Noise in Communication Systems

The term *noise* refers to *unwanted* electrical signals that are always present in electrical systems. The presence of noise superimposed on a signal tends to obscure or mask the signal; it limits the receiver's ability to make correct symbol decisions, and thereby limits the rate of information transmission. Noise arises from a variety of sources, both man made and natural. The *man-made noise* includes such sources as spark-plug ignition noise, switching transients, and other radiating electromagnetic signals. *Natural noise* includes such elements as the atmosphere, the sun, and other galactic sources.

Good engineering design can eliminate much of the noise or its undesirable effect through filtering, shielding, the choice of modulation, and the selection of an optimum receiver site. For example, sensitive radio astronomy measurements are typically located at remote desert locations, far from man-made noise sources. However, there is one natural source of noise, called *thermal* or *Johnson noise*, that cannot be eliminated. Thermal noise [4, 5] is caused by the thermal motion of electrons in all dissipative components—resistors, wires, and so on. The same electrons that are responsible for electrical conduction are also responsible for thermal noise.

We can describe thermal noise as a zero-mean Gaussian random process. A Gaussian process $n(t)$ is a random function whose value n at any arbitrary time t is statistically characterized by the Gaussian probability density function

$$p(n) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{n}{\sigma}\right)^2\right] \tag{1.40}$$

where σ^2 is the variance of n. The *normalized* or *standardized Gaussian density function* of a zero-mean process is obtained by assuming that $\sigma = 1$. This normalized pdf is shown sketched in Figure 1.7.

