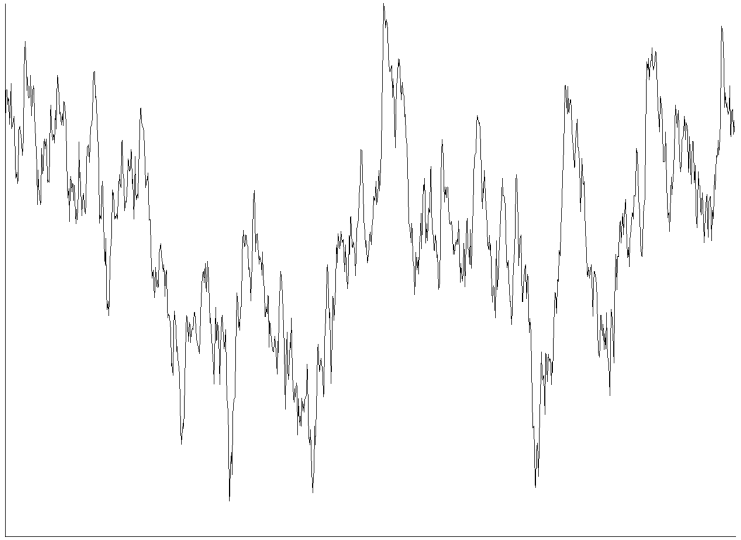
Noise in Communication Systems

The term *noise* refers to *unwanted* electrical signals that are always present in electrical systems. The presence of noise superimposed on a signal tends to obscure or mask the signal; it limits the receiver's ability to make correct symbol decisions, and thereby limits the rate of information transmission. Noise arises from a variety of sources, both man made and natural. The *man-made noise* includes such sources as spark-plug ignition noise, switching transients, and other radiating electromagnetic signals. *Natural noise* includes such elements as the atmosphere, the sun, and other galactic sources.

Good engineering design can eliminate much of the noise or its undesirable effect through filtering, shielding, the choice of modulation, and the selection of an optimum receiver site. For example, sensitive radio astronomy measurements are typically located at remote desert locations, far from man-made noise sources. However, there is one natural source of noise, called *thermal* or *Johnson noise*, that cannot be eliminated. Thermal noise [4, 5] is caused by the thermal motion of electrons in all dissipative components—resistors, wires, and so on. The same electrons that are responsible for electrical conduction are also responsible for thermal noise.

Noise in Communication Systems



- Noise is any unwanted signal, random or deterministic, that interfere with the desired signal in a system.
- sources of noise: man made and naturally occurring.
- There is external noise and internal noise
- The noise is distinguished from interference:
    - signal-to-noise ratio (SNR),
    - signal-to-interference ratio (SIR)
    - signal-to-noise plus interference ratio (SNIR)

Noise in Communication Systems

- Johnson–Nyquist or thermal noise is unavoidable, and generated by the random thermal motion of charge carriers (usually electrons), inside an electrical conductor, which happens regardless of any applied voltage.

- Thermal noise is approximately **white**, meaning that its power spectral density is nearly equal throughout the frequency spectrum. The amplitude of the signal has very nearly a Gaussian probability density function.

- A communication system affected by thermal noise is often modeled as an additive white Gaussian noise (AWGN) channel.

We can describe thermal noise as a zero-mean *Gaussian* random process. A Gaussian process $n(t)$ is a random function whose value n at any arbitrary time t is statistically characterized by the Gaussian probability density function

$$p(n) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{n}{\sigma}\right)^2\right] \tag{1.40}$$

where $\boxed{\sigma^2}$ is the variance of n. The *normalized* or *standardized Gaussian density function* of a zero-mean process is obtained by assuming that $\sigma = 1$. This normalized pdf is shown sketched in Figure 1.7.

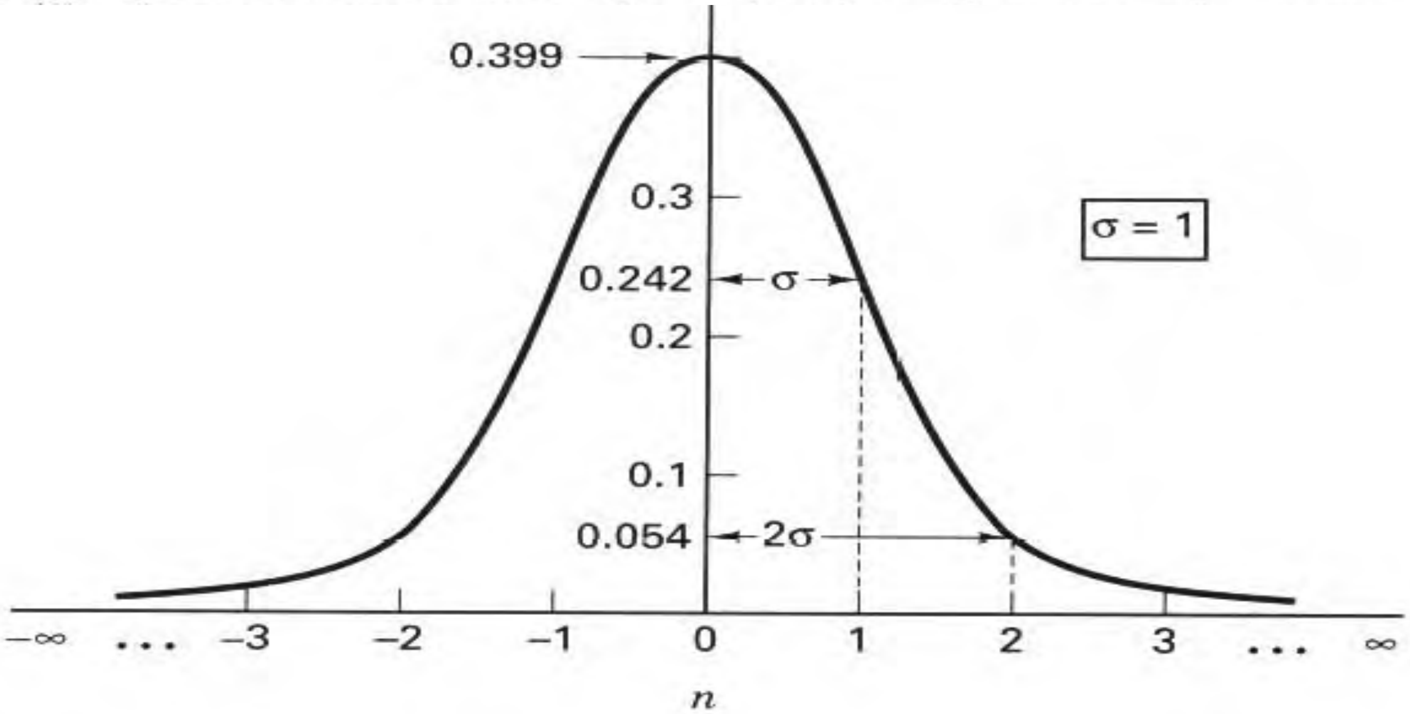


Figure 1.7 Normalized ($\sigma = 1$) Gaussian probability density function.

