EENG473 Mobile Communications
Module 3 : Week # (12)

Mobile Radio Propagation: Small-Scale Path Loss

Introduction

- **Small-scale fading** is used to describe the rapid fluctuation of the amplitude of a radio signal over a short period of time or travel distance.

- **Fading is caused by** interference between two or more versions of the transmitted signal which arrive at the receiver at slightly different times.

- **Multipath waves** consists of a large number of plane waves having randomly distributed amplitudes, phases, and angles of arrival. that causes the signal to distort or fade.

Small-Scale Multipath Propagation

Multipath creates small-scale fading effects such as:

1. **Rapid changes in signal strength over a small travel distance or time interval**

2. **Random frequency modulation due to varying Doppler shifts on different multipath signals.**

3. **Time dispersion caused by multipath propagation delays.**

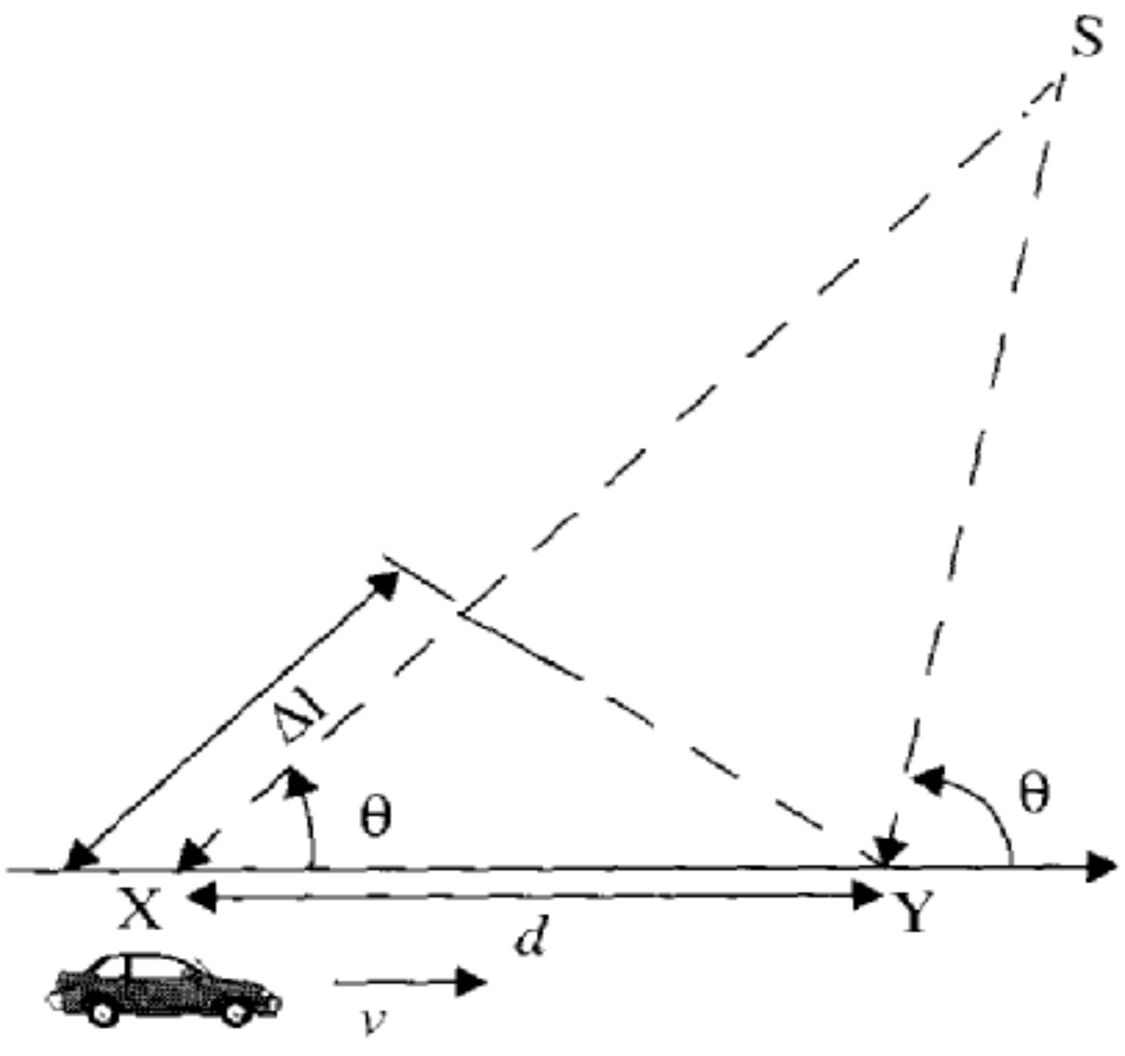
The physical factors leading to these effects are:

- ➔ The presence of objects in the propagation path (buildings, signs, trees and plants, fixed and moving vehicles, etc.) reflects and/or scatters the incident electromagnetic energy leading to multipath.
- ➔ Relative motion between the transmitter and receiver gives rise to doppler effects.
- ➔ Movement of surrounding objects also gives rise to doppler effects.
- ➔ The bandwidith of the channel, especially if the channel bandwidth is less than the signal bandwidth, leads to time dispersion.