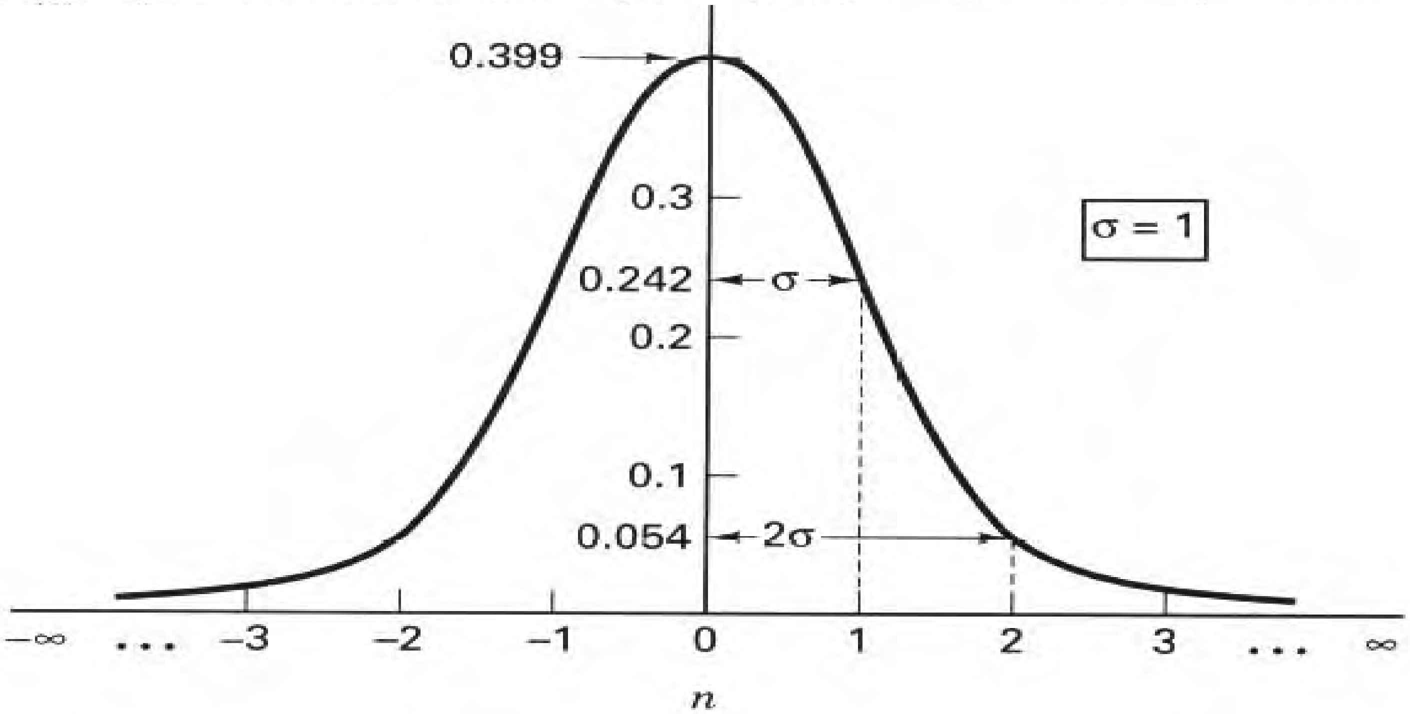


Figure 1.7 Normalized ($\sigma = 1$) Gaussian probability density function.

1.5.5.1 White Noise

The primary spectral characteristic of thermal noise is that its power spectral density is *the same* for all frequencies of interest in most communication systems; in other words, a thermal noise source emanates an equal amount of noise power per unit bandwidth at all frequencies—from dc to about 10^{12} Hz. Therefore, a simple model for thermal noise assumes that its power spectral density $G_n(f)$ is flat for all frequencies, as shown in Figure 1.8a, and is denoted as

$$G_n(f) = \frac{N_0}{2} \quad \text{watts/hertz} \tag{1.42}$$

where the factor of 2 is included to indicate that $G_n(f)$ is a *two-sided* power spectral density.

The adjective "white" is used in the same sense as it is with white light, which contains equal amounts of all frequencies within the visible band of electromagnetic radiation.

The autocorrelation function of white noise is given by the inverse Fourier transform of the noise power spectral density (see Table A.1), denoted as follows:

$$R_n(\tau) = \mathcal{F}^{-1}\{G_n(f)\} = \frac{N_0}{2}\,\delta(\tau) \tag{1.43}$$

Thus the autocorrelation of white noise is a delta function weighted by the factor $N_0/2$ and occurring at $\tau = 0$, as seen in Figure 1.8b. Note that $R_n(\tau)$ is zero for $\tau \neq 0$; that is, any two different samples of white noise, no matter how close together in time they are taken, are uncorrelated.

The average power P_n of white noise is *infinite* because its bandwidth is infinite. This can be seen by combining Equations (1.19) and (1.42) to yield