1.5.5.1 White Noise

The primary spectral characteristic of thermal noise is that its power spectral density is *the same* for all frequencies of interest in most communication systems; in other words, a thermal noise source emanates an equal amount of noise power per unit bandwidth at all frequencies—from dc to about 10^{12} Hz. Therefore, a simple model for thermal noise assumes that its power spectral density $G_n(f)$ is flat for all frequencies, as shown in Figure 1.8a, and is denoted as

$$\boxed{G_n(f) = \frac{N_0}{2} \qquad \text{watts/hertz}} \tag{1.42}$$

where the factor of 2 is included to indicate that $G_n(f)$ is a *two-sided* power spectral density.

The adjective "white" is used in the same sense as it is with white light, which contains equal amounts of all frequencies within the visible band of electromagnetic radiation.

The autocorrelation function of white noise is given by the inverse Fourier transform of the noise power spectral density (see Table A.1), denoted as follows:

$$\boxed{R_n(\tau) = \mathscr{F}^{-1}\{G_n(f)\} = \frac{N_0}{2}\,\delta(\tau)} \tag{1.43}$$

Thus the autocorrelation of white noise is a delta function weighted by the factor $N_0/2$ and occurring at $\tau = 0$, as seen in Figure 1.8b. Note that $R_n(\tau)$ is zero for $\tau \neq 0$; that is, any two different samples of white noise, no matter how close together in time they are taken, are uncorrelated.

The average power P_n of white noise is *infinite* because its bandwidth is infinite. This can be seen by combining Equations (1.19) and (1.42) to yield

$$\boxed{P_n = \int_{-\infty}^{\infty} \frac{N_0}{2}\, df = \infty} \tag{1.44}$$

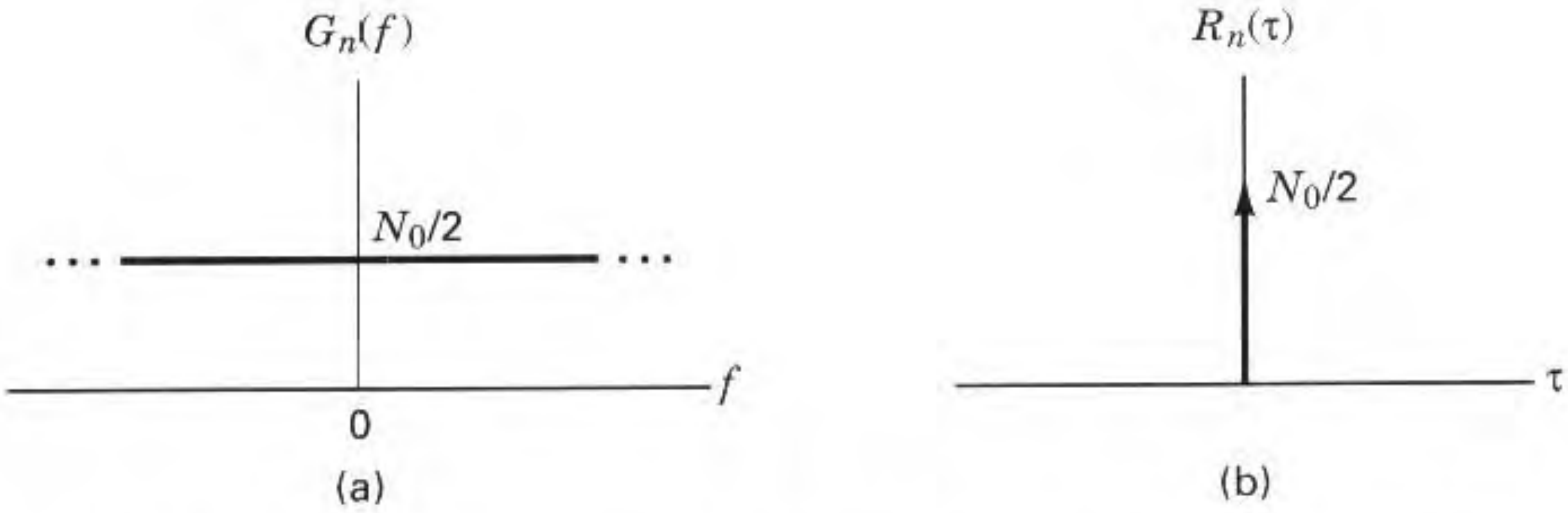


Figure 1.8 (a) Power spectral density of white noise. (b) Autocorrelation function of white noise.

shown in Figure 1.9, can be described either as a time-domain signal, $x(t)$, or by its Fourier transform, $X(f)$. The use of time-domain analysis yields the time-domain output $y(t)$, and in the process, $h(t)$, the characteristic or *impulse response* of the network will be defined. When the input is considered in the frequency domain, we shall define a *frequency transfer function* $H(f)$ for the system, which will determine the frequency-domain output $Y(f)$. The system is assumed to be linear and time invariant. It is also assumed that there is no stored energy in the system at the time the input is applied.

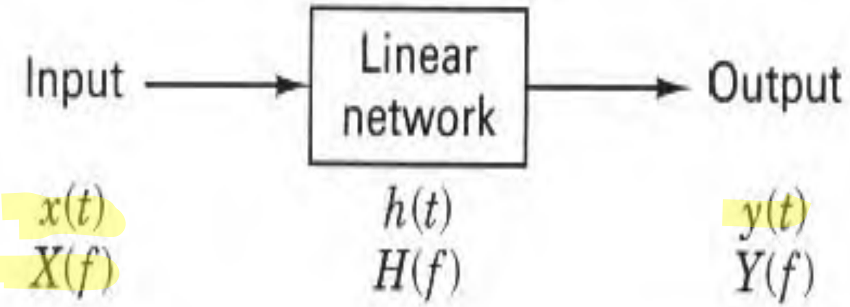


Figure 1.9 Linear system and its key parameters.