The Doppler shift

- The shift in received signal frequency due to motion
- is directly proportional to the **velocity and direction of motion** of the mobile with respect to the direction of arrival of the received multipath wave.

Illustration of Doppler effect



Doppler Shift

Consider a mobile moving at a constant velocity v, along a path segment having length d between points X and Y, while it receives signals from a remote source S, as illustrated in Figure 4.1. The difference in path lengths traveled by the wave from source S to the mobile at points X and Y is $\Delta l = d\cos\theta = v\Delta t\cos\theta$, where Δt is the time required for the mobile to travel from X to Y, and θ is assumed to be the same at points X and Y since the source is assumed to be very far away. The phase change in the received signal due to the difference in path lengths is therefore

$$\Delta\phi = \frac{2\pi\Delta l}{\lambda} = \frac{2\pi v\Delta t}{\lambda}\cos\theta \tag{4.1}$$

and hence the apparent change in frequency, or Doppler shift, is given by f_d, where

$$f_d = \frac{1}{2\pi}\cdot\frac{\Delta\phi}{\Delta t} = \frac{v}{\lambda}\cdot\cos\theta \tag{4.2}$$

- The equation relates the Doppler shift to the mobile velocity and the spatial angle between the direction of motion of the mobile and the direction of arrival of the wave.

- **The Doppler shift is positive** (i.e., the apparent received frequency is increased), if the mobile is moving toward the direction of arrival of the wave.

- **The Doppler shift is negative** (i.e. the apparent received frequency is decreased), if the mobile is moving away from the direction of arrival of the wave.

- multipath components from a CW signal which arrive from different directions contribute to Doppler spreading of the received signal, **thus increasing the signal bandwidth.**

Example 4.1

Consider a transmitter which radiates a sinusoidal carrier frequency of 1850 MHz. For a vehicle moving 60 mph, compute the received carrier frequency if the mobile is moving (a) directly towards the transmitter, (b) directly away from the transmitter, (c) in a direction which is perpendicular to the direction of arrival of the transmitted signal.

Solution to Example 4.1

Given:

Carrier frequency $f_c = 1850\,\text{MHz}$

Therefore, wavelength $\lambda = c/f_c = \dfrac{3 \times 10^8}{1850 \times 10^6} = 0.162\,\text{m}$

Vehicle speed $v = 60$ mph $= 26.82$ m/s

(a) The vehicle is moving directly towards the transmitter.

The Doppler shift in this case is positive and the received frequency is given by equation (4.2)

$$f = f_c + f_d = 1850 \times 10^6 + \frac{26.82}{0.162} = 1850.00016 \text{ MHz}$$

(b) The vehicle is moving directly away from the transmitter.

The Doppler shift in this case is negative and hence the received frequency is given by

$$f = f_c - f_d = 1850 \times 10^6 - \frac{26.82}{0.162} = 1849.999834 \text{ MHz}$$

(c) The vehicle is moving perpendicular to the angle of arrival of the transmitted signal.

In this case, $\theta = 90°$, $\cos\theta = 0$, and there is no Doppler shift.

The received signal frequency is the same as the transmitted frequency of 1850 MHz.

4.2 Impulse Response Model of a Multipath Channel

- To show that a mobile radio channel may be modeled as **a linear filter with a time varying impulse response**, consider the case where time variation is due strictly to receiver motion in space.

