FM modulator?

First a NB-FM is generated with small m_f and then using frequency Multipliers WB-FM is generated with larger m_f

$v_m(t)$ $\quad$ $v_{NB-FM}(t)$ $\quad$ $v_{WB-FM}(t)$

NB FM Generation

Q: What is NB-FM?

Going back to the defining equation:

$$v_{FM}(t) = V_c \cos(2\pi f_c t + 2\pi k_f \int_{-\infty}^{t} v_m(\tau) d\tau)$$

Using trigonometric identities:

$$v_{FM}(t) = V_c \cos(2\pi f_c t) \cos\left(2\pi k_f \int_{-\infty}^{t} v_m(\tau) d\tau\right)$$

$$- V_c \sin(2\pi f_c t) \sin\left(2\pi k_f \int_{-\infty}^{t} v_m(\tau) d\tau\right)$$

NB FM Generation

Q: What if the modulation index m_f is small ?

Then:

$$\cos\left(2\pi k_f \int_{-\infty}^{t} v_m(\tau)d\tau\right) \approx 1$$

$$\sin\left(2\pi k_f \int_{-\infty}^{t} v_m(\tau)d\tau\right) \approx 2\pi k_f \int_{-\infty}^{t} v_m(\tau)d\tau$$

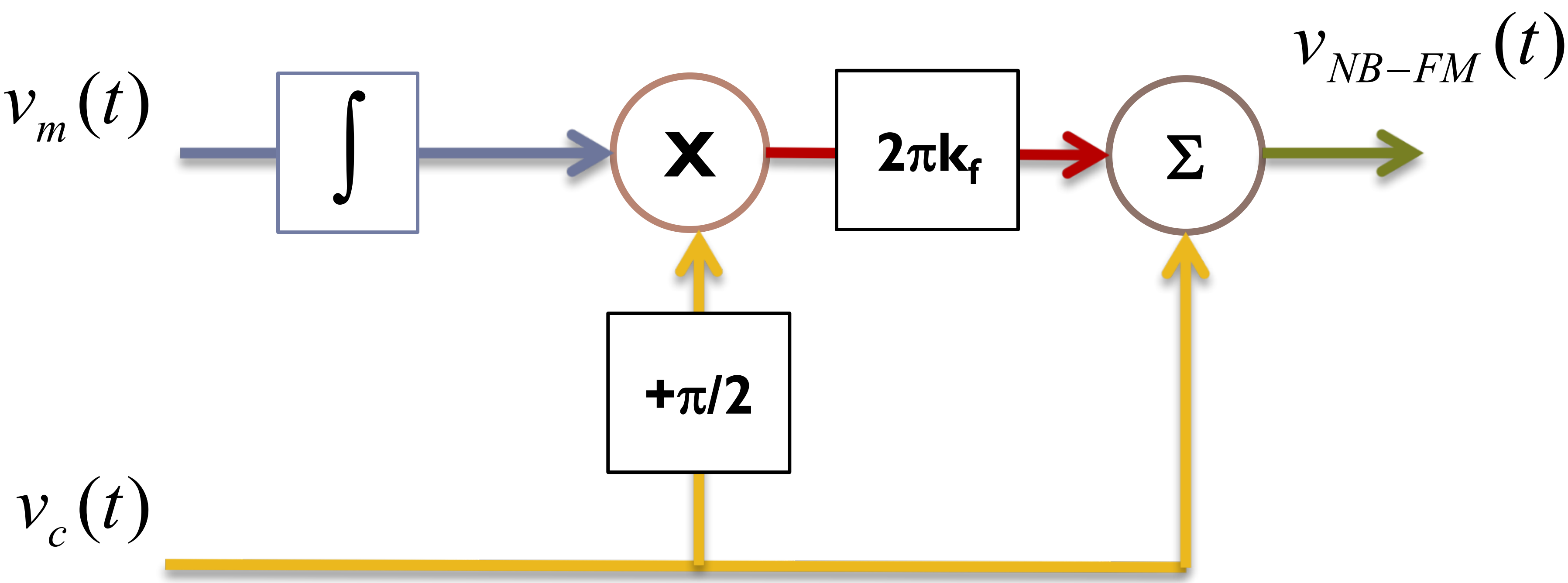
Substituting:

$$v_{FM}(t) \approx V_c \cos(2\pi f_c t) - V_c \sin(2\pi f_c t)\left(2\pi k_f \int_{-\infty}^{t} v_m(\tau)d\tau\right)$$

$$v_{FM}(t) \approx V_c \cos(2\pi f_c t) - 2\pi k_f V_c \sin(2\pi f_c t)\left(\int_{-\infty}^{t} v_m(\tau)d\tau\right)$$

