EEnG473 Mobile Communications
Module 2 : Week # (7)

The Cellular Concept – System Design Fundamentals

Trunking and Grade of Service

- Cellular radio systems rely on **trunking** to accommodate a large number of users in a limited radio spectrum.

- **The concept of trunking:** allows a large number of users to share the relatively small number of channels in a cell by providing access to each user, on demand, from a pool of available channels.

- In a trunked radio system, each user is allocated a channel on a per call basis, and upon termination of the call, the previously occupied channel is immediately returned to the pool of available channels.

- There is a trade-off between **the number of available telephone circuits** and **the likelihood of a user finding that no circuits are available**.

  (As the number of phone lines decreases, it becomes more likely that all circuits will be busy for a particular user.)

- **The user is blocked(denied access) in trunked system**:

  when a particular user requests service and all of the radio channels are already in use.

- In some systems, **a queue may be used** to hold the requesting users until a channel becomes available.

- To design trunked radio systems that can handle a specific capacity at a specific ***"grade of service, GOS"*** it is essential to understand **trunking theory and queuing theory**.

Erlang

- The fundamentals of trunking theory were developed by **Erlang**, a Danish mathematician who, in the late 19th century, embarked on the study of how a large population could be accommodated by a limited number of servers.

Today, the measure of traffic intensity bears his name.

- **One Erlang** represents **the amount of traffic intensity carried by a channel that is completely occupied** (i.e. 1 call-hour per hour or 1 call-minute per minute).

- For example, a radio channel that is occupied for thirty minutes during an hour carries 0.5 Erlangs of traffic.

The grade of service (GOS)

- **(1) The grade of service (GOS)** is a measure of the ability of a user to access a trunked system during the busiest hour.

  (The busy hour is based upon customer demand at the busiest hour during a week, month, or year. The busy hours for cellular radio systems typically occur during rush hours, between 4 p.m. and 6 p.m. on a Thursday or Friday evening).

- (2) The grade of service is a benchmark used to define the desired performance of a particular trunked system.

  (by specifying a desired likelihood of a user obtaining channel access given a specific number of channels available in the system.)

- (3) GOS is typically given as the likelihood that a call is blocked, or the likelihood of a call experiencing a delay greater than a certain queuing time.

A number of definitions listed in Table 2.3 are used in trunking theory to make capacity estimates in trunked systems.

Table 2.3 Definitions of Common Terms Used in Trunking Theory

Set-up Time: The time required to allocate a trunked radio channel to a requesting user.

Blocked Call: Call which cannot be completed at time of request, due to congestion. Also referred to as a *lost call*.

Holding Time: Average duration of a typical call. Denoted by H (in seconds).

Traffic Intensity: Measure of channel time utilization, which is the average channel occupancy measured in Erlangs. This is a dimensionless quantity and may be used to measure the time utilization of single or multiple channels. Denoted by A.

Load: Traffic intensity across the entire trunked radio system, measured in Erlangs.

Grade of Service (GOS): A measure of congestion which is specified as the probability of a call being blocked (for Erlang B), or the probability of a call being delayed beyond a certain amount of time (for Erlang C).

Request Rate: The average number of call requests per unit time. Denoted by λ seconds^{-1}.

Traffic intensity

The traffic intensity offered by each user is equal to the call request rate multiplied by the holding time. That is, each user generates a traffic intensity of A_u Erlangs given by

$$A_u = \lambda H \tag{2.13}$$

where H is the average duration of a call and λ is the average number of call requests per unit time. For a system containing U users and an unspecified number of channels, the total offered traffic intensity A, is given as

$$A = U A_u \tag{2.14}$$

Furthermore, in a C channel trunked system, if the traffic is equally distributed among the channels, then the traffic intensity per channel, A_c, is given as

$$A_c = UA_u/C \tag{2.15}$$

Note that the offered traffic is not necessarily the traffic which is *carried* by the trunked system, only that which is *offered* to the trunked system. When the offered traffic exceeds the maximum capacity of the system, the carried traffic becomes limited due to the limited capacity (i.e. limited number of channels). The maximum possible carried traffic is the total number of channels, C, in Erlangs. The AMPS cellular system is designed for a GOS of 2% blocking. This implies that the channel allocations for cell sites are designed so that 2 out of 100 calls will be blocked due to channel occupancy during the busiest hour.