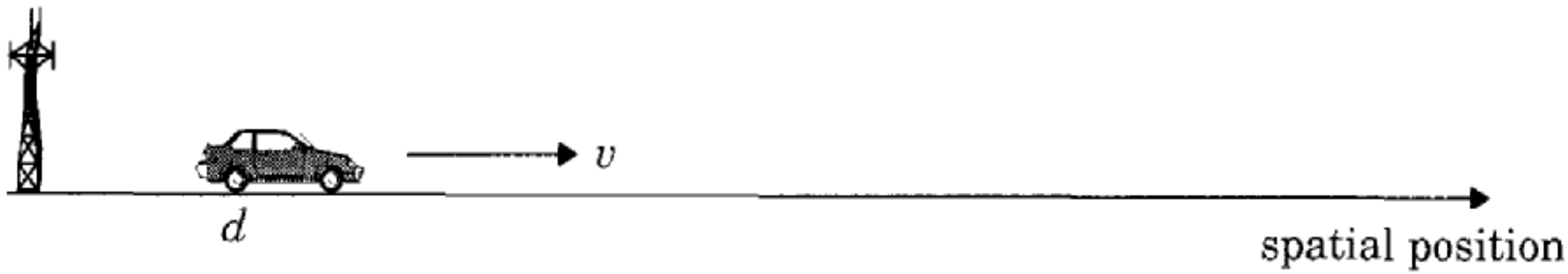


Figure 4.2
The mobile radio channel as a function of time and space.

In Figure 4.2, the receiver moves along the ground at some constant velocity v. For a fixed position d, the channel between the transmitter and the receiver can be modeled as a linear time invariant system. However, due to the different multipath waves which have propagation delays which vary over different spatial locations of the receiver, the impulse response of the linear time invariant channel should be a function of the position of the receiver. That is, the channel impulse response can be expressed as $h(d,t)$. Let $x(t)$ represent the transmitted signal, then the received signal $y(d,t)$ at position d can be expressed as a convolution of $x(t)$ with $h(d,t)$.

$$y(d,t) = x(t) \otimes h(d,t) = \int_{-\infty} x(\tau)h(d,t-\tau)d\tau \quad (4.3)$$

For a causal system, $h(d,t) = 0$ for $t<0$, thus equation (4.3) reduces to

$$y(d,t) = \int_{-\infty}^{t} x(\tau)h(d,t-\tau)d\tau \quad (4.4)$$

Since the receiver moves along the ground at a constant velocity v, the position of the receiver can by expressed as

$$d = vt \quad (4.5)$$

Substituting (4.5) in (4.4), we obtain

t

Since v is a constant, $y(vt,t)$ is just a function of t. Therefore, equation (4.6) can be expressed as

$$y(t) = \int_{-\infty}^{t} x(\tau)h(vt,t-\tau)d\tau = x(t) \otimes h(vt,t) = x(t) \otimes h(d,t) \quad (4.7)$$

From equation (4.7) it is clear that the mobile radio channel can be modeled as a linear time varying channel, where the channel changes with time and distance.

Since v may be assumed constant over a short time (or distance) interval, we may let $x(t)$ represent the transmitted bandpass waveform, $y(t)$ the received waveform, and $h(t, \tau)$ the impulse response of the time varying multipath radio channel. The impulse response $h(t, \tau)$ completely characterizes the channel and is a function of both t and τ. The variable t represents the time variations due to motion, whereas τ represents the channel multipath delay for a fixed value of t. One may think of τ as being a vernier adjustment of time. The received signal $y(t)$ can be expressed as a convolution of the transmitted signal $x(t)$ with the channel impulse response (see Figure 4.3a).

$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t, \tau)d\tau = x(t) \otimes h(t, \tau) \tag{4.8}$$

If the multipath channel is assumed to be a bandlimited bandpass channel, which is reasonable, then $h(t,\tau)$ may be equivalently described by a complex baseband impulse response $h_b(t, \tau)$ with the input and output being the complex envelope representations of the transmitted and received signals, respectively (see Figure 4.3b). That is,