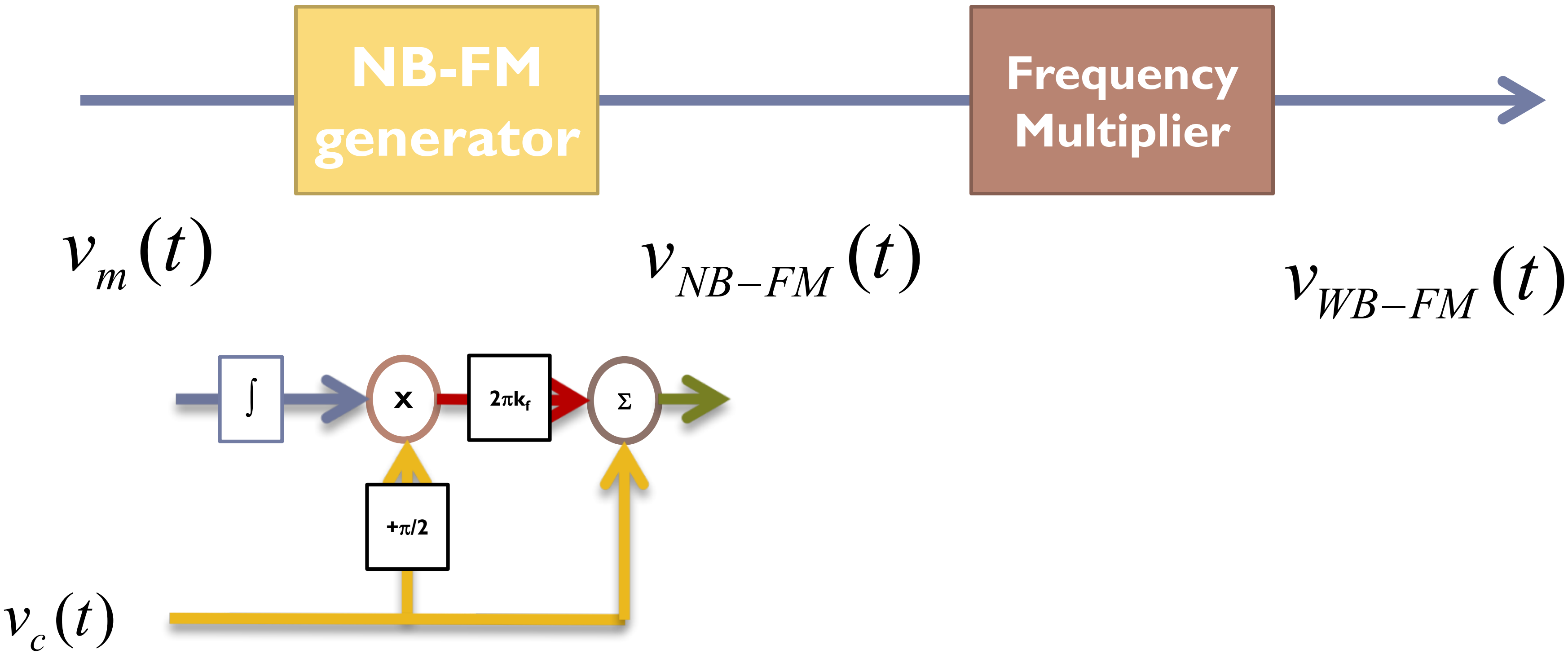
NB FM Generation

$$v_{FM}(t) \approx V_c \cos(2\pi f_c t) - 2\pi k_f V_c \sin(2\pi f_c t)\left(\int_{-\infty}^{t} v_m(\tau)d\tau\right)$$



