Week 5 – Part I.

Signal-Space Analysis

For Digital Modulation Techniques

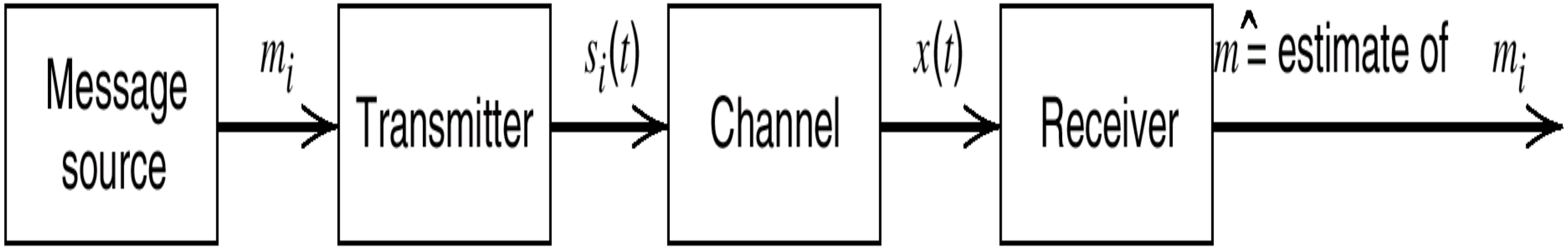


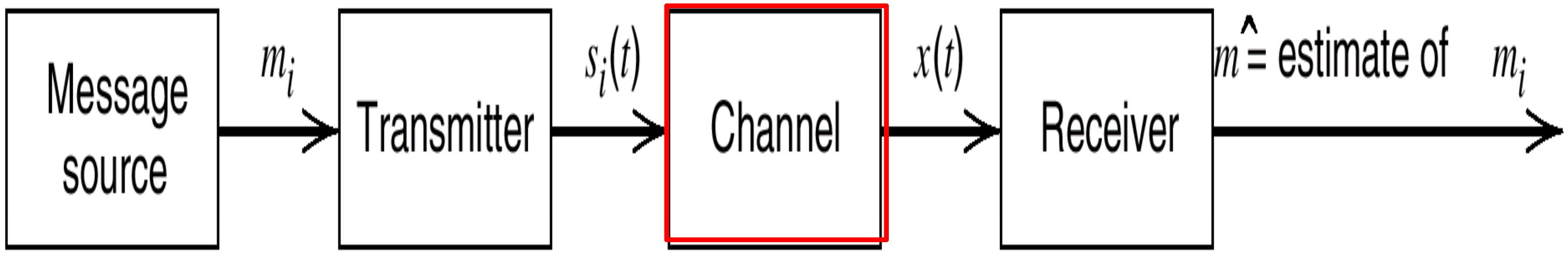
Figure 5.1 Block diagram of a generic digital communication system.

Consider the most basic form of a digital communication system depicted in Figure 5.1. A *message source* emits one *symbol* every T seconds, with the symbols belonging to an alphabet of M symbols denoted by $m_1, m_2, \ldots, m_M$. Consider, for example, the remote connection of two digital computers, with one computer acting as an information source that calculates digital outputs based on observations and inputs fed into it. The resulting computer output is expressed as a sequence of 0s and 1s, which are transmitted to a second computer over a communication channel. In this case, the alphabet consists simply of two binary symbols: 0 and 1. A second example is that of a quaternary PCM encoder with an alphabet consisting of four possible symbols: 00, 01, 10, and 11. In any event, the *a priori* probabilities $p_1, p_2, \ldots, p_M$ specify the message source output. In the absence of prior information, it is customary to assume that the M symbols of the alphabet are *equally likely*. Then we may express the probability that symbol m_i is emitted by the source as *likely*. Then we may express the probability that symbol m_i is emitted by the source as

$$p_i = P(m_i) = \frac{1}{M} \text{ for } i = 1, 2, \ldots, M \tag{5.1}$$

The transmitter takes the message source output m_i and codes it into a *distinct* signal $s_i(t)$ suitable for transmission over the channel. The signal $s_i(t)$ occupies the full duration T allotted to symbol m_i. Most important, $s_i(t)$ is a real-valued *energy signal* (i.e., a signal with finite energy), as shown by

$$E_i = \int_0^T s_i^2(t)\, dt, \qquad i = 1, 2, \ldots, M \tag{5.2}$$



The channel is assumed to have two characteristics:

1. The channel is *linear*, with a bandwidth that is wide enough to accommodate the transmission of signal $s_i(t)$ with negligible or no distortion.
2. The channel noise, $w(t)$, is the sample function of a *zero-mean white Gaussian noise process*. The reasons for this second assumption are that it makes receiver calculations tractable, and it is a reasonable description of the type of noise present in many practical communication systems.

