Conclusions and discussions
Comparisons

Performance of Digital Modulation

- Let us first consider the case of binary signaling with *coherent* detection in an AWGN channel
- For equally likely symbols the minimum probability of error is obtained using a *maximum liklihood receiver*
- The probability of symbol (and equivalently bit) error is

$$P_b = Q\left(\sqrt{\frac{E_b}{N_o}(1-\rho)}\right)$$

P$_b$ = bit err. prob.
P$_s$ = sym. err. prob.

- Where $Q(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}} e^{-\frac{u^2}{2}} du$ is the standard *Q*-function
- E_b/N_o is the *average* energy per bit divided by the one sided power spectral density N_o
- and ρ is correlation coefficient between symbols:

$$\rho = \frac{1}{\sqrt{E_1 E_2}} \int_0^T s_1(t) s_2(t) dt$$

E_1 = energy in symbol 1
E_2 = energy in symbol 2
T = symbol duration

Performance of Binary Digital Modulation

$$P_b = Q\left(\sqrt{\frac{E_b}{N_o}(1-\rho)}\right)$$

- Specific cases:
  - ☐ $\rho = 0$
    - orthogonal modulation (e.g., BFSK, BASK)
  - ☐ $\rho = -1$
    - Antipodal signaling (e.g., BPSK)
    - Best performance for binary modulation
  - ☐ $\rho > 0$ results in worse performance than either case

Non-coherent modulation

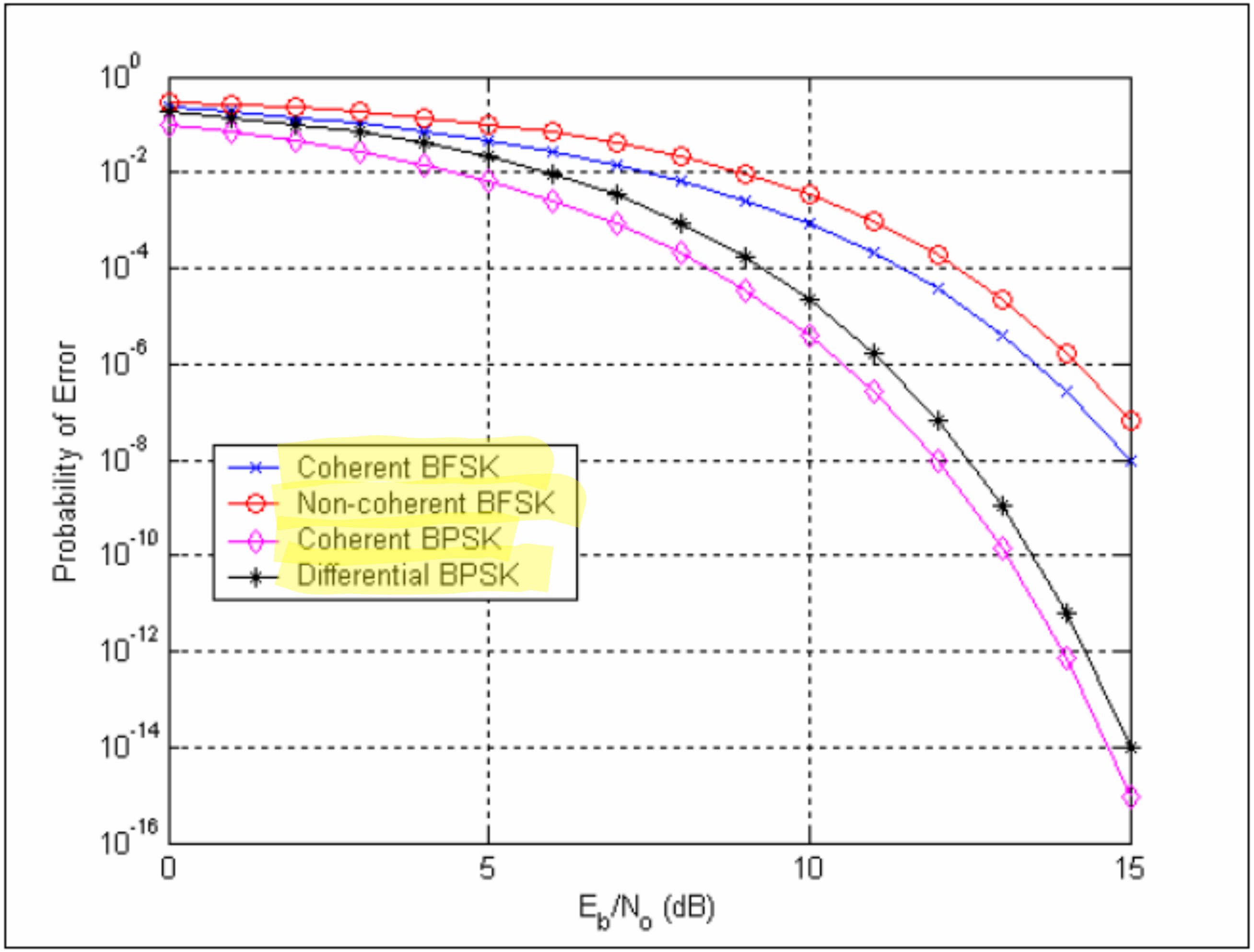
- FSK and ASK do not encode information in the phase of the carrier, thus we do not need to demodulate coherently.
- Most simply, we use an envelope detector
- However, there is a performance penalty
- The bit error rate for non-coherent BASK and BFSK is