$$P_n = \int_{-\infty}^{\infty} \frac{N_0}{2}\,df = \infty \tag{1.44}$$

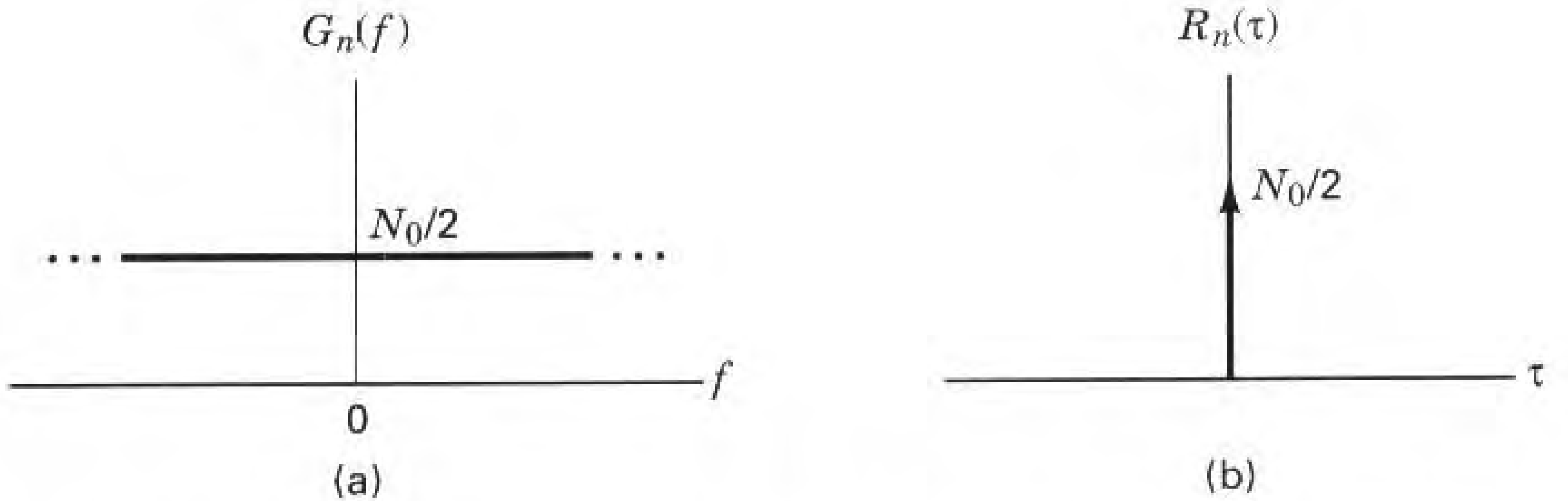
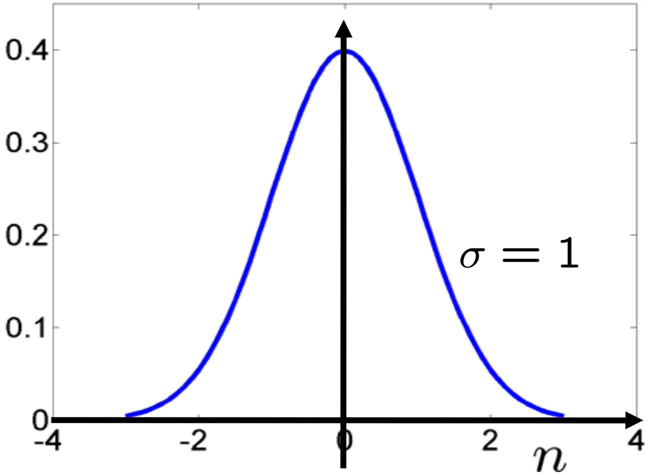


Figure 1.8 (a) Power spectral density of white noise. (b) Autocorrelation function of white noise.

Summary : Noise in communication systems

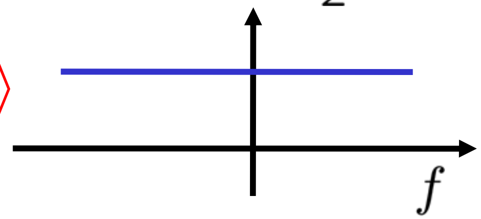
- Thermal noise is described by a zero-mean Gaussian random process, n(t).
- Its PSD is flat, hence, it is called white noise.

$$p(n) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{n^2}{2\sigma^2}\right]$$



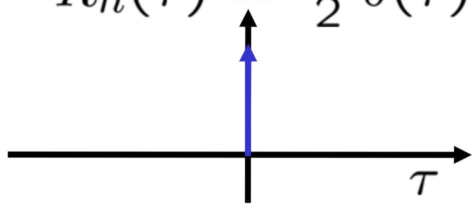
Probability density function

$$G_n(f) = \frac{N_0}{2} \text{ [w/Hz]}$$



Power spectral density

$$R_n(\tau) = \frac{N_0}{2}\delta(\tau)$$



Autocorrelation function

shown in Figure 1.9, can be described either as a time-domain signal, $x(t)$, or by its Fourier transform, $X(f)$. The use of time-domain analysis yields the time-domain output $y(t)$, and in the process, $h(t)$, the characteristic or *impulse response* of the network will be defined. When the input is considered in the frequency domain, we shall define a *frequency transfer function* $H(f)$ for the system, which will determine the frequency-domain output $Y(f)$. The system is assumed to be linear and time invariant. It is also assumed that there is no stored energy in the system at the time the input is applied.

