Digital Modulation Techniques

BPSK

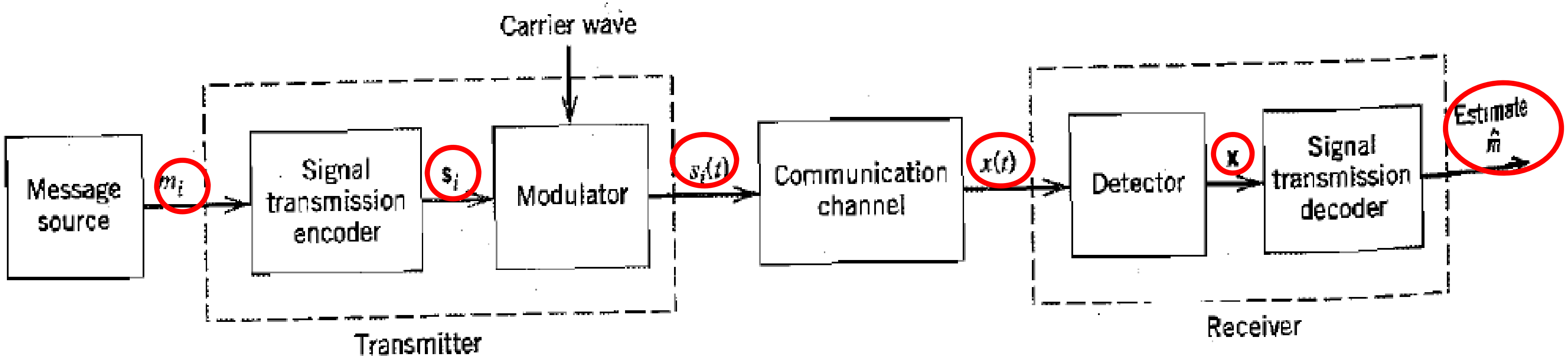


FIGURE 6.2 Functional model of passband data transmission system.

Passband Transmission Model

In a functional sense, we may model a passband data transmission system as shown in Figure 6.2. First, there is assumed to exist a *message source* that emits one *symbol* every T seconds, with the symbols belonging to an alphabet of M symbols, which we denote by $m_1, m_2, \ldots, m_M$. The *a priori probabilities* $P(m_1), P(m_2), \ldots, P(m_M)$ specify the message source output. When the M symbols of the alphabet are *equally likely*, we write

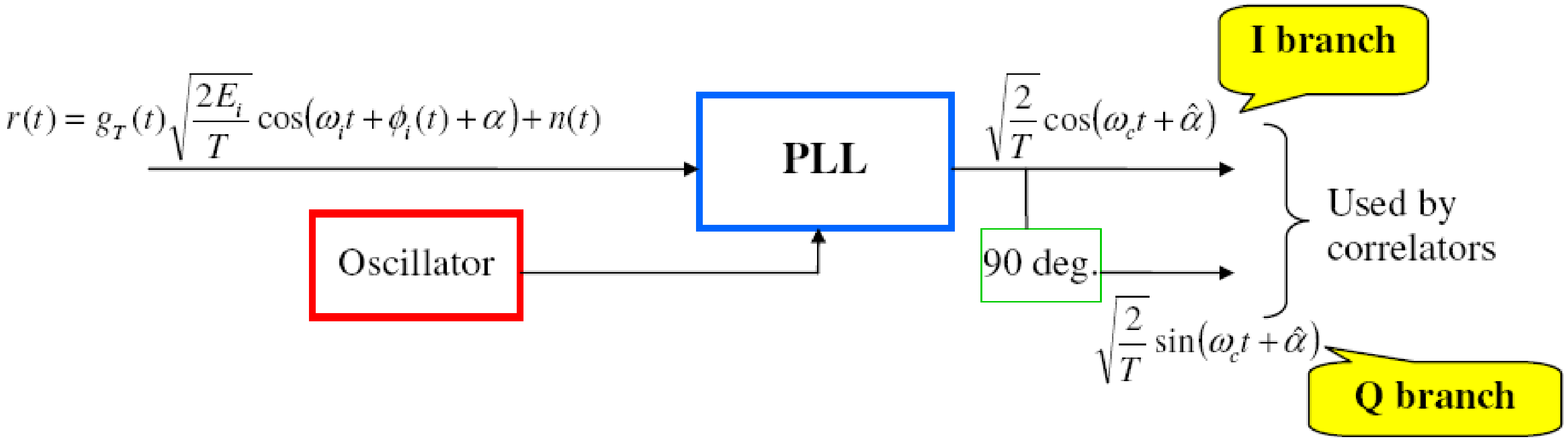
$$\begin{aligned} p_i &= P(m_i) \\ &= \frac{1}{M} \quad \text{for all } i \end{aligned} \tag{6.6}$$

Coherent detection

- Coherent detection
  - requires carrier phase recovery at the receiver and hence, circuits to perform phase estimation.
  - Sources of carrier-phase mismatch at the receiver:
    - Propagation delay causes carrier-phase offset in the received signal.
    - The oscillators at the receiver which generate the carrier signal, are not usually phased locked to the transmitted carrier.