Types of trunked systems

- There are two which are commonly used.
- **The first type** offers no queuing for call requests. **(lost call cleared)**
- **The second kind** is one in which a queue is provided to hold calls which are blocked. **(lost call delayed)**

- **The first type** offers no queuing for call requests.

  That is, for every user who requests service, it is assumed there is no setup time and the user is given immediate access to a channel if one is available.

- If no channels are available, **the requesting user is blocked** without access and is free to try again later.

- This type of trunking **is called blocked calls cleared** and assumes that calls arrive as determined by **a Poisson distribution.**

- Furthermore, it is assumed that:

(a) there are an infinite number of users.

(b) there are memoryless arrivals of requests, implying that all users, including blocked users, may request a channel at any time;

(c) the probability of a user occupying a channel is exponentially distributed, so that longer calls are less likely to occur as described by an exponential distribution;

(d) there are a finite number of channels available in the trunking pool. This is known as an M/M/m queue, and leads to the derivation of the **Erlang B formula** (also known as the blocked calls cleared formula).

<u>The Erlang B</u> formula: **determines the probability that a call is blocked and is a measure of the GOS for a trunked system which provides no queuing for blocked calls.** (Appendix A)

$$Pr[blocking] = \frac{\frac{A^C}{C!}}{\sum_{k=0}^{C} \frac{A^k}{k!}} = GOS$$

<u>C</u> is the number of trunked channels offered by a trunked radio system
<u>A</u> is the total offered traffic.

The capacity of a trunked radio system where blocked calls are lost is tabulated for various values of GOS and numbers of channels in Table 3.4.

Erlang B Trunking GOS

Table 3.4 Capacity of an Erlang B System (Traffic intensity) A ("ρ")

Number of Channels *C*	Capacity (Erlangs) for GOS = 0.01	= 0.005	= 0.002	= 0.001
2	0.153	0.105	0.065	0.046
4	0.869	0.701	0.535	0.439
5	1.36	1.13	0.900	0.762
10	4.46	3.96	3.43	3.09
20	12.0	11.1	10.1	9.41
24	15.3	14.2	13.0	12.2
40	29.0	27.3	25.7	24.5
70	56.1	53.7	51.0	49.2
100	84.1	80.9	77.4	75.2