1.6.1 Impulse Response

The linear time invariant system or network illustrated in Figure 1.9 is characterized in the time domain by an impulse response $h(t)$, which is the response when the input is equal to a unit impulse $\delta(t)$; that is,

$$\boxed{h(t) = y(t) \qquad \text{when } x(t) = \delta(t)} \tag{1.45}$$

The response of the network to an arbitrary input signal $x(t)$ is found by the convolution of $x(t)$ with $h(t)$, expressed as

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t - \tau)\, d\tau \tag{1.46}$$

$$y(t) = \int_{0}^{\infty} x(t - \tau)\, h(\tau)\, d\tau$$

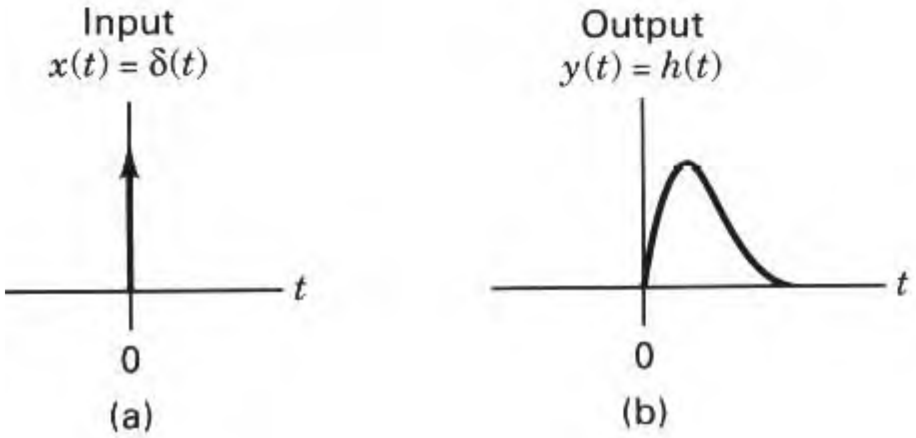
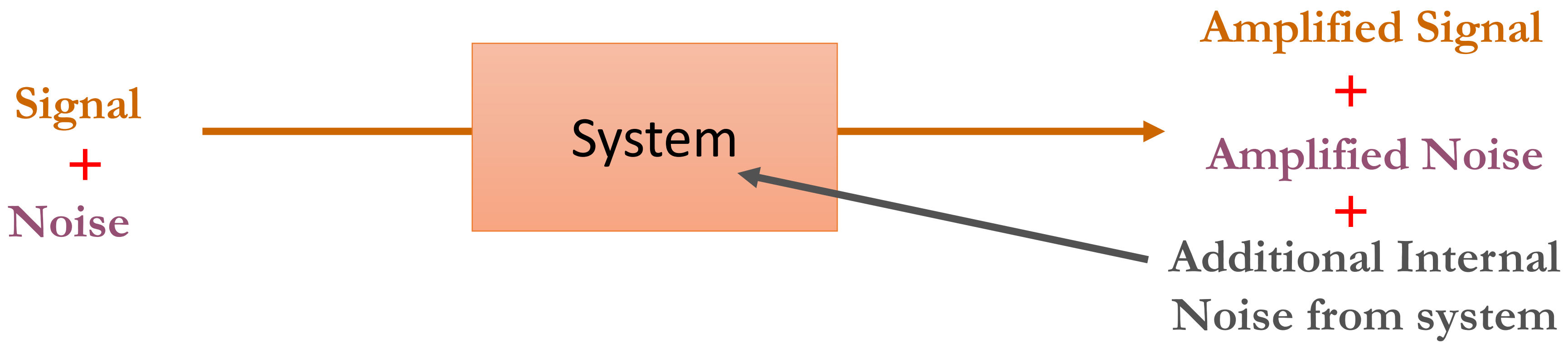


Figure 1.10 (a) Input signal $x(t)$ is a unit impulse function. (b) Output signal $y(t)$ is the system's impulse response $h(t)$.

Noise in Communication Systems



Signal to Noise ratio (SNR)

Q: What is used to " measure" the effect of noise?

The SNR is defined as the average signal power divided by the average noise power at a certain point in the system.

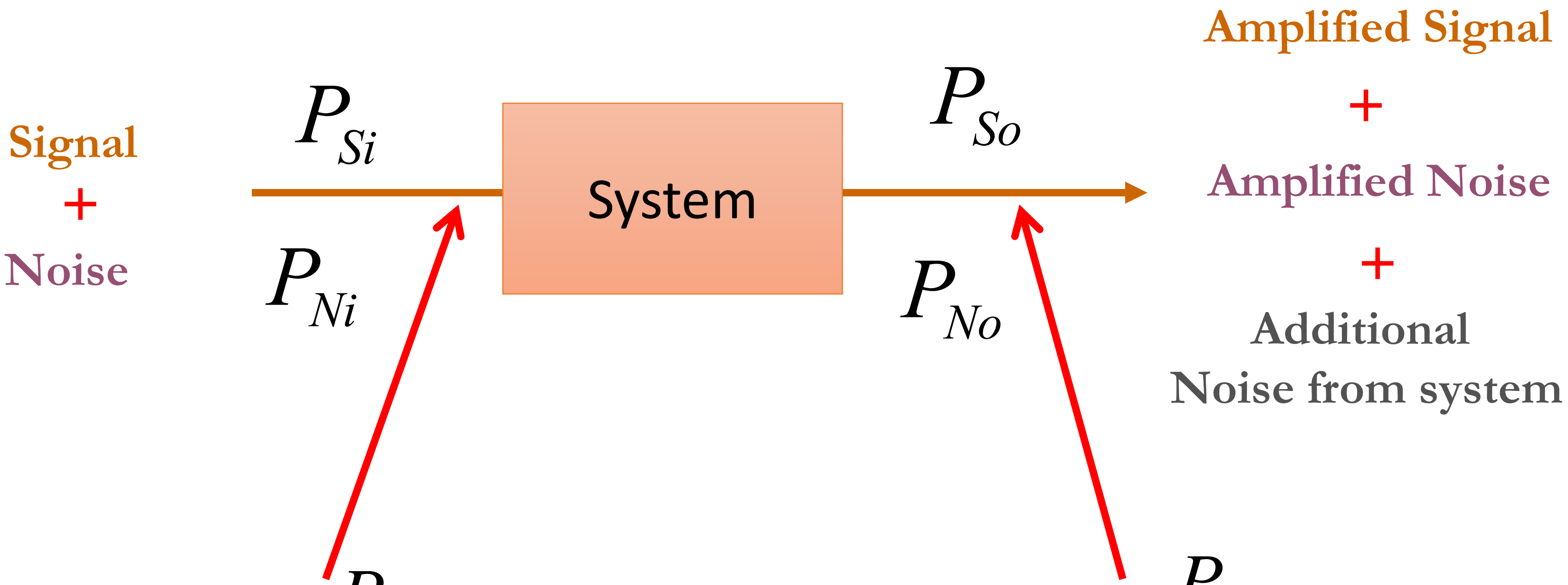
$$SNR = \frac{S}{N} = \frac{average \quad signal \quad power}{average \quad noise \quad power}$$