Coherent detection ..

- Circuits such as Phase-Locked-Loop (PLL) are implemented at the receiver for carrier phase estimation ($\alpha \approx \hat{\alpha}$).



BPSK

BINARY PHASE-SHIFT KEYING

In a coherent binary PSK system, the pair of signals $s_1(t)$ and $s_2(t)$ used to represent binary symbols 1 and 0, respectively, is defined by

$$s_1(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t) \tag{6.8}$$

$$s_2(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t + \pi) = -\sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t) \tag{6.9}$$

where $0 \leq t \leq T_b$, and E_b is the *transmitted signal energy per bit*. To ensure that each transmitted bit contains an integral number of cycles of the carrier wave, the carrier frequency f_c is chosen equal to n_c/T_b for some fixed integer n_c. A pair of sinusoidal waves that differ only in a relative phase-shift of 180 degrees, as defined in Eqautions (6.8) and (6.9), are referred to as *antipodal signals*.

From this pair of equations it is clear that, in the case of binary PSK, there is only one basis function of unit energy, namely,

$$\phi_1(t) = \sqrt{\frac{2}{T_b}} \cos(2\pi f_c t), \qquad 0 \leq t < T_b \tag{6.10}$$

Then we may express the transmitted signals $s_1(t)$ and $s_2(t)$ in terms of $\phi_1(t)$ as follows:

$$s_1(t) = \sqrt{E_b}\phi_1(t), \qquad 0 \le t < T_b \tag{6.11}$$

and

$$s_2(t) = -\sqrt{E_b}\phi_1(t), \qquad 0 \le t < T_b \tag{6.12}$$

$$s_{11} = \int_0^{T_b} s_1(t)\phi_1(t)\, dt = +\sqrt{E_b}$$

$$s_{21} = \int_0^{T_b} s_2(t)\phi_1(t)\, dt = -\sqrt{E_b}$$

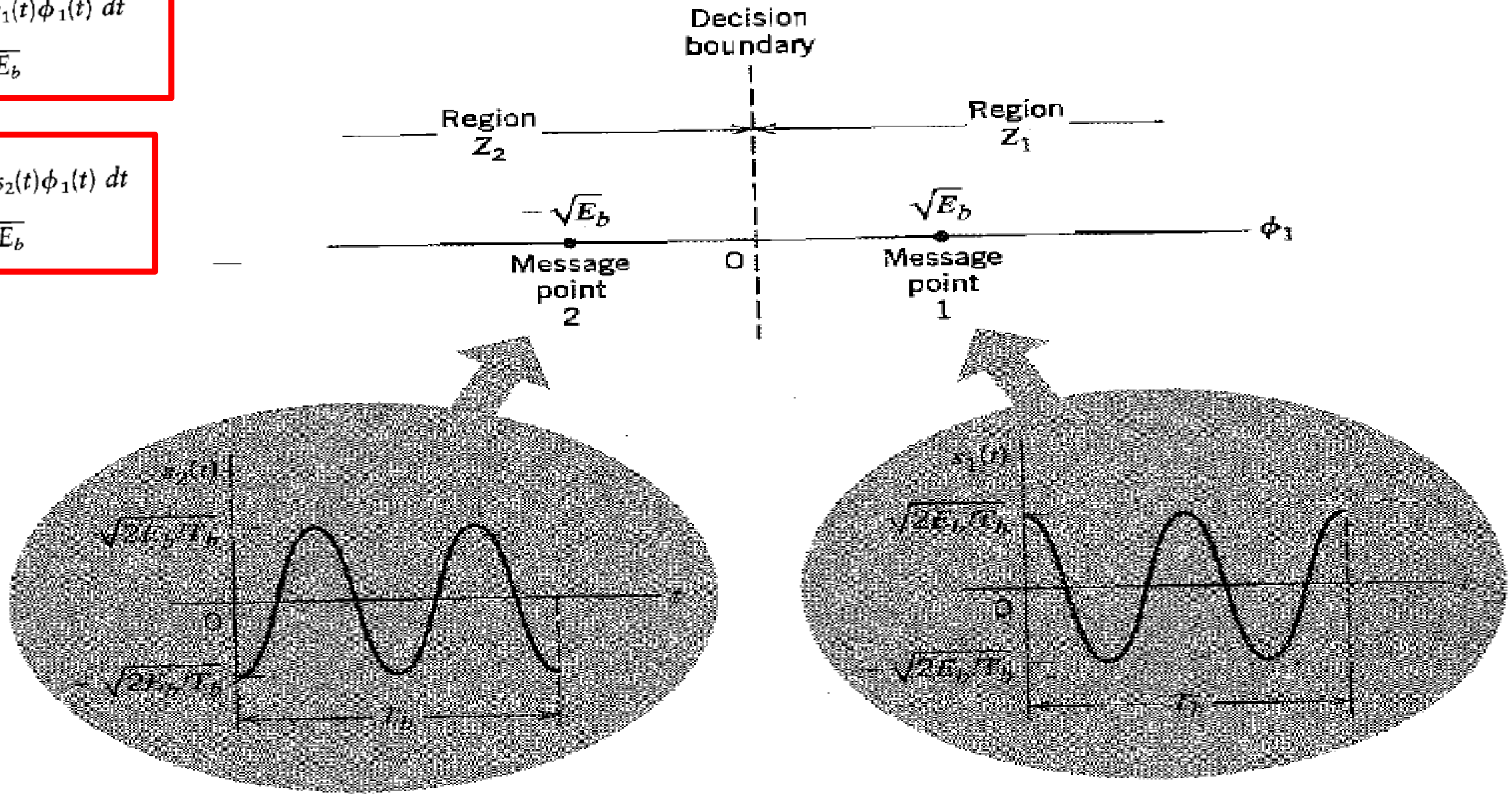


FIGURE 6.3 Signal-space diagram for coherent binary PSK system. The waveforms depicting the transmitted signals $s_1(t)$ and $s_2(t)$, displayed in the inserts, assume $n_c = 2$.

Error Probability of Binary PSK

To realize a *rule for making a decision* in favor of symbol 1 or symbol 0, we apply Equation (5.59) of Chapter 5. Specifically, we partition the signal space of Figure 6.3 into two regions:

- The set of points closest to message point 1 at $+\sqrt{E_b}$.
- The set of points closest to message point 2 at $-\sqrt{E_b}$.

To calculate the probability of making an error of the first kind, we note from Figure 6.3 that the decision region associated with symbol 1 or signal $s_1(t)$ is described by

$$Z_1: 0 < x_1 < \infty$$

where the observable element x_1 is related to the received signal $x(t)$ by

$$x_1 = \int_0^{T_b} x(t)\phi_1(t)\, dt \qquad (6.15)$$

The conditional probability density function of random variable X_1, given that symbol 0 [i.e., signal $s_2(t)$] was transmitted, is defined by

$$\begin{aligned} f_{X_1}(x_1|0) &= \frac{1}{\sqrt{\pi N_0}} \exp\left[-\frac{1}{N_0}(x_1 - s_{21})^2\right] \\ &= \frac{1}{\sqrt{\pi N_0}} \exp\left[-\frac{1}{N_0}(x_1 + \sqrt{E_b})^2\right] \end{aligned} \tag{6.16}$$

The conditional probability of the receiver deciding in favor of symbol 1, given that symbol 0 was transmitted, is therefore

$$\begin{aligned} p_{10} &= \int_0^\infty f_{X_1}(x_1|0)\, dx_1 \\ &= \frac{1}{\sqrt{\pi N_0}} \int_0^\infty \exp\left[-\frac{1}{N_0}(x_1 + \sqrt{E_b})^2\right] dx_1 \end{aligned} \tag{6.17}$$

Putting

$$z = \frac{1}{\sqrt{N_0}}(x_1 + \sqrt{E_b}) \tag{6.18}$$

and changing the variable of integration from x_1 to z, we may rewrite Equation (6.17) in the compact form

$$\begin{aligned} p_{10} &= \frac{1}{\sqrt{\pi}} \int_{\sqrt{E_b/N_0}}^{\infty} \exp(-z^2)\, dz \\ &= \frac{1}{2} \operatorname{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right) \end{aligned} \tag{6.19}$$

where erfc(·) is the complementary error function.