Indirect FM Generation

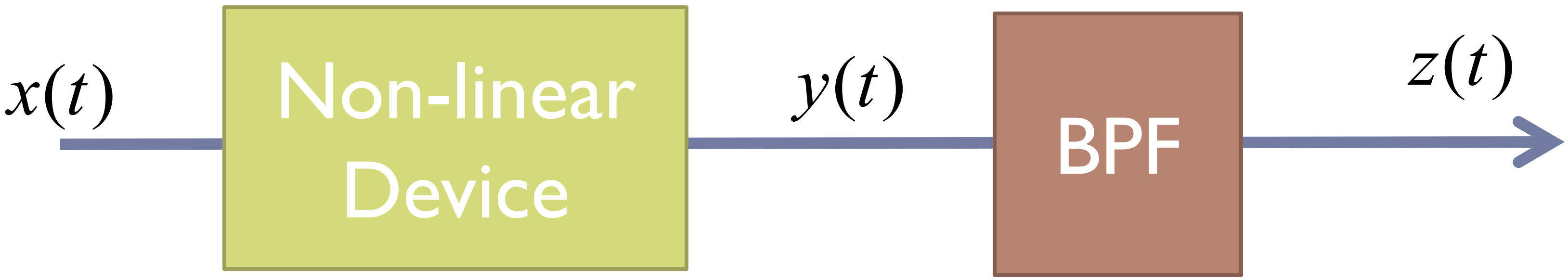
Q: What is an indirect FM modulator?



Frequency Multiplier

Q: What is a Frequency Multiplier?

It is a non linear device followed by a Band pass filter (BPF)



The output of the NLD is:

Signal multiplied by itself

$$y(t) = a_0 + a_1 x(t) + a_2 x^2(t) + a_3 x^3(t) + ...$$

$$y(t) \approx a_2 x^2(t)$$

Frequency Multiplier

Q: What if x(t) is a NB-FM signal?

The output of the NLD will be

$$y(t) \approx a_2 x^2(t) \approx a_2 v_{NB-FM}{}^2(t)$$

$$y(t) \approx a_2 \left(V_c \cos(2\pi(f_c t + k_f \int_{-\infty}^{t} v_m(\tau) d\tau)) \right)^2$$

$$y(t) \approx a_2 V_c{}^2 \cos^2(2\pi(f_c t + k_f \int_{-\infty}^{t} v_m(\tau) d\tau))$$

$$y(t) \approx \frac{a_2 V_c{}^2}{2} \left(1 + \cos\left(4\pi \left(f_c t + k_f \int_{-\infty}^{t} v_m(\tau) d\tau \right) \right) \right)$$

$$y(t) \approx \frac{a_2 V_c{}^2}{2} \left(1 + \cos\left(2\pi \left(2 f_c t + 2k_f \int_{-\infty}^{t} v_m(\tau) d\tau \right) \right) \right)$$

an FM signal with double the carrier frequency and double the modulation index:

Frequency Multiplier

Q: What is the output of the NLD?

The output of the NLD is an FM signal with double the carrier frequency and double the modulation index:

$$y(t) \approx \frac{a_2 V^2{}_c}{2} + \frac{a_2 V_c^{\,2}}{2} \cos\left(2\pi f_c' t + 2\pi k_f' \int_{-\infty}^{t} v_m(\tau) d\tau \right)$$

$$k_f' = 2k_f$$

$$f_c' = 2 f_c \qquad \Rightarrow$$

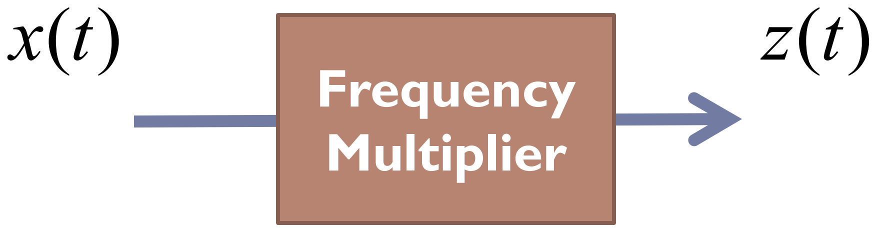
$$m_f' = 2m_f$$

After a BPF centered at $2f_c$ and with BW $2\Delta f$

$$z(t) = \frac{a_2 V_c^{\,2}}{2} \cos\left(2\pi f_c' t + 2\pi k_f' \int_{-\infty}^{t} v_m(\tau) d\tau \right)$$