$$SNR = \frac{P_S}{P_N}$$

$$SNR_{dB} = P_{S\_dBm} - P_{N\_dBm}$$

Signal to Noise ratio (SNR)

Q: What is the System SNR?



$$SNR_i = \frac{P_{Si}}{P_{Ni}}$$

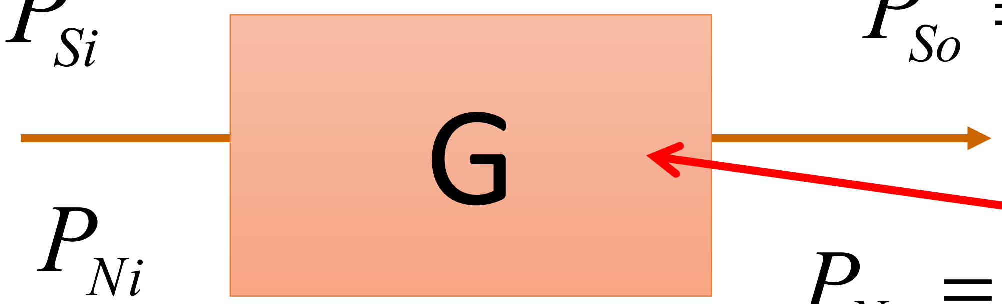
$$SNR_o = \frac{P_{So}}{P_{No}}$$

Signal to Noise ratio (SNR)

Q: What is the System SNR?

$$P_{Si}$$

$$P_{Ni}$$



$$P_{So} = GP_{Si}$$

$$P_{No} = GP_{Ni} + P_A$$

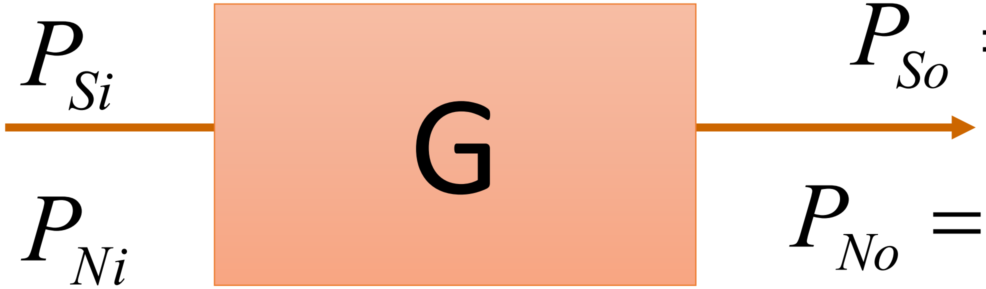
$$SNR_i = \frac{P_{Si}}{P_{Ni}}$$

$$SNR_o = \frac{P_{So}}{P_{No}} = \frac{GP_{Si}}{GP_{Ni} + P_A} < SNR_i$$

Noise Figure

Q: What is the Noise Figure of System?

It is an indication of how much noise the system adds:



$$P_{So} = GP_{Si}$$

$$P_{No} = GP_{Ni} + P_A$$

$$NF = \frac{SNR_i}{SNR_o} \quad = \frac{GP_{Ni} + P_A}{GP_{Ni}} \quad > 1$$

$$NF_{dB} = SNR_{i\_dB} - SNR_{o\_dB} \quad > 0dB$$

Time Averaged Noise Representation

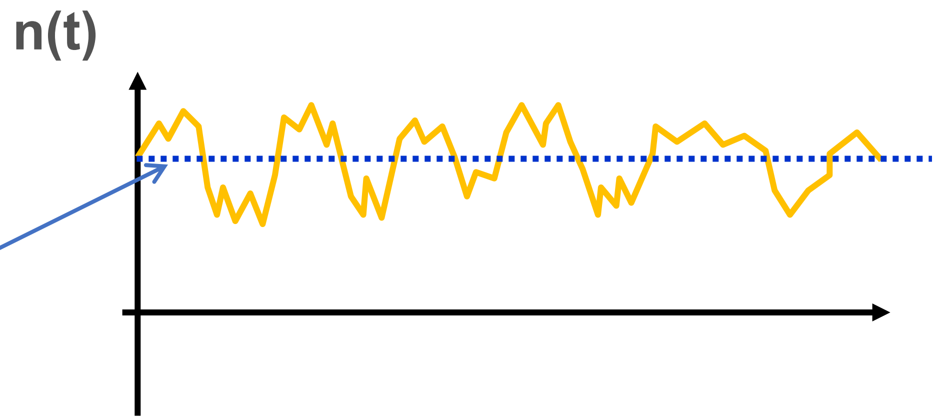
Q: How is Noise represented ?

- Random noise is usually represented in the form of averages as it is difficult to obtain an closed form expression for noise.

Suppose n(t) is noise:

1. Mean value of noise:

$$\overline{n(t)} = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} n(t)dt$$



This is referred to as the DC or average of the noise.