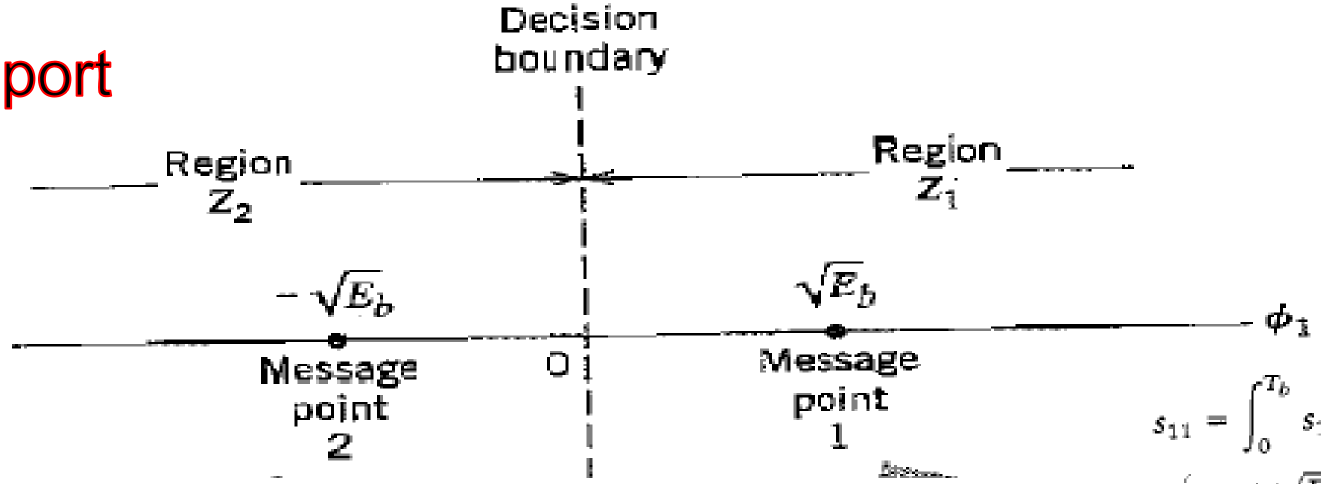
Consider next an error of the second kind. We note that the signal space of Figure 6.3 is symmetric with respect to the origin. It follows therefore that p_{01}, the conditional probability of the receiver deciding in favor of symbol 0, given that symbol 1 was transmitted, also has the same value as in Equation (6.19).

Thus, averaging the conditional error probabilities p_{10} and p_{01}, we find that the *average probability of symbol error* or, equivalently, the *bit error rate for coherent binary PSK* is (assuming equiprobable symbols)

$$P_e = \frac{1}{2} \operatorname{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right) \tag{6.20}$$

As we increase the transmitted signal energy per bit, E_b, for a specified noise spectral density N_0, the message points corresponding to symbols 1 and 0 move further apart, and the average probability of error P_e is correspondingly reduced in accordance with Equation (6.20), which is intuitively satisfying.

Rappaport



$$s_{11} = \int_0^{T_b} s_1(t)\phi_1(t)\,dt = +\sqrt{E_b}$$

Z_1: $0 < x_1 < \infty$

$$x_1 = \int_0^{T_b} x(t)\phi_1(t)\,dt$$

$$s_{21} = \int_0^{T_b} s_2(t)\phi_1(t)\,dt = -\sqrt{E_b}$$

The conditional probability density function of random variable X_1, given that symbol 0 [i.e., signal $s_2(t)$] was transmitted, is defined by

$$f_{X_1}(x_1|0) = \frac{1}{\sqrt{\pi N_0}} \exp\left[-\frac{1}{N_0}(x_1 - s_{21})^2\right]$$
$$= \frac{1}{\sqrt{\pi N_0}} \exp\left[-\frac{1}{N_0}(x_1 + \sqrt{E_b})^2\right] \qquad (6.16)$$

$$p_{10} = \int_0^{\infty} f_{X_1}(x_1|0)\,dx_1$$
$$= \frac{1}{\sqrt{\pi N_0}} \int_0^{\infty} \exp\left[-\frac{1}{N_0}(x_1 + \sqrt{E_b})^2\right] dx_1$$

Putting $$z = \frac{1}{\sqrt{N_0}}(x_1 + \sqrt{E_b})$$

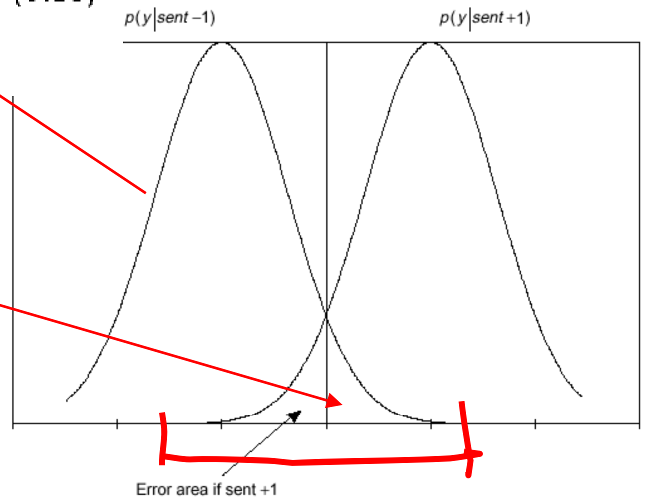


Figure 3.16 Conditional probability density function for the received signals.