Erlang B

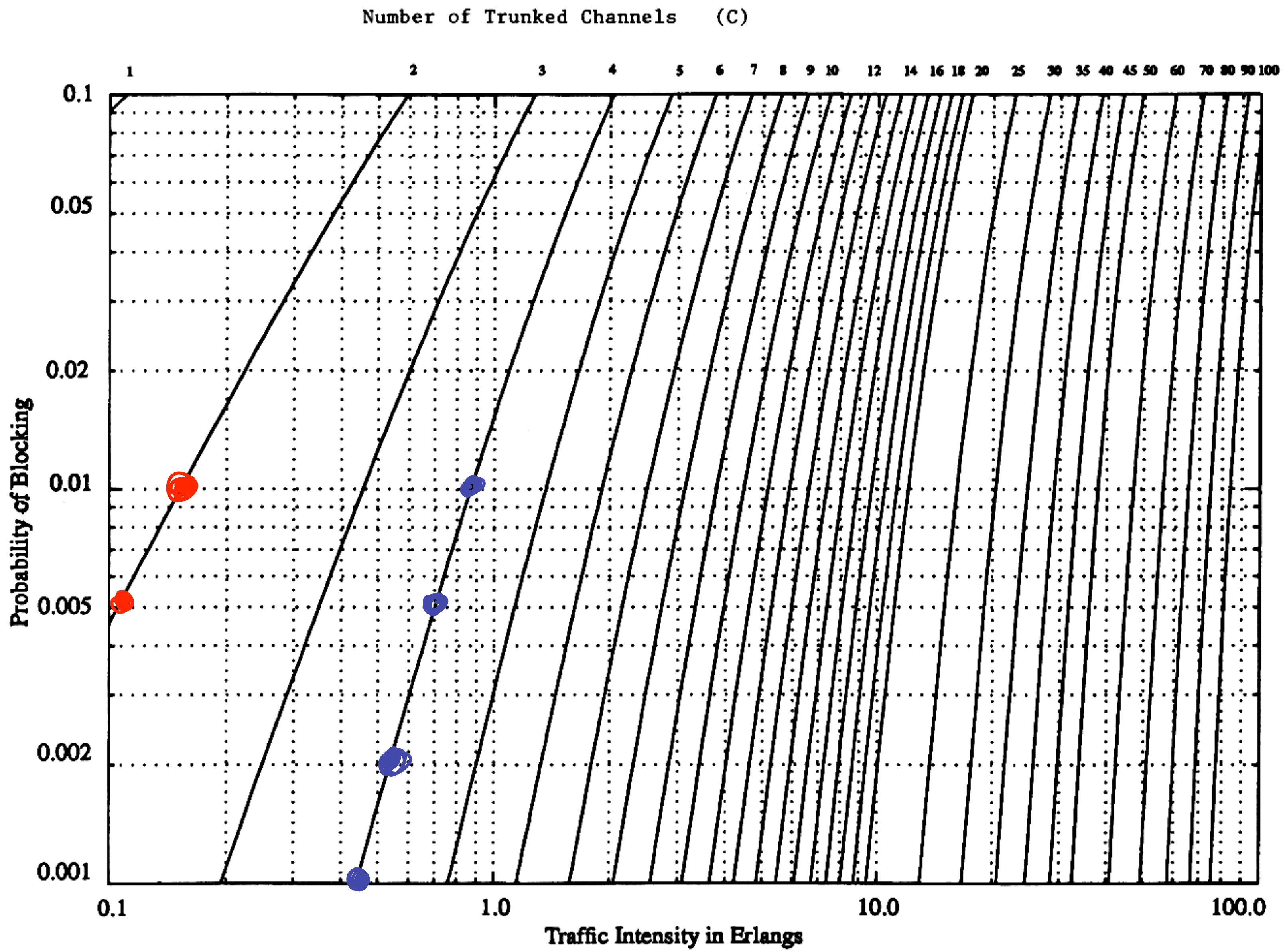

Figure 3.6 The Erlang B chart showing the probability of blocking as functions of the number of channels and traffic intensity in Erlangs.

- **The second kind of trunked system is one in which a queue is provided to hold calls which are blocked.**

- If a channel is not available immediately, the call request may be delayed until a channel becomes available. This type of trunking is called **Blocked Calls Delayed,** and its measure of **GOS** is defined as **the probability that a call is blocked after waiting a specific length of time in the queue.** To find the GOS, it is first necessary to find the likelihood that a call is initially denied access to the system.

- The likelihood of a call not having immediate access to a channel is determined by the **Erlang C** formula derived in Appendix A

$$Pr[delay > 0] = \frac{A^C}{A^C + C!\left(1 - \frac{A}{C}\right)\sum_{k=0}^{C-1}\frac{A^k}{k!}}$$