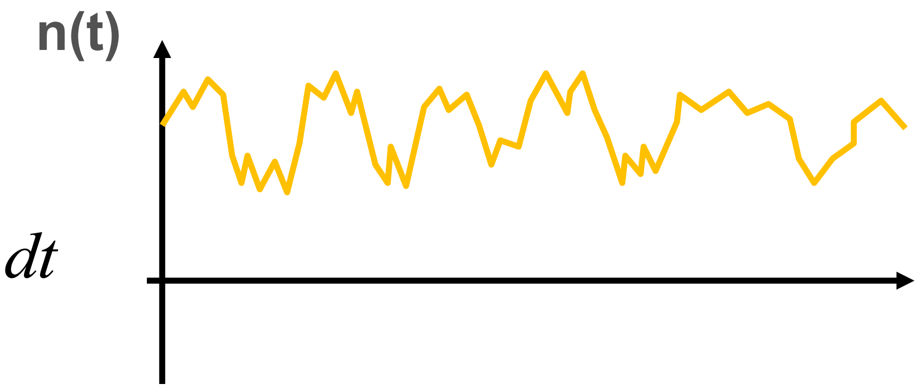
T is finite for a good estimate for the noise.

Time Averaged Noise Representation

Q: How is Noise represented ?

2. Mean Square Value:

$$P_N = \overline{n^2(t)} = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} |n(t)|^2\, dt$$



This is the time averaged power of the noise.

The square root of this is called the root mean square (rms)

The average power can be determined using the noise PSD:

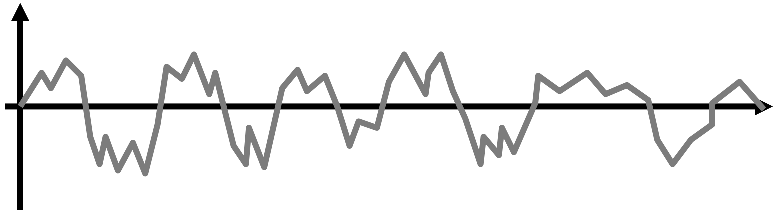
$$P_N = \overline{n^2(t)} = \int_{-\infty}^{\infty} S_n(f)\, df$$

Time Averaged Noise Representation

Q: How is Noise represented ?

3. <u>AC component :</u>

$$\sigma(t) = n(t) - \overline{n(t)}$$



This is AC fluctuating component in the Noise signal.

The average power in the AC component is:

$$P_{AC} = \overline{\sigma^2(t)} = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} |\sigma(t)|^2 \, dt$$

Time Averaged Noise Representation

Q: How is Noise represented ?

So the average power in the noise can be written as:

$$P_N = \overline{n^2(t)} = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} \left|\overline{n(t)} + \sigma(t)\right|^2 dt$$

$$P_N = \overline{n^2(t)} = \lim_{T\to\infty} \underbrace{\frac{1}{T}\int_{-T/2}^{T/2} \left|\overline{n(t)}\right|^2 dt}_{\text{DC power}} + \lim_{T\to\infty} \underbrace{\frac{1}{T}\int_{-T/2}^{T/2} \left|\sigma(t)\right|^2 dt}_{\text{AC power}}$$

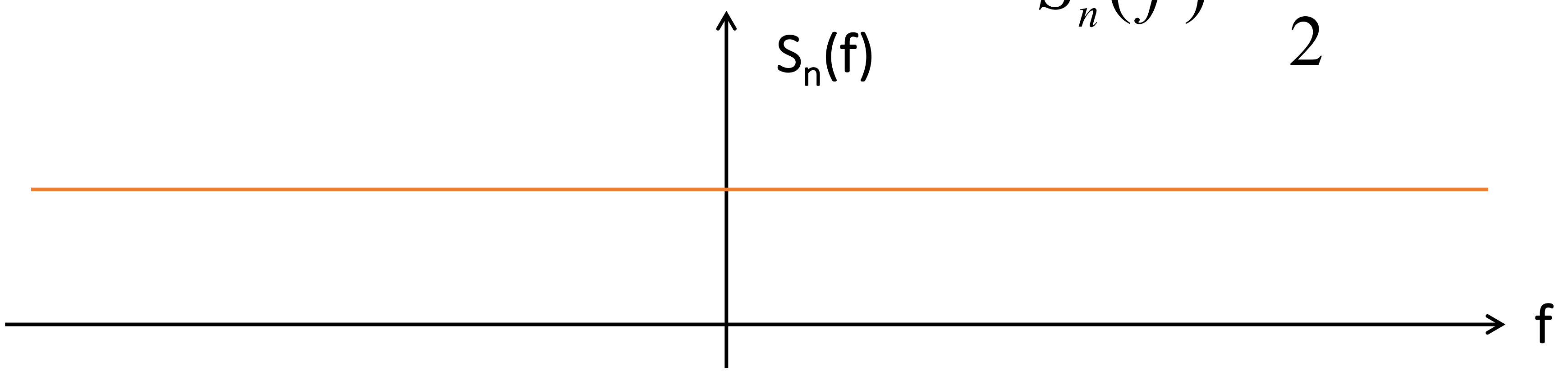
White Noise

Q: What information does the power spectral density of Noise convey?

It shows the frequency components present in the Noise signal.