$$r(t) = c(t) \otimes \frac{1}{2}h_b(t, \tau) \tag{4.9}$$

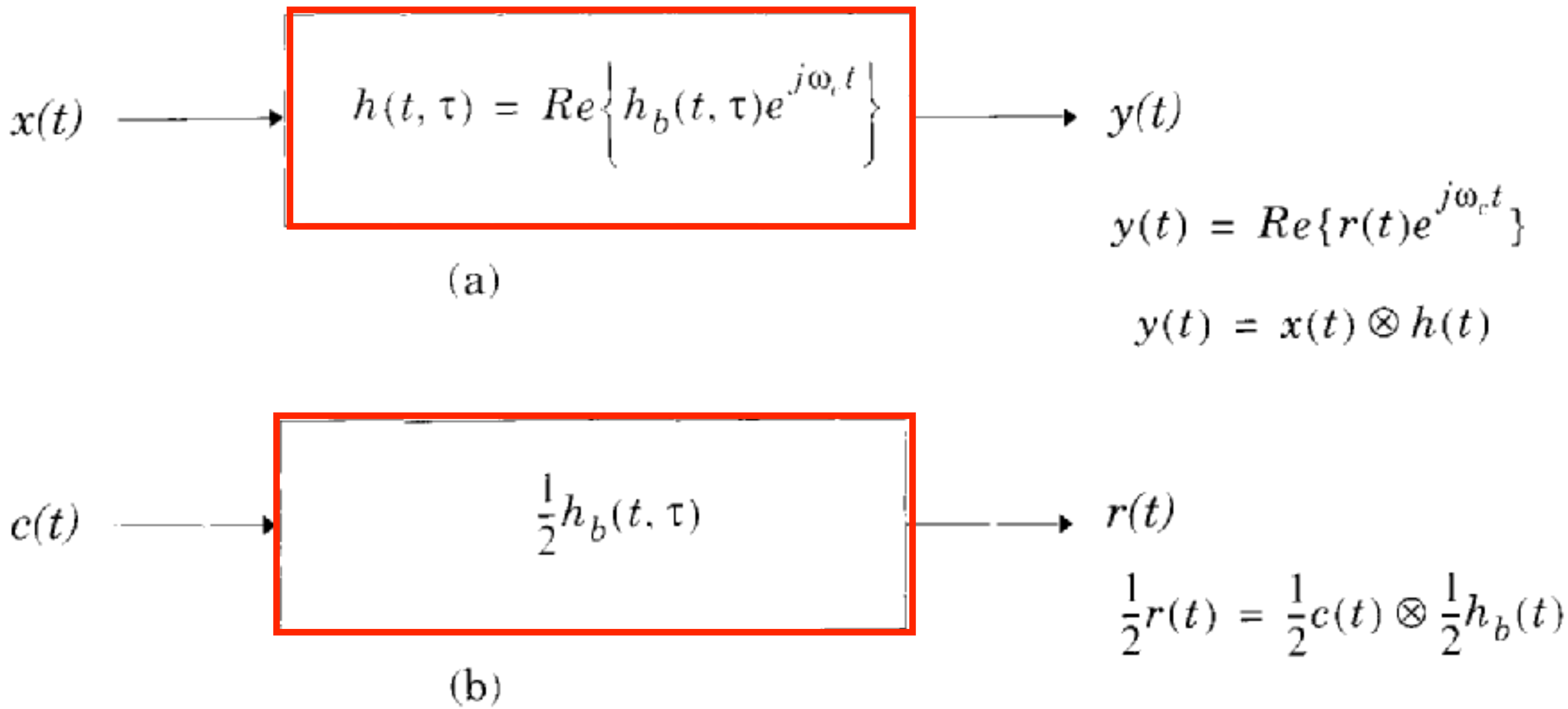


Figure 4.3

(a) Bandpass channel impulse response model.

(b) Baseband equivalent channel impulse response model.

where $c(t)$ and $r(t)$ are the complex envelopes of $x(t)$ and $y(t)$, defined as

$$x(t) = Re\{c(t)\exp(j2\pi f_c t)\} \tag{4.10}$$

$$y(t) = Re\{r(t)\exp(j2\pi f_c t)\} \tag{4.11}$$

The factor of $1/2$ in equation (4.9) is due to the properties of the complex envelope, in order to represent the passband radio system at baseband. The low-pass characterization removes the high frequency variations caused by the carrier, making the signal analytically easier to handle. It is shown by Couch [Cou93] that the average power of a bandpass signal $\overline{x^2(t)}$ is equal to $\frac{1}{2}\overline{|c(t)|^2}$, where the overbar denotes ensemble average for a stochastic signal, or time average for a deterministic or ergodic stochastic signal.

It is useful to discretize the multipath delay axis τ of the impulse response into equal time delay segments called *excess delay bins*, where each bin has a time delay width equal to $\tau_{i+1} - \tau_i$, where τ_0 is equal to 0, and represents the first arriving signal at the receiver. Letting i = 0, it is seen that $\tau_1 - \tau_0$ is equal to the time delay bin width given by $\Delta\tau$. For convention, $\tau_0 = 0$, $\tau_1 = \Delta\tau$, and $\tau_i = i\Delta\tau$, for $i = 0$ to $N - 1$, where N represents the total number of possible equally-spaced multipath components, including the first arriving component. Any number of multipath signals received within the i th bin are represented by a single resolvable multipath component having delay τ_i. This technique of quantizing the delay bins determines the time delay resolution of the channel model, and the useful frequency span of the model can be shown to be $1/(2\Delta\tau)$. That is, the model may be used to analyze transmitted signals having band-widths which are less than $1/(2\Delta\tau)$. Note that $\tau_0 = 0$ is the excess time delay of the first arriving multipath component, and neglects the propagation delay between the transmitter and receiver. *Excess delay* is the relative delay of the i th multipath component as compared to the first arriving component and is given by τ_i. The *maximum excess delay* of the channel is given by $N\Delta\tau$.