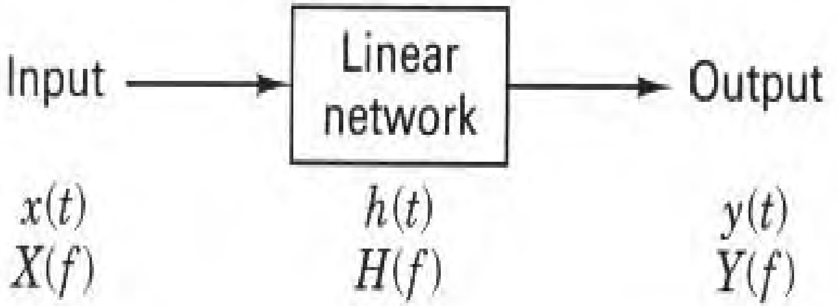


Figure 1.9 Linear system and its key parameters.

1.6.1 Impulse Response

The linear time invariant system or network illustrated in Figure 1.9 is characterized in the time domain by an impulse response $h(t)$, which is the response when the input is equal to a unit impulse $\delta(t)$; that is,

$$h(t) = y(t) \qquad \text{when } x(t) = \delta(t) \tag{1.45}$$

The response of the network to an arbitrary input signal $x(t)$ is found by the convolution of $x(t)$ with $h(t)$, expressed as

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t - \tau)\, d\tau \tag{1.46}$$

$$y(t) = \int_{0}^{\infty} x(t - \tau)\, h(\tau)\, d\tau$$

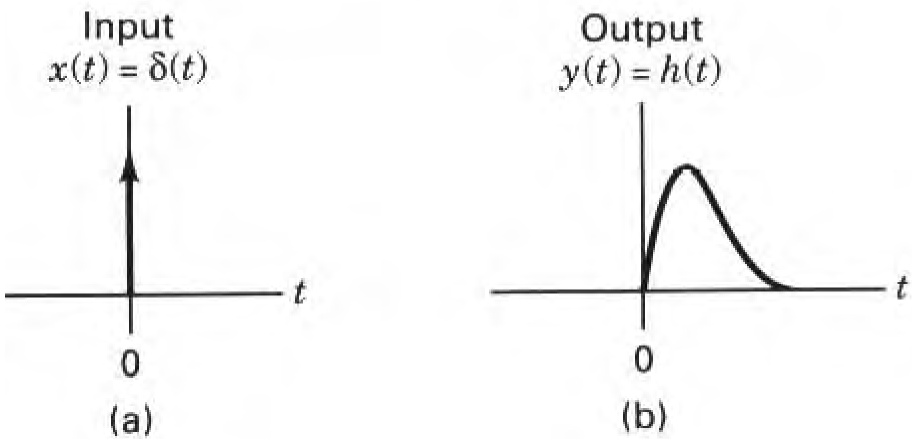


Figure 1.10 (a) Input signal $x(t)$ is a unit impulse function. (b) Output signal $y(t)$ is the system's impulse response $h(t)$.

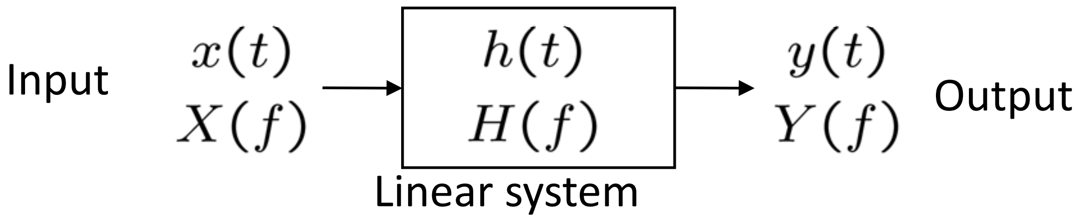
1.6.2.1 Random Processes and Linear Systems