Erlang C

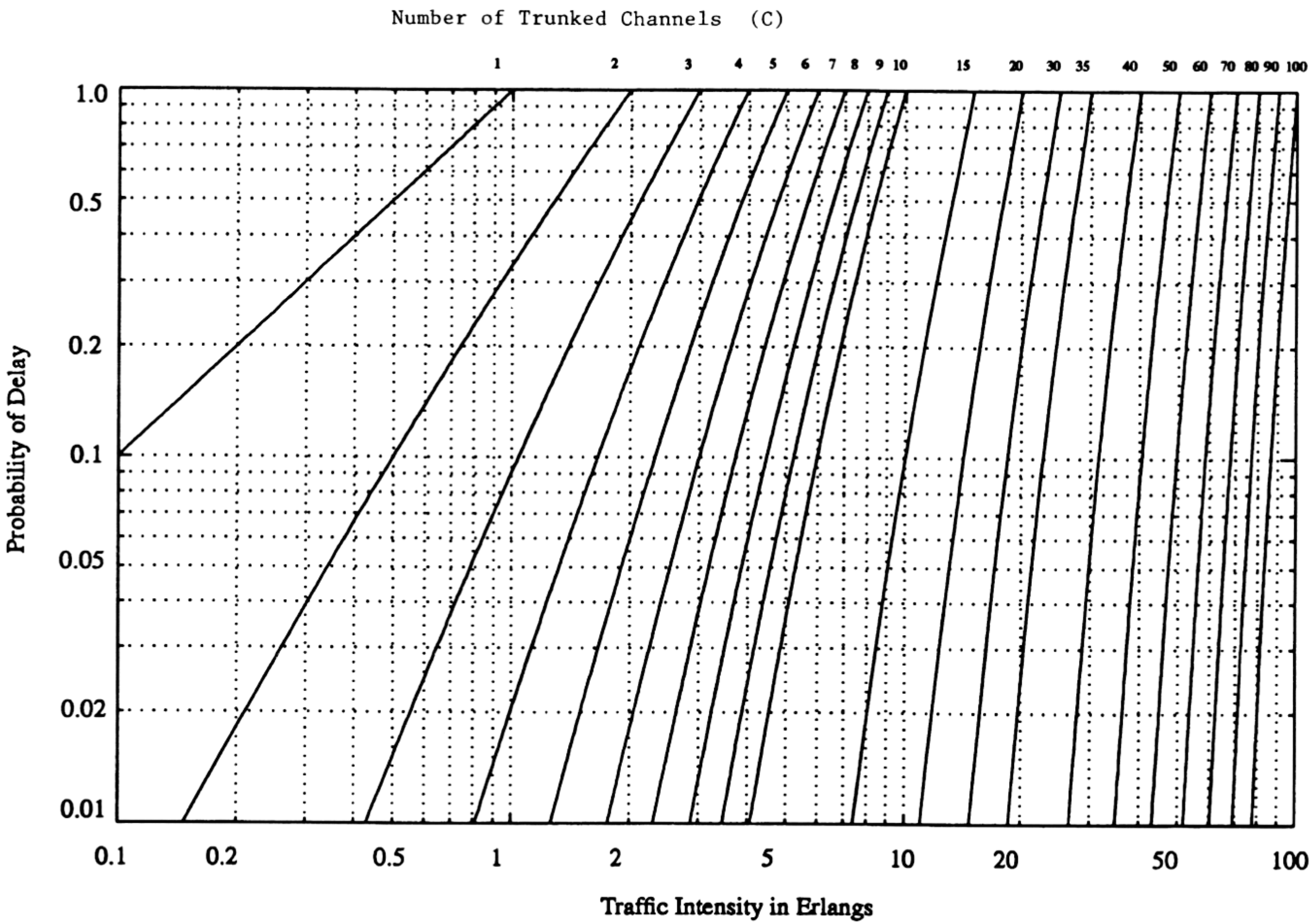

Figure 3.7 The Erlang C chart showing the probability of a call being delayed as a function of the number of channels and traffic intensity in Erlangs.

The **GOS** of a trunked system where blocked calls are delayed is

$$Pr[delay > t] = Pr[delay > 0] Pr[delay > t | delay > 0] \tag{2.18}$$
$$= Pr[delay > 0] \exp(-(C-A)t/H)$$

The average delay D for all calls in a queued system is given by

$$D = Pr[delay > 0] \frac{H}{C-A} \tag{2.19}$$

where the average delay for those calls which are queued is given by **H/(C—A).**

- **Trunking efficiency:** is a measure of the number of users which can be offered a particular GOS with a particular configuration of fixed channels.

- **important note:**

- The way in which channels are grouped can substantially alter the number of users handled by a trunked system.
- For example, from Table 3.4, **10 trunked channels** at a GOS of 0.01 can support 4.46 Erlangs of traffic,
- whereas **2 groups of 5** truncated channels can support 2x1.36=2.72 Erlangs of traffic.
- Clearly, 10 channels trunked together support 60% more traffic at a specific GOS than do two 5 channel trunks!