We refer to such a channel as an *additive white Gaussian noise* (AWGN) *channel*. Accordingly, we may express the *received signal* $x(t)$ as

$$x(t) = s_i(t) + w(t), \qquad \begin{cases} 0 \le t \le T \\ i = 1, 2, \ldots, M \end{cases} \tag{5.3}$$

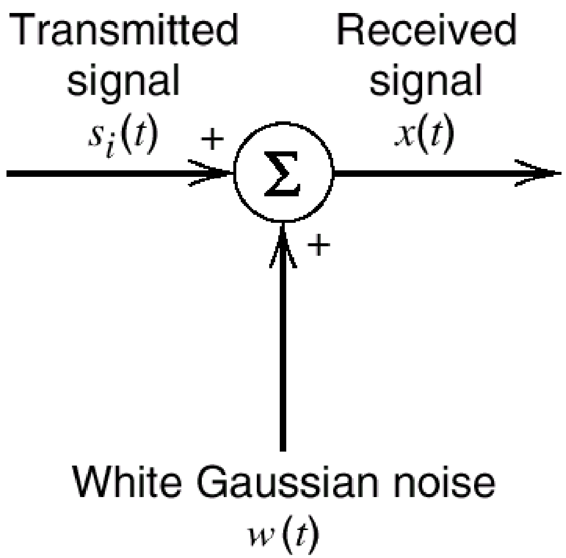
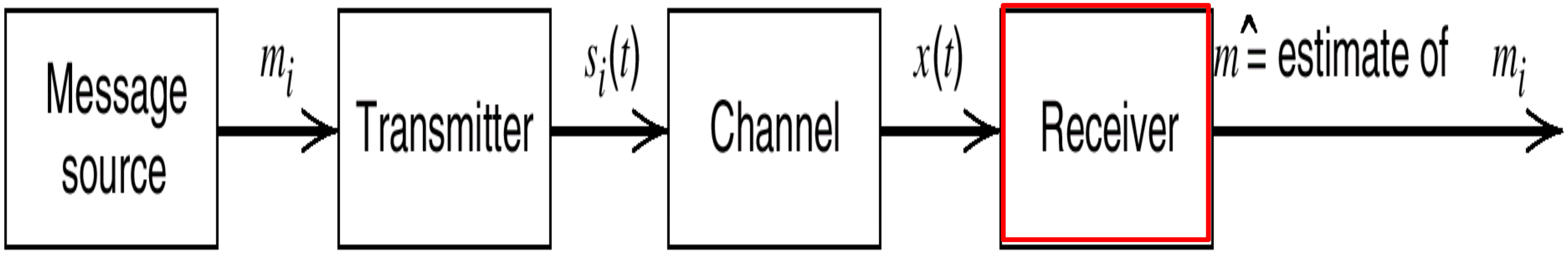


Figure 5.2 Additive white Gaussian noise (AWGN) model of a channel.



The receiver has the task of observing the received signal $x(t)$ for a duration of T seconds and making a best *estimate* of the transmitted signal $s_i(t)$ or, equivalently, the symbol m_i. However, owing to the presence of channel noise, this decision-making process is statistical in nature, with the result that the receiver will make occasional errors. The requirement is therefore to design the receiver so as to minimize the *average probability of symbol error*, defined as

$$P_e = \sum_{i=1}^{M} p_i \, P(\hat{m} \neq m_i \,|m_i) \tag{5.4}$$

where m_i is the transmitted symbol, $\hat{m}$ is the estimate produced by the receiver, and $P(\hat{m} \neq m_i \,|m_i)$ is the conditional error probability given that the *i*th symbol was sent. The resulting receiver is said to be *optimum in the minimum probability of error* sense.

This model provides a basis for the design of the optimum receiver, for which we will use geometric representation of the known set of transmitted signals, $\{s_i(t)\}$. This method, discussed in Section 5.2, provides a great deal of insight, with considerable simplification of detail.