If a random process forms the input to a time-invariant linear system, the output will also be a random process. That is, each sample function of the input process yields a sample function of the output process. The input power spectral density $G_X(f)$ and the output power spectral density $G_Y(f)$ are related as follows:

$$G_Y(f) = G_X(f)\,|H(f)|^2 \tag{1.53}$$

Equation (1.53) provides a simple way of finding the power spectral density out of a time-invariant linear system when the input is a random process.

Signal transmission through linear systems



– Deterministic signals: $Y(f) = X(f)H(f)$

– Random signals: $G_Y(f) = G_X(f)|H(f)|^2$

What is required of a network for it to behave like an *ideal* transmission line? The output signal from an ideal transmission line may have some time delay compared with the input, and it may have a different amplitude than the input (just a scale change), but otherwise it must have no distortion—it must have the same shape as the input. Therefore, for ideal distortionless transmission, we can describe the output signal as

$$y(t) = Kx(t - t_0) \tag{1.54}$$

where K and t_0 are constants. Taking the Fourier transform of both sides (see Section A.3.1), we write

$$Y(f) = KX(f)e^{-j2\pi f t_0} \tag{1.55}$$

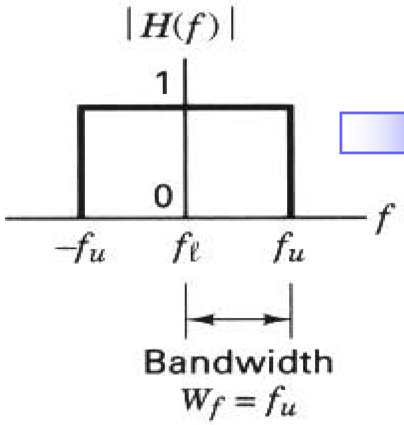
Substituting the expression (1.55) for $Y(f)$ into Equation (1.49), we see that the required system transfer function for distortionless transmission is

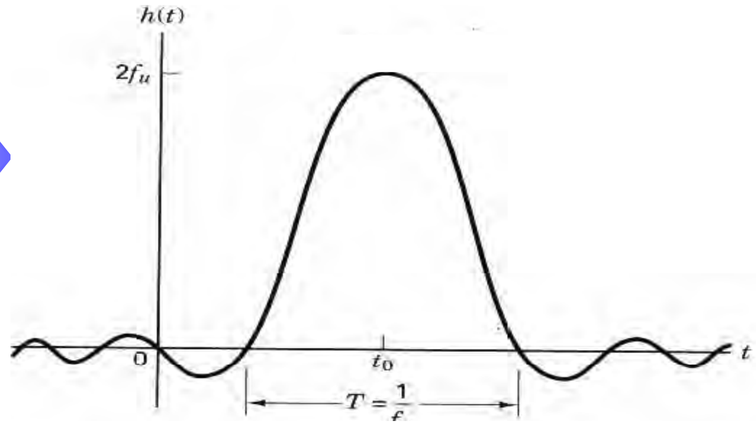
$$H(f) = Ke^{-j2\pi f t_0} \tag{1.56}$$

<u>Ideal distortion less transmission</u>:

All the frequency components of the signal not only arrive with an identical time delay, but also are amplified or attenuated equally.