If all the frequency components have the same power, this is called white noise

$$S_n(f) = \frac{\eta}{2}$$

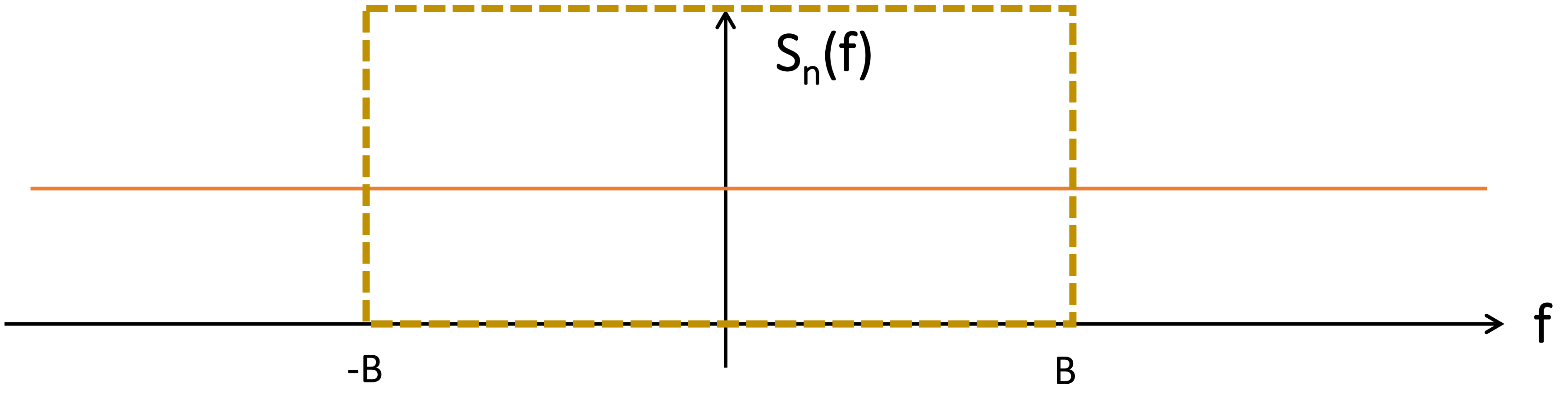


Bandlimited White Noise

Q: What is Bandlimited White Noise?

If the system has a bandwidth of B, then the noise in bandlimited white noise

$$P_N = \int_{-B}^{B} S_n(f)df = \int_{-B}^{B} \frac{\eta}{2} df \quad = \eta B$$

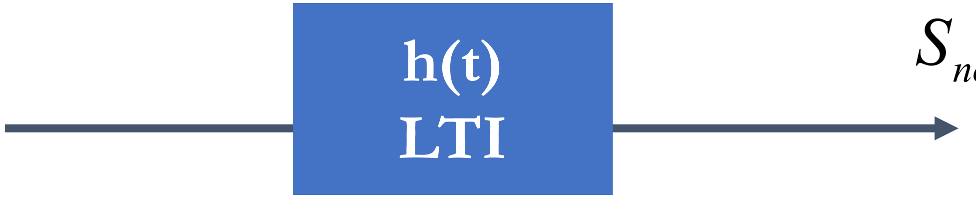


Bandlimited White Noise

Q: How is Bandlimited White Noise transmitted in an LTI system?

Similar to what we have previously:

$$S_{ni}(f) = \frac{\eta}{2}$$



$$S_{no}(f) = \frac{\eta}{2}\left|H(f)\right|^2$$

$$P_{N_{out}} = \int_{-\infty}^{\infty} S_{ni}(f)\left|H(f)\right|^2 df$$

For white Noise:

$$P_{N_{out}} = \frac{\eta}{2}\int_{-\infty}^{\infty} \left|H(f)\right|^2 df$$

Noise Temperature

- Q: What is the Noise Temperature

Since k and B are constant, then the noise power is fully specified by T

$$P_{NT\_available} = k\,\boxed{T}\,B$$

Constant

Constant

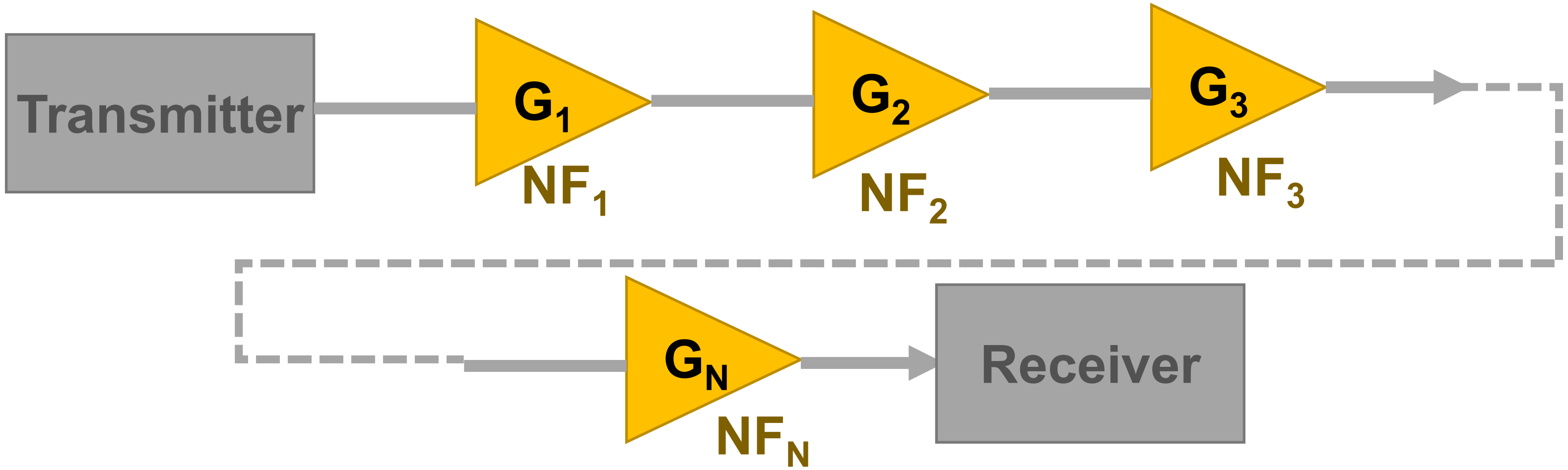
Noise Temperature

Exercise: A given amplifier has a 4dB noise figure and a bandwidth of 500kHz, and an input resistance of 50Ω . Calculate the rms signal input that gives an output signal to noise ratio of unity when the amplifier is connected to a 50Ω resistor at 290K

Noise in Amplifiers

Q: What is the total NF of a cascade of systems? (optical & Satellite com)

For the system shown below the total Noise Factor NF is:



$$NF = NF_1 + \frac{NF_2 - 1}{G_1} + \frac{NF_3 - 1}{G_1 G_2} + \cdots + \frac{NF_N - 1}{\prod_{i=1}^{N-1} G_i}$$

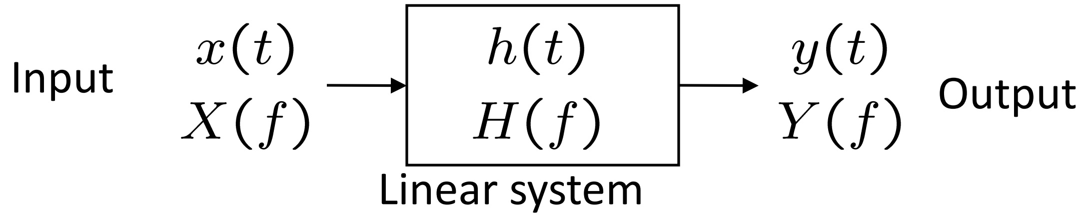
1.6.2.1 Random Processes and Linear Systems

If a random process forms the input to a time-invariant linear system, the output will also be a random process. That is, each sample function of the input process yields a sample function of the output process. The input power spectral density $G_X(f)$ and the output power spectral density $G_Y(f)$ are related as follows:

$$\boxed{G_Y(f) = G_X(f)\,|H(f)|^2} \tag{1.53}$$

Equation (1.53) provides a simple way of finding the power spectral density out of a time-invariant linear system when the input is a random process.