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Error Probability of Binary PSK

$$p_{10} = \frac{1}{\sqrt{\pi}} \int_{\sqrt{E_b/N_0}}^{\infty} \exp(-z^2)\, dz = \frac{1}{2} \operatorname{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right) \quad (6.19)$$

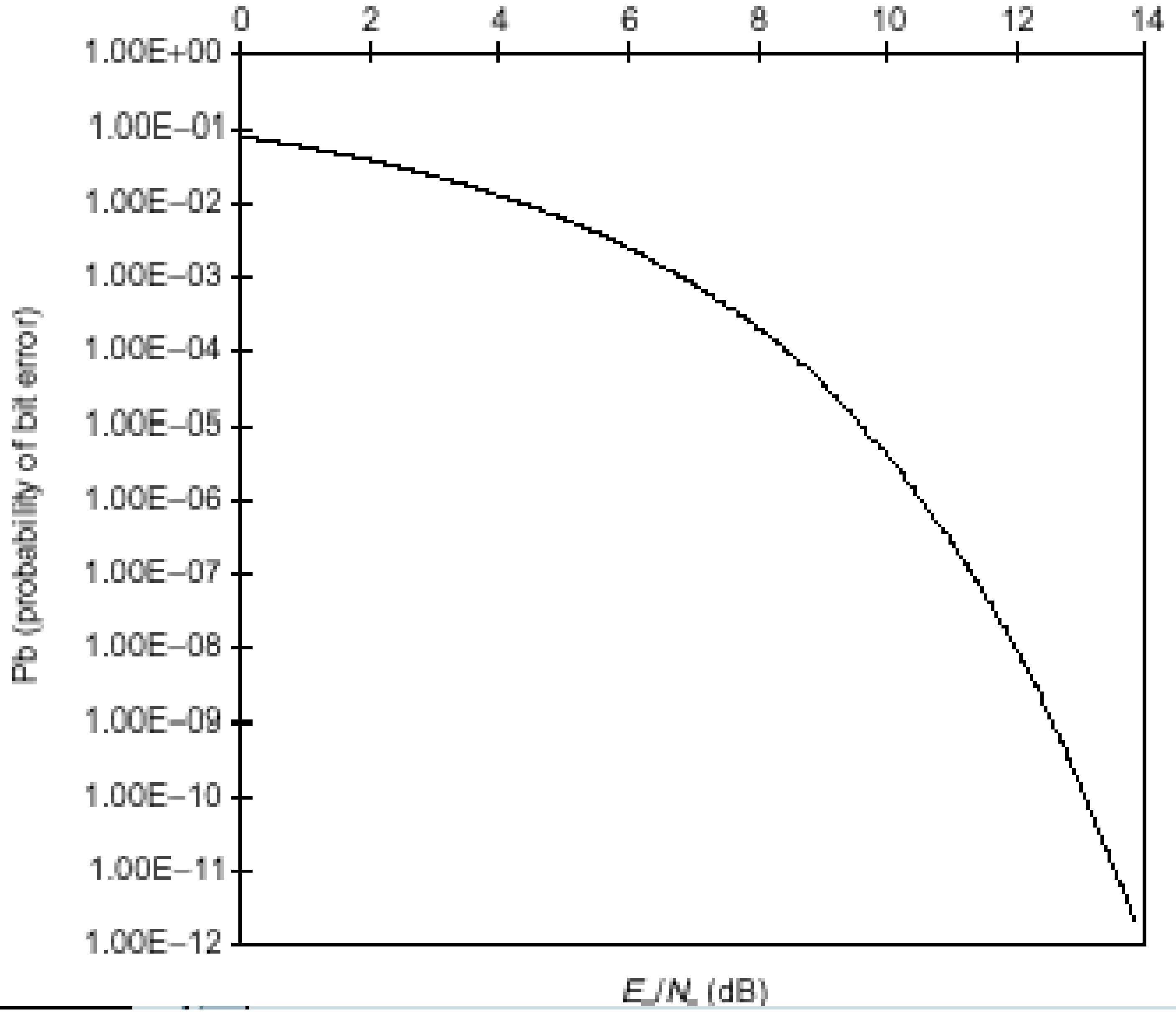
where erfc(·) is the complementary error function.

$$P_{e,\text{ BPSK}} = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

of an error distance d in signal space:

$$P_{b,\text{BPSK}} = Q\left(\frac{d}{\sqrt{2N_o}}\right)$$

Bit error performance of coherent BPSK system



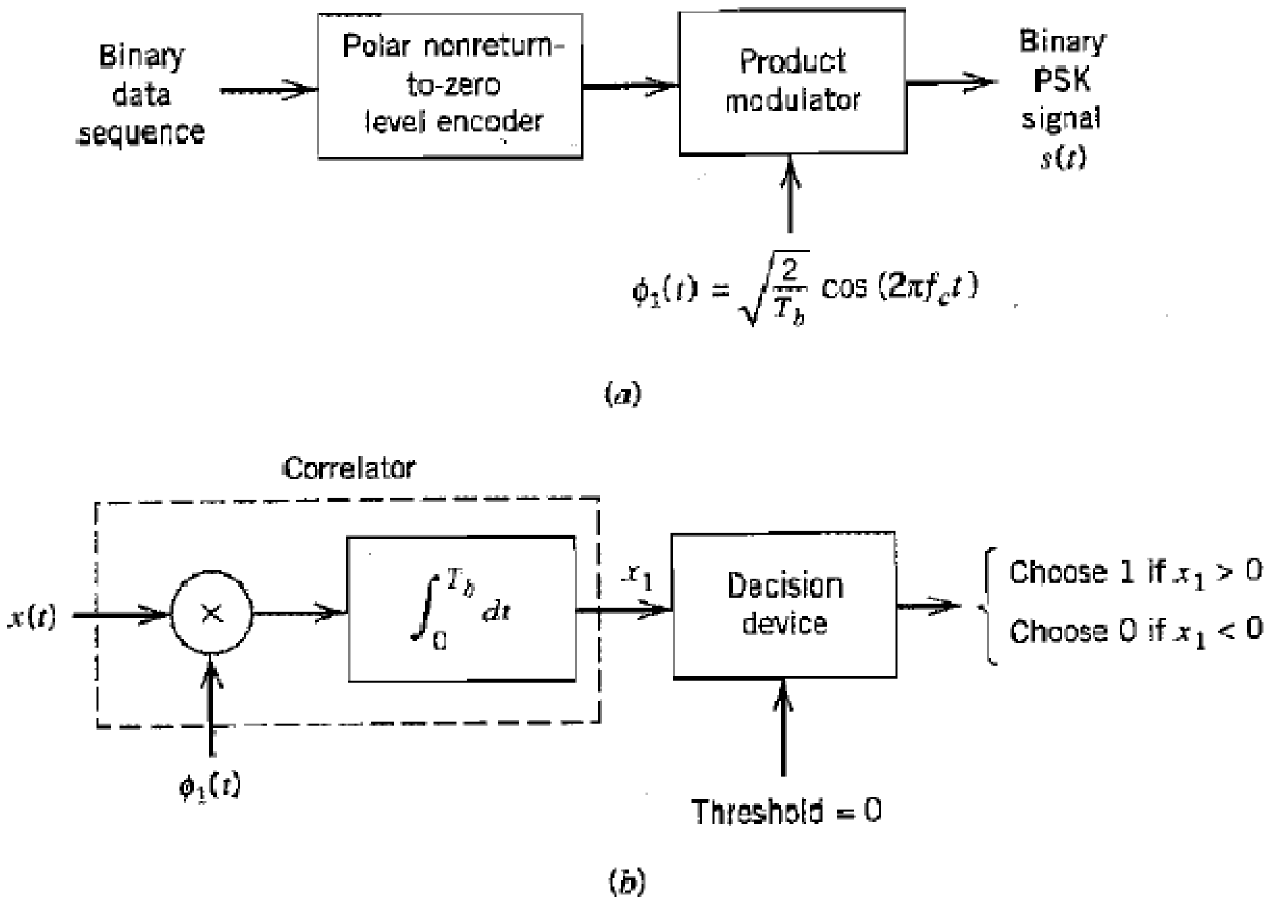


FIGURE 6.4 Block diagrams for (*a*) binary PSK transmitter and (*b*) coherent binary PSK receiver.

Power Spectra of Binary PSK Signals

From the modulator of Figure 6.4*a*, we see that the complex envelope of a binary PSK wave consists of an in-phase component only. Furthermore, depending on whether we have symbol 1 or symbol 0 at the modulator input during the signaling interval $0 \le t \le T_b$, we find that this in-phase component equals $+g(t)$ or $-g(t)$, respectively, where $g(t)$ is the *symbol shaping function* defined by

$$g(t) = \begin{cases} \sqrt{\dfrac{2E_b}{T_b}}, & 0 \le t \le T_b \\ 0, & \text{otherwise} \end{cases} \tag{6.21}$$

We assume that the input binary wave is random, with symbols 1 and 0 equally likely and the symbols transmitted during the different time slots being statistically independent. In Example 1.6 of Chapter 1 it is shown that the power spectral density of a random binary wave so described is equal to the energy spectral density of the symbol shaping function divided by the symbol duration. The energy spectral density of a Fourier transformable signal $g(t)$ is defined as the squared magnitude of the signal's Fourier transform. Hence, the baseband power spectral density of a binary PSK signal equals

$$\begin{aligned} S_B(f) &= \frac{2E_b \sin^2(\pi T_b f)}{(\pi T_b f)^2} \\ &= 2E_b \operatorname{sinc}^2(T_b f) \end{aligned} \tag{6.22}$$

This power spectrum falls off as the inverse square of frequency, as shown in Figure 6.5.

Figure 6.5 also includes a plot of the baseband power spectral density of a binary FSK signal, details of which are presented in Section 6.5. Comparison of these two spectra is deferred to that section.