$$y(t) = Kx(t - t_0) \text{ or } H(f) = Ke^{-j2\pi f t_0}$$





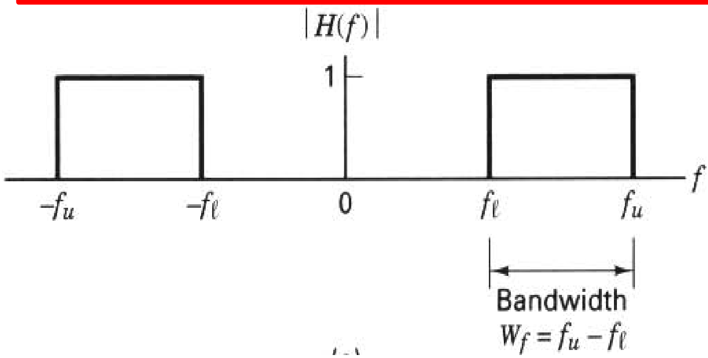
$$H(f) = |H(f)|\, e^{-j\theta(f)}$$

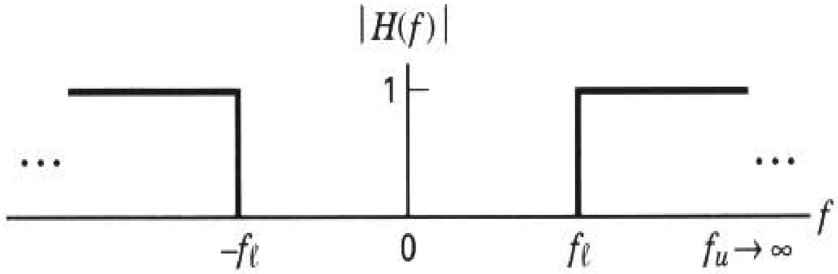
$$|H(f)| = \begin{cases} 1 & \text{for } |f| < f_u \\ 0 & \text{for } |f| \geq f_u \end{cases}$$

$$e^{-j\theta(f)} = e^{-j2\pi f t_0}$$

$$h(t) = \mathcal{F}^{-1}\{H(f)\} = \int_{-\infty}^{\infty} H(f) e^{j2\pi f t}\, df = \int_{-f_u}^{f_u} e^{-j2\pi f t_0} e^{j2\pi f t}\, df$$

$$= \int_{-f_u}^{f_u} e^{j2\pi f(t - t_0)}\, df = 2f_u \frac{\sin 2\pi f_u(t - t_0)}{2\pi f_u(t - t_0)} = 2f_u \operatorname{sinc} 2f_u(t - t_0)$$





(c)

Figure 1.11 Ideal filter transfer function. (a) Ideal bandpass filter. (b) Ideal low-pass filter. (c) Ideal high-pass filter.

1.6.3.2 Realizable Filters

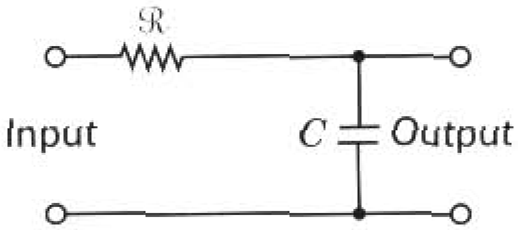
The very simplest example of a realizable low-pass filter is made up of resistance ($\mathcal{R}$) and capacitance (C), as shown in Figure 1.13a; it is called an $\mathcal{R}C$ *filter*, and its transfer function can be expressed as [7]

$$H(f) = \frac{1}{1 + j2\pi f\mathcal{R}C} = \frac{1}{\sqrt{1 + (2\pi f\mathcal{R}C)^2}} e^{-j\theta(f)} \tag{1.63}$$