Since the received signal in a multipath channel consists of a series of attenuated, time-delayed, phase shifted replicas of the transmitted signal, the baseband impulse response of a multipath channel can be expressed as

$$h_b(t, \tau) = \sum_{i=0}^{N-1} a_i(t, \tau)\exp[j(2\pi f_c \tau_i(t) + \phi_i(t, \tau))]\delta(\tau - \tau_i(t)) \tag{4.12}$$

where $a_i(t, \tau)$ and $\tau_i(t)$ are the real amplitudes and excess delays, respectively, of i th multipath component at time t [Tur72]. The phase term $2\pi f_c \tau_i(t) + \phi_i(t, \tau)$ in (4.12) represents the phase shift due to free space propagation of the i th multipath component, plus any additional phase shifts which are encountered in the channel. In general, the phase term is simply represented by encountered in the channel. In general, the phase term is simply represented by a single variable $\theta_i(t, \tau)$ which lumps together all the mechanisms for phase shifts of a single multipath component within the ith excess delay bin. Note that some excess delay bins may have no multipath at some time t and delay τ_i, since $a_i(t, \tau)$ may be zero. In equation (4.12), N is the total possible number of multipath components (bins), and $\delta(\bullet)$ is the unit impulse function which determines the specific multipath bins that have components at time t and excess delays τ_i. Figure 4.4 illustrates an example of different snapshots of $h_b(t, \tau)$, where t varies into the page, and the time delay bins are quantized to widths of $\Delta\tau$.