Frequency Multiplier

Q: What is a Frequency Multiplier?



$$x(t) \approx V_c \cos\left(2\pi f_c t + 2\pi k_f \int_{-\infty}^{t} v_m(\tau)d\tau\right)$$

$$z(t) \approx \frac{a_2 V_c^2}{2} \cos\left(2\pi f_c' t + 2\pi k_f' \int_{-\infty}^{t} v_m(\tau)d\tau\right)$$

$$f_c$$

$$f_c' = 2 f_c$$

$$m_f = \frac{k_f V_m}{f_m}$$

$$m_f' = 2 m_f$$

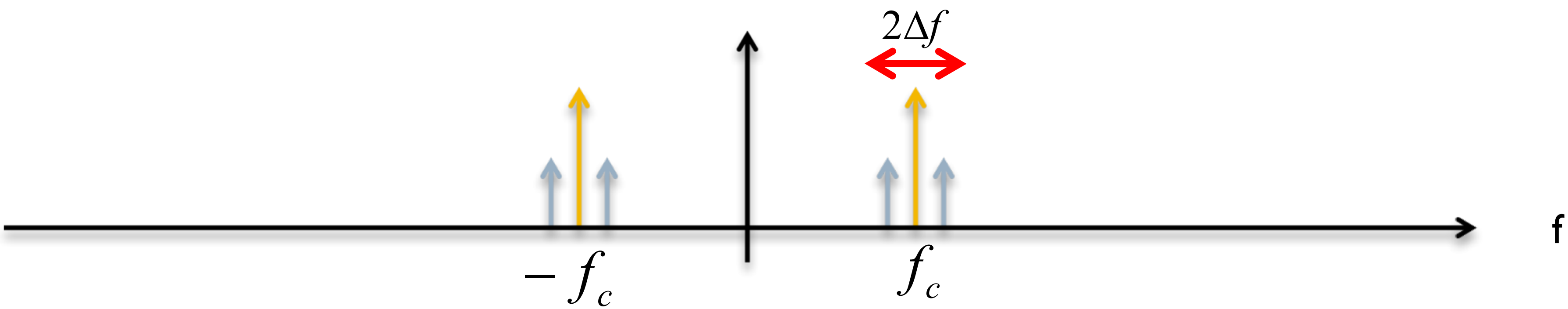
$$\Delta f = k_f V_m$$

$$\Delta f' = 2\Delta f$$

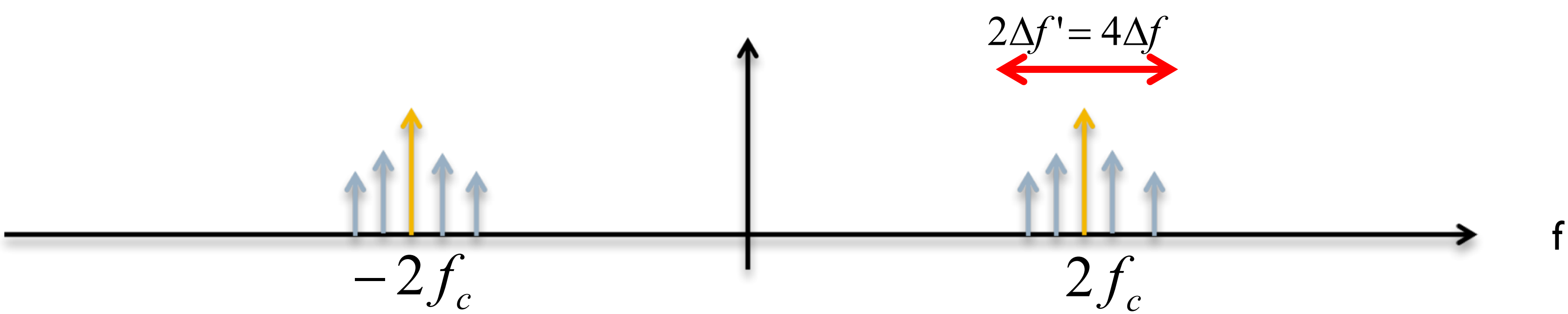
Frequency Multiplier

Q: What is the output of the multiplier for single modulation?