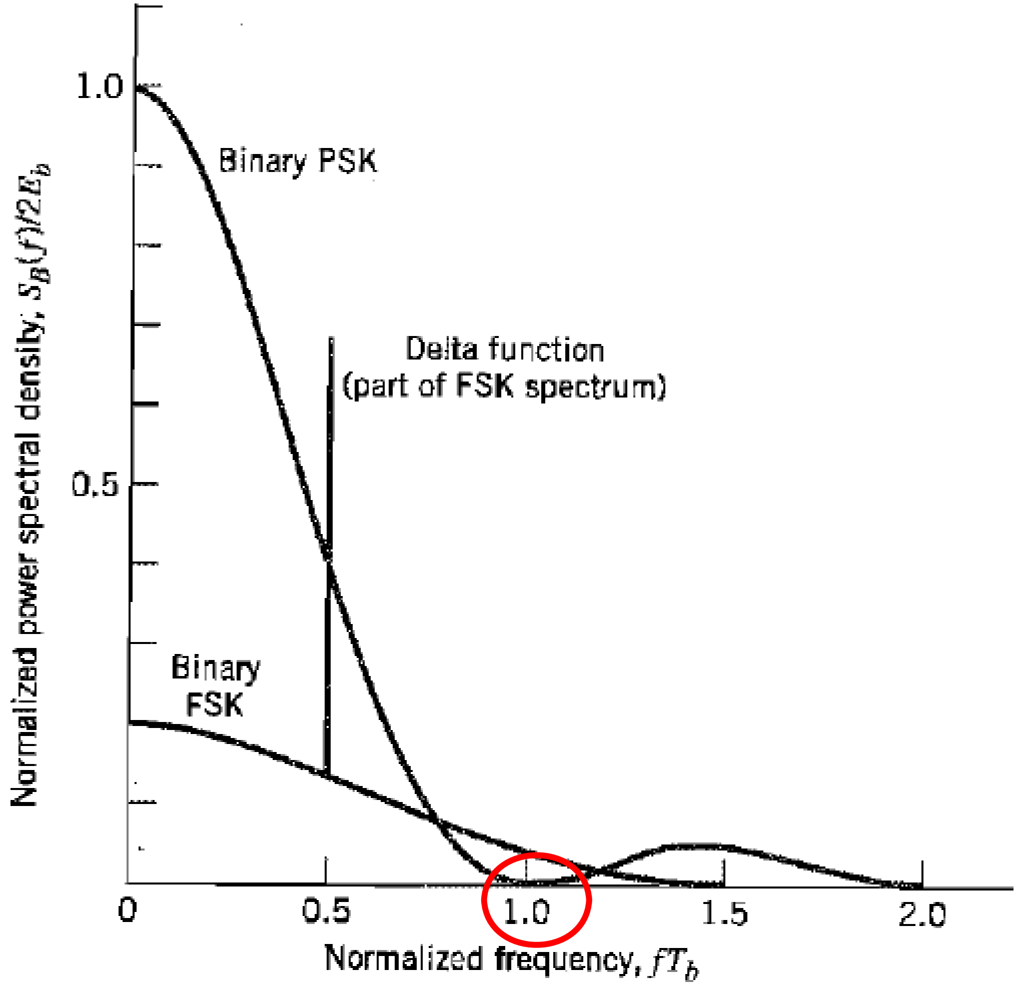


FIGURE 6.5 Power spectra of binary PSK and FSK signals.

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BPSK Receiver

If no multipath impairments are induced by the channel, the received BPSK signal can be expressed as

$$S_{\mathrm{BPSK}}(t) = m(t)\sqrt{\frac{2E_b}{T_b}}\cos\left(2\pi f_c t + \theta_c + \theta_{ch}\right) \tag{5.72}$$

$$= m(t)\sqrt{\frac{2E_b}{T_b}}\cos\left(2\pi f_c t + \theta\right)$$

where θ_{ch} is the phase shift corresponding to the time delay in the channel. BPSK uses *coherent* or *synchronous* demodulation, which requires that information about the phase and frequency of the carrier be available at the receiver. If a low level pilot carrier signal is transmitted along with the BPSK signal, then the carrier phase and frequency may be recovered at the receiver using a phase

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Spectrum and Bandwidth of BPSK

The BPSK signal using a polar baseband data waveform *m(t)* can be expressed in complex envelope form as

$$S_{BPSK} = Re\{g_{BPSK}(t)\exp(j2\pi f_c t)\} \quad (5.68)$$

where $g_{BPSK}(t)$ is the complex envelope of the signal given by

$$g_{BPSK}(t) = \sqrt{\frac{2E_b}{T_b}}m(t)e^{j\theta_c} \quad (5.69)$$

The power spectral density (PSD) of the complex envelope can be shown to be

$$P_{g_{BPSK}}(f) = 2E_b\left(\frac{\sin\pi f T_b}{\pi f T_b}\right)^2 \quad (5.70)$$

The PSD for the BPSK signal at RF can be evaluated by translating the baseband spectrum to the carrier frequency using the relation given in equation (5.41).