It should be clear that the allocation of channels in a trunked radio system has a major impact on overall system capacity.

Example 2.4

How many users can be supported for 0.5% blocking probability for the following number of trunked channels in a blocked calls cleared system? (a) 1, (b) 5, (c) 10, (d) 20, (e) 100. Assume each user generates 0.1 Erlangs of traffic.

Solution to Example 2.4

From Table 2.4 we can find the total capacity in Erlangs for the 0.5% GOS for different numbers of channels. By using the relation $A = UA_u$, we can obtain the total number of users that can be supported in the system.

(a) Given $C = 1$, $A_u = 0.1$, $GOS = 0.005$
From Figure 2.6, we obtain $A = 0.005$.
Therefore, total number of users, $U = A/A_u = 0.005/0.1 = 0.05$ users.
But, actually one user could be supported on one channel. So, $U = 1$.

(b) Given $C = 5$, $A_u = 0.1$, $GOS = 0.005$
From Figure 2.6, we obtain $A = 1.13$.
Therefore, total number of users, $U = A/A_u = 1.13/0.1 \approx 11$ users.

(c) Given $C = 10$, $A_u = 0.1$, $GOS = 0.005$
From Figure 2.6, we obtain $A = 3.96$.
Therefore, total number of users, $U = A/A_u = 3.96/0.1 \approx 39$ users.

(d) Given $C = 20$, $A_u = 0.1$, $GOS = 0.005$
From Figure 2.6, we obtain $A = 11.10$.
Therefore, total number of users, $U = A/A_u = 11.1/0.1 = 110$ users.

(e) Given $C = 100$, $A_u = 0.1$, $GOS = 0.005$
From Figure 2.6, we obtain $A = 80.9$.
Therefore, total number of users, $U = A/A_u = 80.9/0.1 = 809$ users.

Example 2.5

An urban area has a population of 2 million residents. Three competing trunked mobile networks (systems A, B, and C) provide cellular service in this area. System A has 394 cells with 19 channels each, system B has 98 cells with 57 channels each, and system C has 49 cells, each with 100 channels. Find the number of users that can be supported at 2% blocking if each user averages 2 calls per hour at an average call duration of 3 minutes. Assuming that all three trunked systems are operated at maximum capacity, compute the percentage market penetration of each cellular provider.

Solution to Example 2.5

System A

Given:

Probability of blocking = 2% = 0.02

Number of channels per cell used in the system, $C = 19$

Traffic intensity per user, $A_u = \lambda H = 2 \times (3/60) = 0.1$ Erlangs

For $GOS = 0.02$ and $C = 19$, from the Erlang B chart, the total carried traffic, A, is obtained as 12 Erlangs.

Therefore, the number of users that can be supported per cell is

$U = A/A_u = 12/0.1 = 120.$

Since there are 394 cells, the total number of subscribers that can be supported by System A is equal to $120 \times 394 = 47280.$

System B

Given:

Probability of blocking = 2% = 0.02

Number of channels per cell used in the system, $C = 57$

Traffic intensity per user, $A_u = \lambda H = 2 \times (3/60) = 0.1$ Erlangs

For $GOS = 0.02$ and $C = 57$, from the Erlang B chart, the total carried traffic, A, is obtained as 45 Erlangs.

Therefore, the number of users that can be supported per cell is

$$U = A/A_u = 45/0.1 = 450.$$

Since there are 98 cells, the total number of subscribers that can be supported by System B is equal to $450 \times 98 = 44100$.

System C

Given:

Probability of blocking = 2% = 0.02

Number of channels per cell used in the system, $C = 100$

Traffic intensity per user, $A_u = \lambda H = 2 \times (3/60) = 0.1$ Erlangs

For $GOS = 0.02$ and $C = 100$, from the Erlang B chart, the total carried traffic, A, is obtained as 88 Erlangs.

Therefore, the number of users that can be supported per cell is

$$U = A/A_u = 88/0.1 = 880.$$

Since there are 49 cells, the total number of subscribers that can be supported by System C is equal to $880 \times 49 = 43120$

Therefore, total number of cellular subscribers that can be supported by these three systems are $47280 + 44100 + 43120 = 134500$ users.