The output of the multiplier is : $V_{NB-FM}(f)$



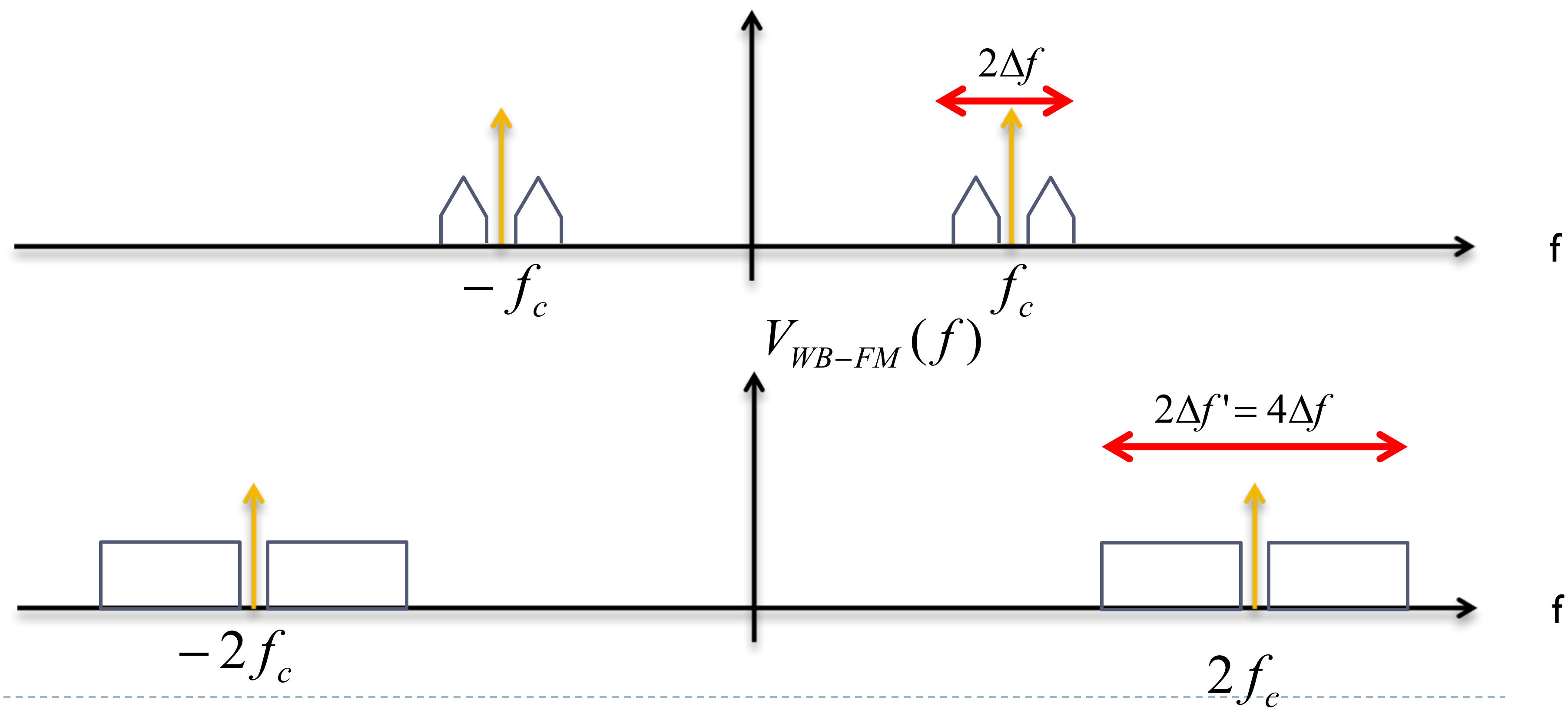
$V_{WB-FM}(f)$



Frequency Multiplier

Q: What is the output of the multiplier for any message signal?

The output of the multiplier is : $V_{NB-FM}(f)$



Indirect FM Generation

Exercise: What is the problem with this method and how can this be solved?

Armstrong FM Modulator