Signal transmission through linear systems



– Deterministic signals: $\boxed{Y(f) = X(f)H(f)}$

– Random signals: $\boxed{G_Y(f) = G_X(f)|H(f)|^2}$

What is required of a network for it to behave like an *ideal* transmission line? The output signal from an ideal transmission line may have some time delay compared with the input, and it may have a different amplitude than the input (just a scale change), but otherwise it must have no distortion—it must have the same shape as the input. Therefore, for ideal distortionless transmission, we can describe the output signal as

$$y(t) = Kx(t - t_0) \tag{1.54}$$

where K and t_0 are constants. Taking the Fourier transform of both sides (see Section A.3.1), we write

$$Y(f) = KX(f)e^{-j2\pi f t_0} \tag{1.55}$$

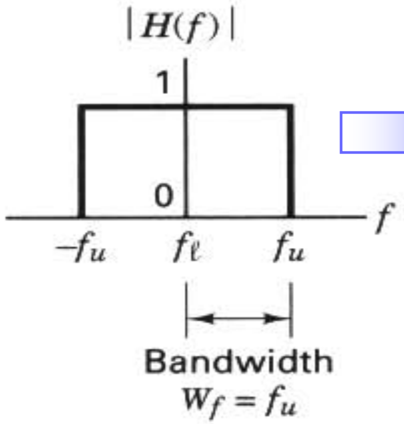
Substituting the expression (1.55) for $Y(f)$ into Equation (1.49), we see that the required system transfer function for distortionless transmission is

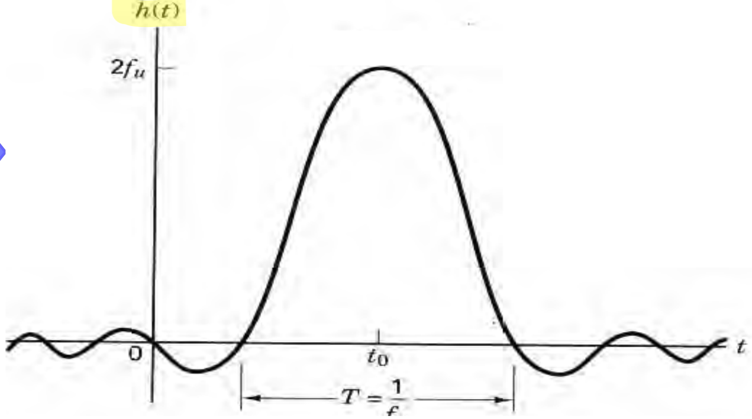
$$H(f) = Ke^{-j2\pi f t_0} \tag{1.56}$$

Ideal distortion less transmission:

All the frequency components of the signal not only arrive with an identical time delay, but also are amplified or attenuated equally.

$y(t) = Kx(t - t_0)$ or $H(f) = Ke^{-j2\pi f t_0}$





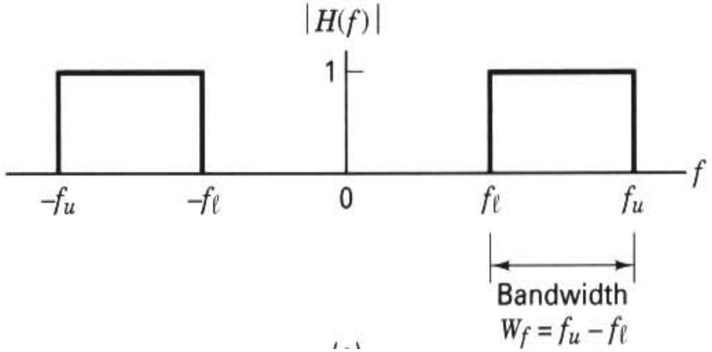
$$H(f) = |H(f)|\, e^{-j\theta(f)}$$

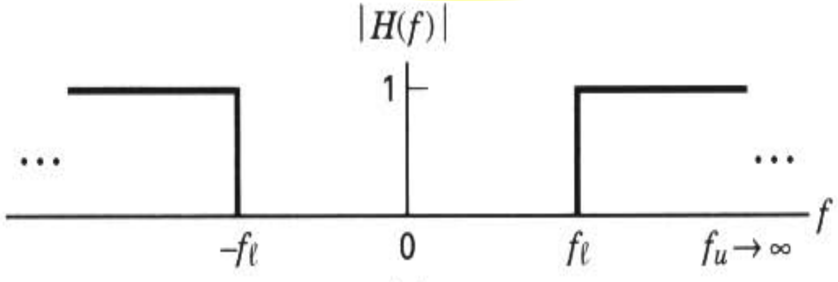
$$|H(f)| = \begin{cases} 1 & \text{for } |f| < f_u \\ 0 & \text{for } |f| \geq f_u \end{cases}$$

$$e^{-j\theta(f)} = e^{-j2\pi f t_0}$$

$$h(t) = \mathcal{F}^{-1}\{H(f)\} = \int_{-\infty}^{\infty} H(f) e^{j2\pi f t}\, df = \int_{-f_u}^{f_u} e^{-j2\pi f t_0} e^{j2\pi f t}\, df$$

$$= \int_{-f_u}^{f_u} e^{j2\pi f(t-t_0)}\, df = 2f_u \frac{\sin 2\pi f_u(t-t_0)}{2\pi f_u(t-t_0)} = 2f_u \operatorname{sinc} 2f_u(t-t_0)$$





(c)

Figure 1.11 Ideal filter transfer function. (a) Ideal bandpass filter. (b) Ideal low-pass filter. (c) Ideal high-pass filter.