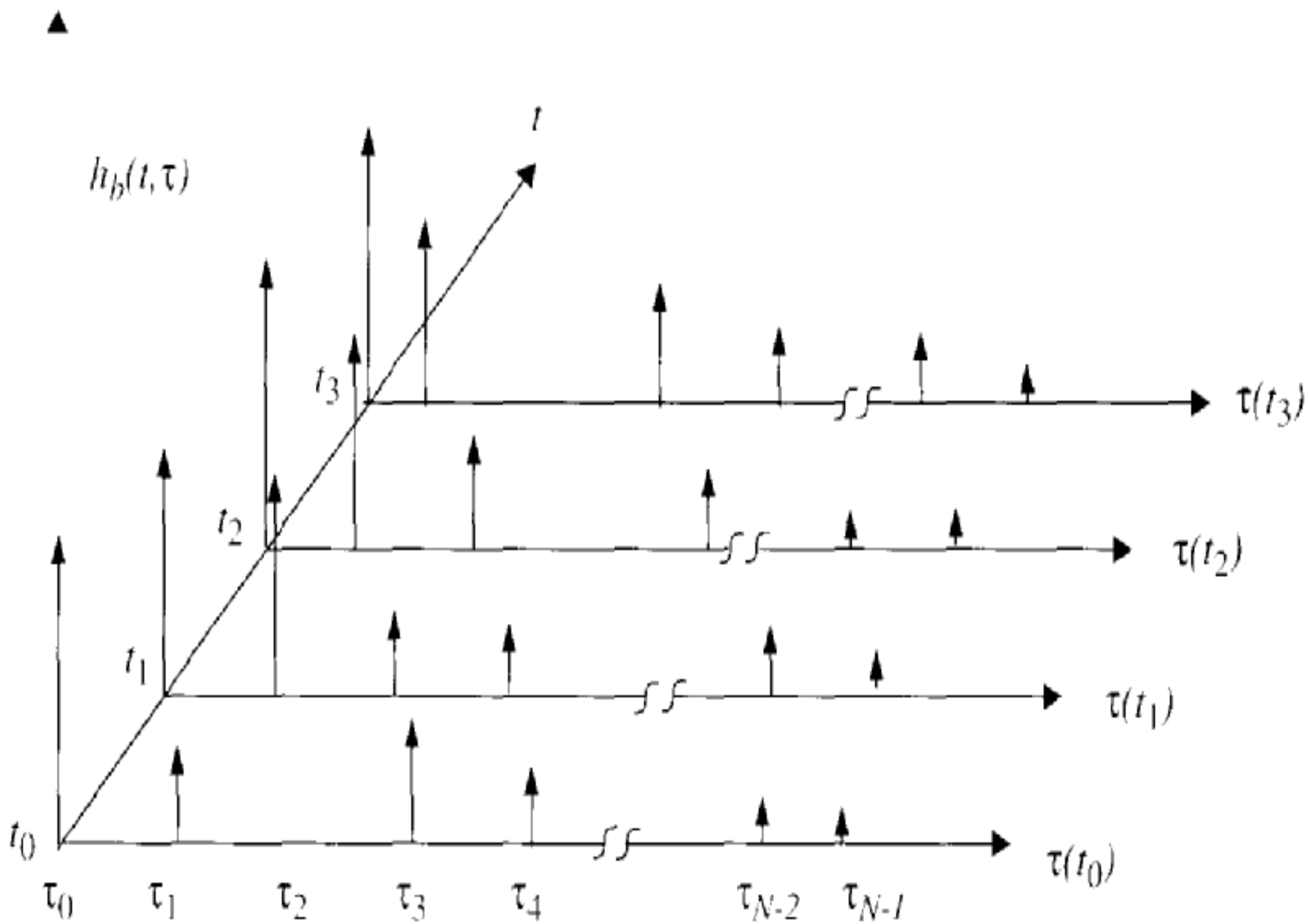


Figure 4.4

An example of the time varying discrete-time impulse response model for a multipath radio channel.

If the channel impulse response is assumed to be time invariant, or is at least wide sense stationary over a small-scale time or distance interval, then the channel impulse response may be simplified as

$$h_b(\tau) = \sum_{i=0}^{N-1} a_i \exp(-j\theta_i)\,\delta(\tau - \tau_i) \tag{4.13}$$

When measuring or predicting $h_b(\tau)$, a probing pulse $p(t)$ which approximates a delta function is used at the transmitter. That is,

$$p(t) \approx \delta(t - \tau) \tag{4.14}$$

is used to sound the channel to determine $h_b(\tau)$.

Power Delay Profile

For small-scale channel modeling, the *power delay profile* of the channel is found by taking the spatial average of $|h_b(t;\tau)|^2$ over a local area. By making several local area measurements of $|h_b(t;\tau)|^2$ in different locations, it is possible to build an ensemble of power delay profiles, each one representing a possible small-scale multipath channel state [Rap91a].

Based on work by Cox [Cox72], [Cox75], if $p(t)$ has a time duration much smaller than the impulse response of the multipath channel, $p(t)$ does not need to be deconvolved from the received signal $r(t)$ in order to determine relative multipath signal strengths. The received power delay profile in a local area is given by

$$P(t;\tau) \approx k|h_b(t;\tau)|^2 \tag{4.15}$$

and many snapshots of $|h_b(t;\tau)|^2$ are typically averaged over a local (small-scale) area to provide a single time-invariant multipath power delay profile $P(\tau)$. The gain k in equation (4.15) relates the transmitted power in the probing pulse $p(t)$ to the total power received in a multipath delay profile.

4.4 Parameters of Mobile Multipath Channels

- Many multipath channel parameters are derived from the power delay profile.

- Depending on the time resolution of the probing pulse and the type of multipath channels studied, researchers often choose to sample at spatial separations of a quarter of a wavelength and over receiver movements no greater than 6 m in outdoor channels and no greater than 2 m in indoor channels in the 450 MHz - 6 GHz range. This small-scale sampling avoids averaging bias in the resulting small-scale statistics.

- Figure 4.9 shows typical power delay profile plots from outdoor and indoor channels, determined from a large number of closely sampled instantaneous profiles.

4.4.1 Time Dispersion Parameters

In order to compare different multipath channels and to develop some general design guidelines for wireless systems, parameters which grossly quantify the multipath channel are used. The *mean excess delay, rms delay spread*, and *excess delay spread (X* dB*)* are multipath channel parameters that can be determined from a power delay profile. The time dispersive properties of wide band multipath channels are most commonly quantified by their mean excess delay ($\bar{\tau}$) and rms delay spread (σ_τ). The mean excess delay is the first moment of the power delay profile and is defined to be