Hence the PSD of a BPSK signal at RF is given by

$$P_{BPSK} = \frac{E_b}{2}\left[\left(\frac{\sin\pi(f-f_c)T_b}{\pi(f-f_c)T_b}\right)^2 + \left(\frac{\sin\pi(-f-f_c)T_b}{\pi(-f-f_c)T_b}\right)^2\right] \quad (5.71)$$

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The PSD of the BPSK signal for both rectangular and raised cosine rolloff pulse shapes is plotted in Figure 5.22. The null-to-null bandwidth is found to be equal to twice the bit rate ($BW = 2R_b = 2/T_b$). From the plot, it can also be shown that 90% of the BPSK signal energy is contained within a bandwidth approximately equal to $1.6R_b$ for rectangular pulses, and all of the energy is within $1.5R_b$ for pulses with $\alpha = 0.5$ raised cosine filtering.

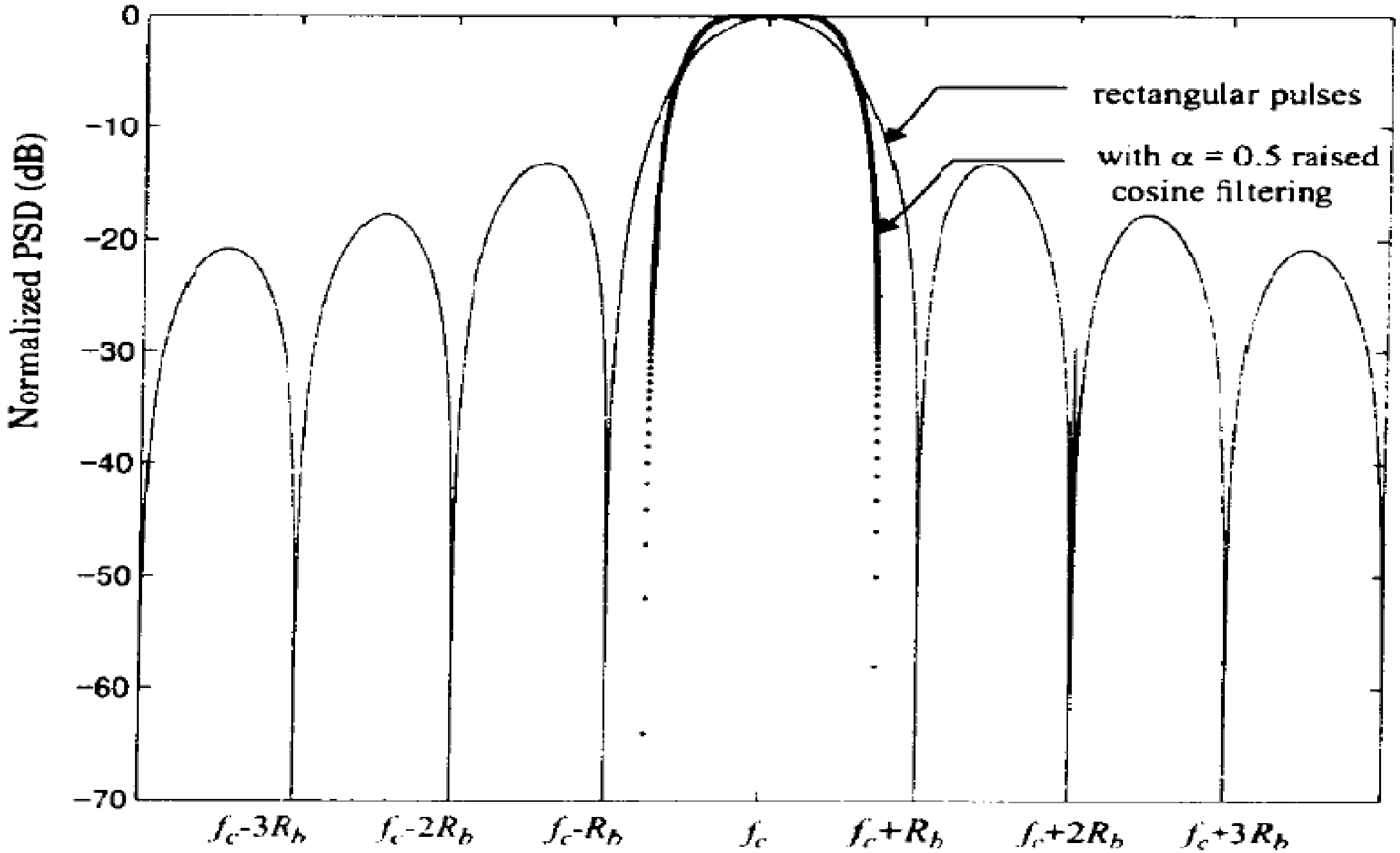

Figure 5.22
Power Spectral Density (PSD) of a BPSK signal.