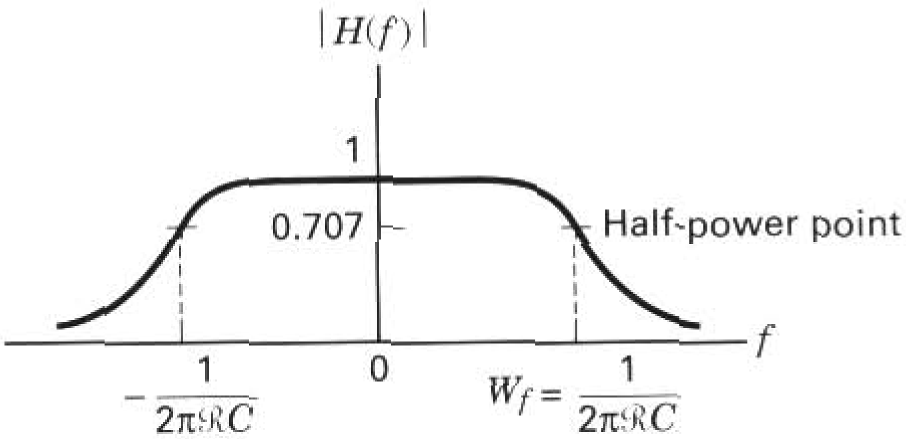
where $\theta(f) = \tan^{-1} 2\pi f\mathcal{R}C$. The magnitude characteristic $|H(f)|$ and the phase characteristic $\theta(f)$ are plotted in Figures 1.13b and c, respectively. The low-pass filter bandwidth is defined to be its half-power point; this point is the frequency at which the output signal power has fallen to one-half of its peak value, or the frequency at which the magnitude of the output voltage has fallen to $1/\sqrt{2}$ of its peak value.

$$H(f) = \frac{1}{1+j2\pi f\mathcal{R}C}$$

$$|H_n(f)| = \frac{1}{\sqrt{1+(f/f_u)^{2n}}}$$



(a)



(b)

Example 1.2 Effect of an Ideal Filter on White Noise

White noise with power spectral density $G_n(f) = N_0/2$, shown in Figure 1.8a, forms the input to the ideal low-pass filter shown in Figure 1.11b. Find the power spectral density $G_Y(f)$ and the autocorrelation function $R_Y(\tau)$ of the output signal.

Solution

$$G_Y(f) = G_n(f)\,|H(f)|^2$$

$$= \begin{cases} \dfrac{N_0}{2} & \text{for } |f| < f_u \\ 0 & \text{otherwise} \end{cases}$$

The autocorrelation is the inverse Fourier transform of the power spectral density and is given by (see Table A.1)

$$R_Y(\tau) = N_0 f_u \frac{\sin 2\pi f_u \tau}{2\pi f_u \tau}$$

$$= N_0 f_u \operatorname{sinc} 2 f_u \tau$$

Comparing this result with Equation (1.62), we see that $R_Y(\tau)$ has the same shape as the impulse response of the ideal low-pass filter shown in Figure 1.12. In this example the ideal low-pass filter transforms the autocorrelation function of white noise (defined by the delta function) into a sinc function. After filtering, we no longer have white noise. The output noise signal will have zero correlation with shifted copies of itself, only at shifts of $\tau = n/2f_u$, where n is any integer other than zero.

Example 1.3 Effect of an $\mathcal{R}C$ Filter on White Noise

White noise with spectral density $G_n(f) = N_0/2$, shown in Figure 1.8a, forms the input to the $\mathcal{R}C$ filter shown in Figure 1.13a. Find the power spectral density $G_Y(f)$ and the autocorrelation function $R_Y(\tau)$ of the output signal.

Solution

$$G_Y(f) = G_n(f)\,|H(f)|^2$$

$$= \frac{N_0}{2}\,\frac{1}{1 + (2\pi f \mathcal{R}C)^2}$$

$$R_Y(\tau) = \mathcal{F}^{-1}\{G_Y(f)\}$$

Using Table A.1, we find that the inverse Fourier transform of $G_Y(f)$ is

$$R_Y(\tau) = \frac{N_0}{4\mathcal{R}C}\exp\left(-\frac{|\tau|}{\mathcal{R}C}\right)$$

As might have been predicted, we no longer have white noise after filtering. The $\mathcal{R}C$ filter transforms the input autocorrelation function of white noise (defined by the delta function) into an exponential function. For a narrowband filter (a large $\mathcal{R}C$ product), the output noise will exhibit higher correlation between noise samples of a fixed time shift than will the output noise from a wideband filter.