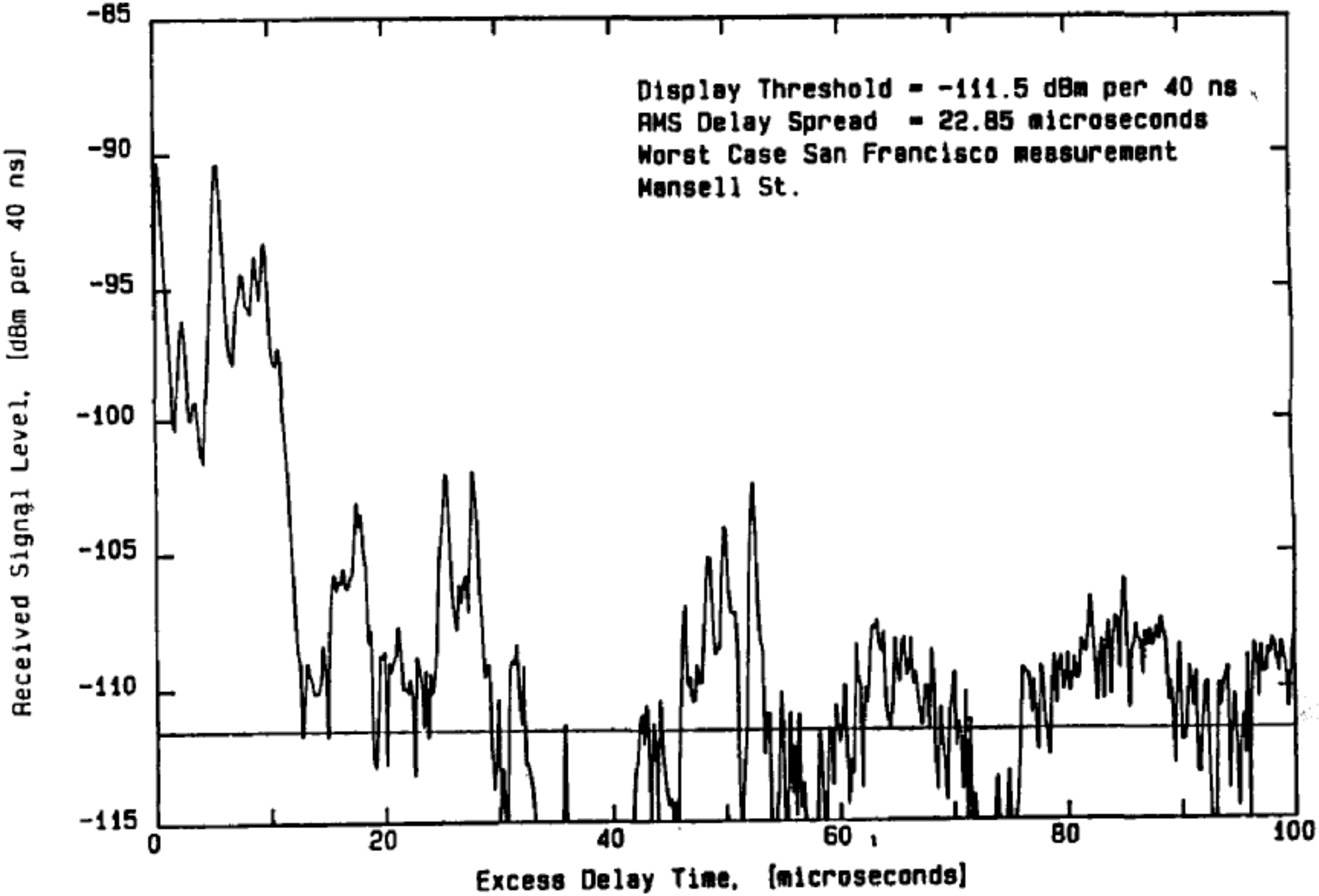
$$\bar{\tau} = \frac{\sum_k a_k^2 \tau_k}{\sum_k a_k^2} = \frac{\sum_k P(\tau_k)\tau_k}{\sum_k P(\tau_k)} \tag{4.35}$$

The rms delay spread is the square root of the second central moment of the power delay profile and is defined to be