$$\boxed{P_b \approx \frac{1}{2} e^{-\frac{1}{2}\frac{E_b}{N_o}}}$$

Compare this to coherent BASK/BFSK which has a performance of

$$P_b = Q\left(\sqrt{\frac{E_b}{N_o}}\right) \approx \frac{1}{\sqrt{2\pi\, E_b / N_o}} e^{-\frac{1}{2}\frac{E_b}{N_o}}$$

Coherent vs. Non-coherent Demodulation



- Approximately 1dB loss for non-coherent demodulation
- Approximately 3dB loss going from PSK to FSK/ASK

HW1

Bandwidth Efficiency

- For rectangular pulses, the spectrum of PSK and ASK is a *sinc* function. The bandwidth of this signal is usually specified as *null-to-null* bandwidth

$$\boxed{W = 2R_s}$$

- For FSK, the minimum spacing between phase synchronous carriers for coherent demodulation is

$$\Delta f_{\min} = \frac{1}{2T_s} = \frac{R_s}{2}$$

  - Since the individual carriers are modulation by square pulses, the null-to-null bandwidth is $R_s + R_s/2 + R_s$ or

$$\boxed{W = 2.5R_s}$$

Note: For non-phase synchronous carriers $W=3R_s$

Bandwidth Efficiency

Modulation Type	**Bandwidth Efficiency (b/s/Hz)**
BPSK	0.5
BASK	0.5
BFSK	0.4

- The bandwidth efficiency of binary modulation is not very good. Thus, we typically use *M*-ary modulation schemes to improve spectral efficiency
- Pulse shaping is also used to control bandwidth

M-ary Modulation

- *M*-PSK → $\log_2(M)$ bits are transmitted per symbol by sending one of *M* carrier phases
- *M*-FSK → $\log_2(M)$ bits are transmitted per symbol by sending one of *M* carrier frequencies
- *QAM* → $\log_2(M)$ bits are transmitted per symbol by sending one of *M* combinations of amplitudes and phases

Performance of *M*-ary Modulation

- *M*-PSK

$$P_s \le 2Q\left(\sqrt{\frac{2E_b}{N_o}\log_2(M)}\,\sin\left(\frac{\pi}{M}\right)\right)$$

Performance degrades with increasing *M*

- *M*-FSK

$$P_s \le (M-1)Q\left(\sqrt{\frac{E_b}{N_o}\log_2(M)}\right)$$

Performance improves with increasing *M*

- *QAM (assuming M is power of 4)*

$$P_s = 1 - \frac{1}{M}\left[\left(\sqrt{M}-2\right)^2\left(1-2Q\left(\sqrt{\frac{6\log_2(M)}{2(M-1)}\frac{E_b}{N_o}}\right)\right)^2 + 4\left(\sqrt{M}-2\right)\left(1-2Q\left(\sqrt{\frac{6\log_2(M)}{2(M-1)}\frac{E_b}{N_o}}\right)\right)\left(1-Q\left(\sqrt{\frac{6\log_2(M)}{2(M-1)}\frac{E_b}{N_o}}\right)\right) + 4\left(1-Q\left(\sqrt{\frac{6\log_2(M)}{2(M-1)}\frac{E_b}{N_o}}\right)\right)^2\right]$$