Since there are 2 million residents in the given urban area and the total number of cellular subscribers in System A is equal to 47280, the percentage market penetration is equal to

$$47280/2000000 = 2.36\ \%$$

Similarly, market penetration of System B is equal to

$$44100/2000000 = 2.205\ \%$$

and the market penetration of System C is equal to

$$43120/2000000 = 2.156\ \%$$

The market penetration of the three systems combined is equal to

$$134500/2000000 = 6.725\ \%$$

Example 2.6

A certain city has an area of 1,300 square miles and is covered by a cellular system using a 7-cell reuse pattern. Each cell has a radius of 4 miles and the city is allocated 40 MHz of spectrum with a full duplex channel bandwidth of 60 kHz. Assume a GOS of 2% for an Erlang B system is specified. If the offered traffic per user is 0.03 Erlangs, compute (a) the number of cells in the service area, (b) the number of channels per cell, (c) traffic intensity of each cell, (d) the maximum carried traffic, (e) the total number of users that can be served for 2% GOS, (f) the number of mobiles per channel, and (g) the theoretical maximum number of users that could be served at one time by the system.

Solution to Example 2.6

(a) Given:

Total coverage area = 1300 miles

Cell radius = 4 miles

The area of a cell (hexagon) can be shown to be $2.5981R^2$, thus each cell covers

$$2.5981 \times (4)^2 = 41.57 \text{ sq mi.}$$

Hence, the total number of cells are $N_c = 1300/41.57 = 31$ cells.

(b) The total number of channels per cell (C)

= allocated spectrum / (channel width × frequency reuse factor)

$= 40,000,000/(60,000 \times 7) = 95$ channels/cell

(c) Given:

$C = 95$, and $GOS = 0.02$

From the Erlang B chart, we have

traffic intensity per cell $A = 84$ Erlangs/cell

(d) Maximum carried traffic = number of cells × traffic intensity per cell

$= 31 \times 84 = 2604$ Erlangs.

(e) Given traffic per user = 0.03 Erlangs

Total number of users = Total traffic / traffic per user

= 2604 / 0.03 = 86,800 users.

(f) Number of mobiles per channel = number of users/number of channels

= 86,800 / 666 = 130 mobiles/channel.

(g) The theoretical maximum number of served mobiles is the number of available channels in the system (all channels occupied)

$= C \times N_C = 95 \times 31 = 2945$ users, which is 3.4% of the customer base.

Example 2.7

A hexagonal cell within a 4-cell system has a radius of 1.387 km. A total of 60 channels are used within the entire system. If the load per user is 0.029 Erlangs, and $\lambda = 1$ call/hour, compute the following for an Erlang C system that has a 5% probability of a delayed call:

(a) How many users per square kilometer will this system support?

(a) What is the probability that a delayed call will have to wait for more than 10 s?

(c) What is the probability that a call will be delayed for more than 10 seconds?

Solution to Example 2.7

Given,

Cell radius, $R = 1.387$ km

Area covered per cell is $2.598 \times (1.387)^2 = 5$ sq km

Number of cells per cluster = 4

Total number of channels = 60

Therefore, number of channels per cell = 60 / 4 = 15 channels.

(a) From Erlang C chart, for 5% probability of delay with $C = 15$, traffic intensity = 9.0 Erlangs.

Therefore, number of users = total traffic intensity / traffic per user

$= 9.0/0.029 = 310$ users

$= 310$ users$/5$ sq km $= 62$ users/sq km

(b) Given $\lambda = 1$, holding time

$H = A_u/\lambda = 0.029$ hour $= 104.4$ seconds.

The probability that a delayed call will have to wait for more than 10 s is

$$Pr[delay > t|delay] = \exp(-(C-A)t/H)$$

$$= \exp(-(15-9.0)10/104.4) = 56.29\ \%$$

(c) Given $Pr[delay > 0] = 5\% = 0.05$

Probability that a call is delayed more than 10 seconds,

$$Pr[delay > 10] = Pr[delay > 0]Pr[delay > t|delay]$$

$$= 0.05 \times 0.5629 = 2.81\ \%$$