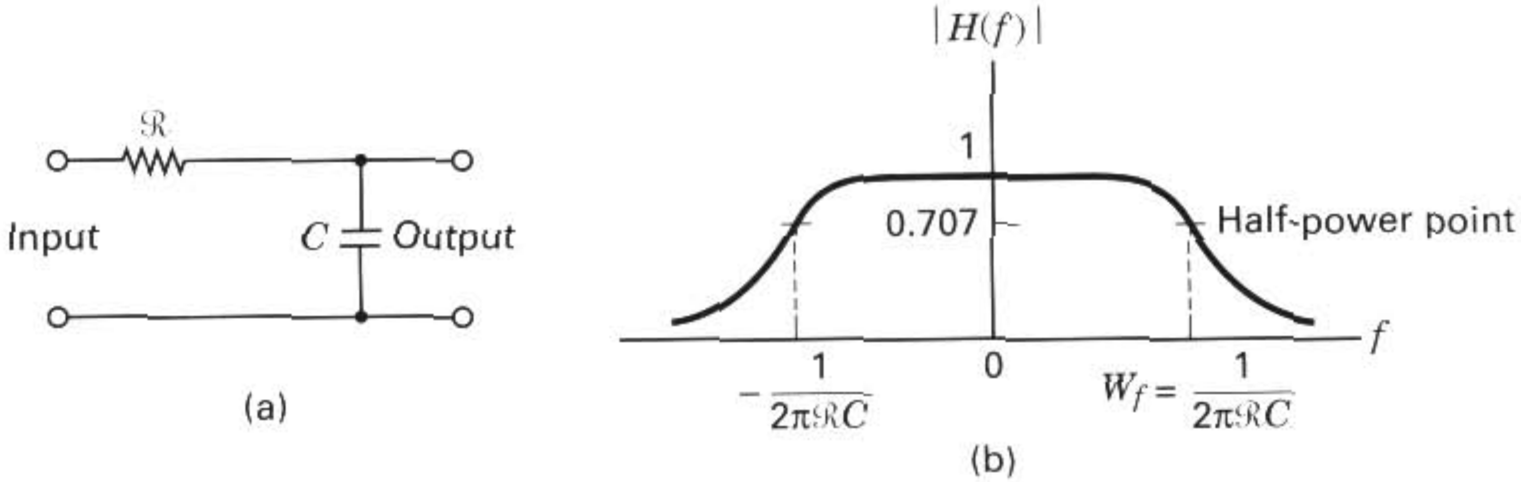
1.6.3.2 Realizable Filters

The very simplest example of a realizable low-pass filter is made up of resistance ($\mathcal{R}$) and capacitance (C), as shown in Figure 1.13a; it is called an $\mathcal{R}C$ *filter*, and its transfer function can be expressed as [7]

$$H(f) = \frac{1}{1 + j2\pi f\mathcal{R}C} = \frac{1}{\sqrt{1 + (2\pi f\mathcal{R}C)^2}}\, e^{-j\theta(f)} \tag{1.63}$$

where $\theta(f) = \tan^{-1} 2\pi f\mathcal{R}C$. The magnitude characteristic $|H(f)|$ and the phase characteristic $\theta(f)$ are plotted in Figures 1.13b and c, respectively. The low-pass filter bandwidth is defined to be its half-power point; this point is the frequency at which the output signal power has fallen to one-half of its peak value, or the frequency at which the magnitude of the output voltage has fallen to $1/\sqrt{2}$ of its peak value.

$$H(f) = \frac{1}{1+j2\pi f\mathcal{R}C} \qquad |H_n(f)| = \frac{1}{\sqrt{1+(f/f_u)^{2n}}}$$



Example 1.2 Effect of an Ideal Filter on White Noise

White noise with power spectral density $G_n(f) = N_0/2$, shown in Figure 1.8a, forms the input to the ideal low-pass filter shown in Figure 1.11b. Find the power spectral density $G_Y(f)$ and the autocorrelation function $R_Y(\tau)$ of the output signal.

Solution

$$G_Y(f) = G_n(f)\,|H(f)|^2$$

$$= \begin{cases} \dfrac{N_0}{2} & \text{for } |f| < f_u \\ 0 & \text{otherwise} \end{cases}$$

The autocorrelation is the inverse Fourier transform of the power spectral density and is given by (see Table A.1)

$$\boxed{\begin{aligned} R_Y(\tau) &= N_0 f_u \frac{\sin 2\pi f_u \tau}{2\pi f_u \tau} \\ &= N_0 f_u \operatorname{sinc} 2 f_u \tau \end{aligned}}$$

Comparing this result with Equation (1.62), we see that $R_Y(\tau)$ has the same shape as the impulse response of the ideal low-pass filter shown in Figure 1.12. In this example the ideal low-pass filter transforms the autocorrelation function of white noise (defined by the delta function) into a sinc function. After filtering, we no longer have white noise. The output noise signal will have zero correlation with shifted copies of itself, only at shifts of $\tau = n/2f_u$, where n is any integer other than zero.

Example 1.3 Effect of an $\mathcal{R}C$ Filter on White Noise

White noise with spectral density $G_n(f) = N_0/2$, shown in Figure 1.8a, forms the input to the $\mathcal{R}C$ filter shown in Figure 1.13a. Find the power spectral density $G_Y(f)$ and the autocorrelation function $R_Y(\tau)$ of the output signal.

Solution

$$\begin{aligned} G_Y(f) &= G_n(f)\,|H(f)|^2 \\ &= \frac{N_0}{2}\,\frac{1}{1 + (2\pi f \mathcal{R}C)^2} \end{aligned}$$

$$R_Y(\tau) = \mathcal{F}^{-1}\{G_Y(f)\}$$

Using Table A.1, we find that the inverse Fourier transform of $G_Y(f)$ is

$$R_Y(\tau) = \frac{N_0}{4\mathcal{R}C}\exp\left(-\frac{|\tau|}{\mathcal{R}C}\right)$$

As might have been predicted, we no longer have white noise after filtering. The $\mathcal{R}C$ filter transforms the input autocorrelation function of white noise (defined by the delta function) into an exponential function. For a narrowband filter (a large $\mathcal{R}C$ product), the output noise will exhibit higher correlation between noise samples of a fixed time shift than will the output noise from a wideband filter.