Performance degrades with increasing *M*

Non-coherent *M*-ary Modulation Performance

- DPSK

$$P_s \le \pi \frac{\cos(\pi/2M)}{\sqrt{\cos(\pi/M)}} Q\left(2\sqrt{\frac{E_b}{N_o}\log_2(M)}\sin\left(\frac{\pi}{M}\right)\right)$$

Performance degrades with increasing *M*

- Non-coherent *M*-FSK

$$P_s = \sum_{k=1}^{M-1} \binom{M-1}{k} \frac{(-1)^{k+1}}{k+1} \exp\left(-\frac{k}{k+1}\frac{E_b}{N_o}\log_2(M)\right)$$

Performance improves with increasing *M*

Bandwidth Efficiency

Modulation Type	**Bandwidth Efficiency (b/s/Hz)**
MPSK/MDPSK/MQAM	$\frac{\log_2(M)}{2}$
BASK	0.5
Coherent MFSK	$\frac{2\log_2(M)}{M+3}$
Non-coherent MFSK	$\frac{\log_2(M)}{M+1}$

- Bandwidth efficiency increases with *M* for PSK and QAM. It decreases with *M* for FSK.
- Pulse shape also heavily influences bandwidth

Power Efficiency

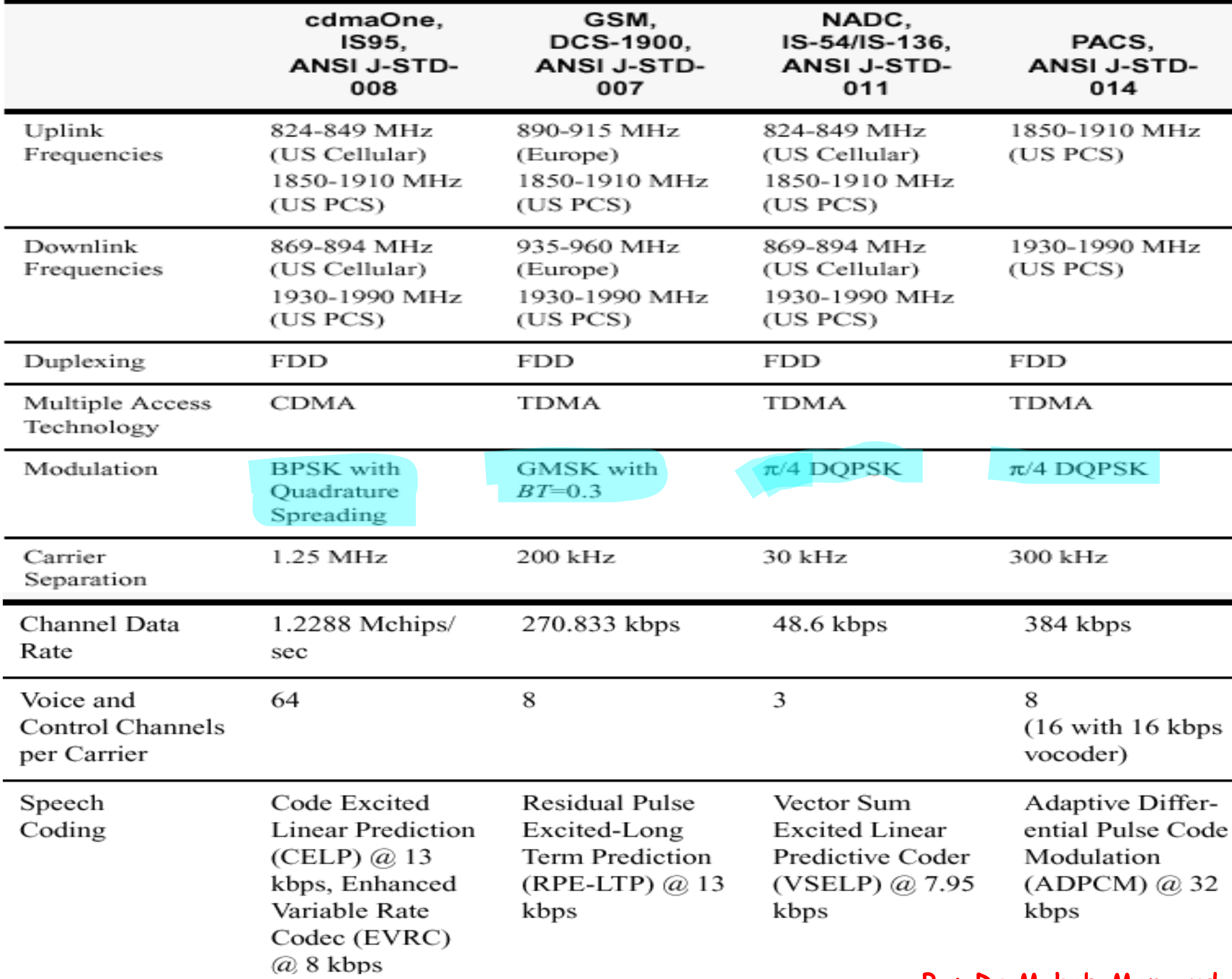
We desire small η_E

Modulation Type	**Power Efficiency (E_b/N_o for $P_e = 10^{-5}$)**
BPSK/QPSK (DBPSK)	9.5dB (10.5dB) ✓
8-PSK (8-DPSK)	13.5dB (16.5dB)
16-PSK (16-DPSK)	18dB (21dB)
32-PSK (32-DPSK)	23dB (26dB)
Coherent BFSK (Non-coherent)	12.5dB (13.5dB)
Coherent 4-FSK (Non-coherent)	10dB (11dB)
Coherent 8-FSK (Non-coherent)	8dB (9dB)
Coherent 16-FSK (Non-coherent)	7dB (8dB)
4-QAM	9.5dB
16-QAM	13dB
64-QAM	17.5dB

- Note: For PSK/QAM we can use Gray coding and thus $P_b = \dfrac{P_s}{\log_2 M}$
- For FSK $P_b = \dfrac{MP_s}{2(M-1)}$
- For PSK/QAM BER increases with *M*
- For FSK BER decreases with *M*
- Non-coherent demodulation suffers 1-3dB penalty for PSK but little penalty for FSK (at low error rates)

Conclusions

- Today we briefly reviewed digital communication systems with an emphasis on modulation schemes
- In general we can increase bandwidth (spectral) efficiency by increasing the order of the modulation scheme (opposite is true for FSK)
- In general the power efficiency is degraded by increasing modulation order (opposite is true for FSK)
- Thus, we have a classic trade-off in communication systems between bandwidth and energy efficiency
  - Note that bandwidth efficiency is not the primary concern for spread spectrum as we will see
- Non-coherent demodulation allows us to trade off complexity for energy efficiency



	cdmaOne, IS95, ANSI J-STD-008	GSM, DCS-1900, ANSI J-STD-007	NADC, IS-54/IS-136, ANSI J-STD-011	PACS, ANSI J-STD-014
Uplink Frequencies	824-849 MHz (US Cellular) 1850-1910 MHz (US PCS)	890-915 MHz (Europe) 1850-1910 MHz (US PCS)	824-849 MHz (US Cellular) 1850-1910 MHz (US PCS)	1850-1910 MHz (US PCS)
Downlink Frequencies	869-894 MHz (US Cellular) 1930-1990 MHz (US PCS)	935-960 MHz (Europe) 1930-1990 MHz (US PCS)	869-894 MHz (US Cellular) 1930-1990 MHz (US PCS)	1930-1990 MHz (US PCS)
Duplexing	FDD	FDD	FDD	FDD
Multiple Access Technology	CDMA	TDMA	TDMA	TDMA
Modulation	BPSK with Quadrature Spreading	GMSK with $BT=0.3$	$\pi/4$ DQPSK	$\pi/4$ DQPSK
Carrier Separation	1.25 MHz	200 kHz	30 kHz	300 kHz
Channel Data Rate	1.2288 Mchips/sec	270.833 kbps	48.6 kbps	384 kbps
Voice and Control Channels per Carrier	64	8	3	8 (16 with 16 kbps vocoder)
Speech Coding	Code Excited Linear Prediction (CELP) @ 13 kbps, Enhanced Variable Rate Codec (EVRC) @ 8 kbps	Residual Pulse Excited-Long Term Prediction (RPE-LTP) @ 13 kbps	Vector Sum Excited Linear Predictive Coder (VSELP) @ 7.95 kbps	Adaptive Differential Pulse Code Modulation (ADPCM) @ 32 kbps

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