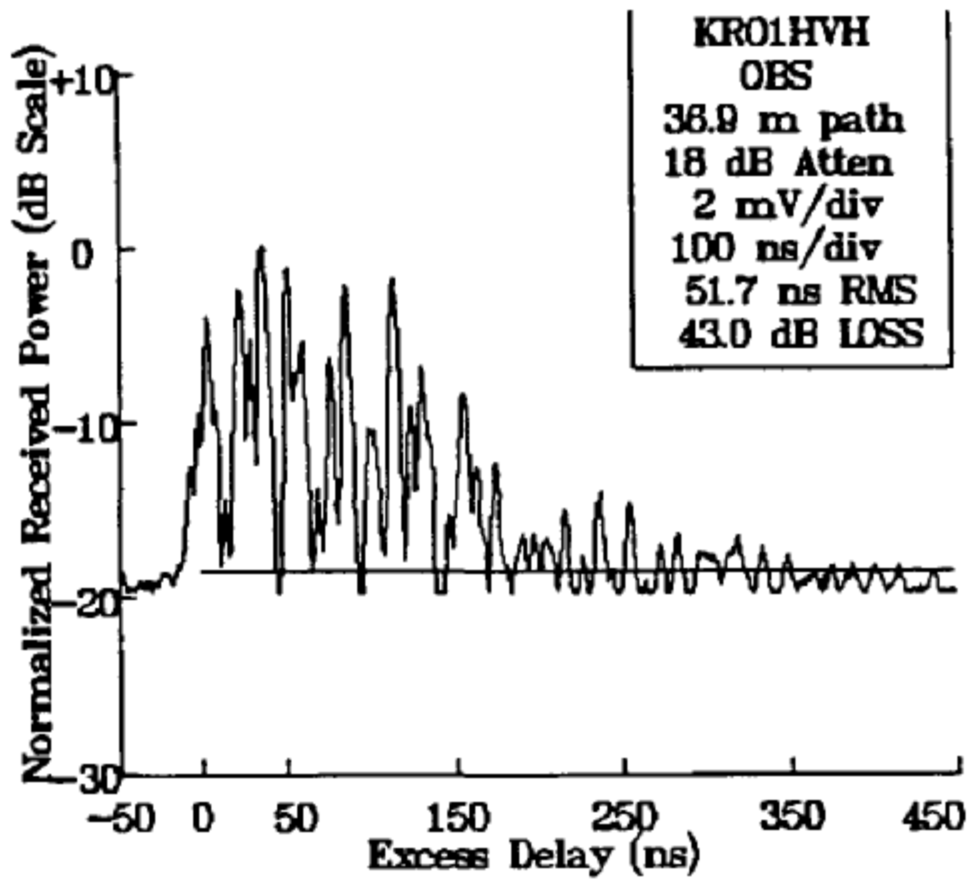
$$\sigma_\tau = \sqrt{\overline{\tau^2} - (\bar{\tau})^2} \tag{4.36}$$

where

$$\overline{\tau^2} = \frac{\sum_k a_k^2 \tau_k^2}{\sum_k a_k^2} = \frac{\sum_k P(\tau_k)\tau_k^2}{\sum_k P(\tau_k)} \tag{4.37}$$



Display Threshold = -111.5 dBm per 40 ns
RMS Delay Spread = 22.85 microseconds
Worst Case San Francisco measurement
Mansell St.
-85
-90
-95
-100
-105
-110
-115
Received Signal Level, [dBm per 40 ns]
0
20
40
60
80
100
Excess Delay Time, [microseconds]



(b)

Figure 4.9

Measured multipath power delay profiles

a) From a 900 MHz cellular system in San Francisco [From [Rap90] © IEEE].

b) Inside a grocery store at 4 GHz [From [Haw91] © IEEE].

Timer Dispersion Parameters

Determined from a power delay profile.

Mean excess delay($\bar{\tau}$):

$$\bar{\tau} = \frac{\sum_k a_k^2 \tau_k}{\sum_k a_k^2} = \frac{\sum_k P(\tau_k)(\tau_k)}{\sum_k P(\tau_k)}$$

Rms delay spread (s_t):

$$\sigma_\tau = \sqrt{\overline{\tau^2} - (\bar{\tau})^2}$$

$$\overline{\tau^2} = \frac{\sum_k a_k^2 \tau_k^2}{\sum_k a_k^2} = \frac{\sum_k P(\tau_k)(\tau_k^2)}{\sum_k P(\tau_k)}$$

The *maximum excess delay (X* dB*)* of the power delay profile is defined to be the time delay during which multipath energy falls to X dB below the maximum. In other words, the maximum excess delay is defined as $\tau_X - \tau_0$, where τ_0 is the first arriving signal and τ_X is the maximum delay at which a multipath component is within X dB of the strongest arriving multipath signal (which does not necessarily arrive at τ_0). Figure 4.10 illustrates the computation of the maximum excess delay for multipath components within 10 dB of the maximum. The maximum excess delay (X dB) defines the temporal extent of the multipath that is above a particular threshold. The value of τ_X is sometimes called the *excess delay spread* of a power delay profile, but in all cases must be specified with a threshold that relates the multipath noise floor to the maximum received multipath component.

- Table 4.1 shows the typical measured values of rms delay spread.
- Typical values of rms delay spread are on the order of microseconds in outdoor mobile radio channels and on the order of nanoseconds in indoor radio channels.
- It is important to note that the rms delay spread and mean excess delay are defined from a single power delay profile which is the temporal or spatial average of consecutive impulse response measurements collected and averaged over a local area.

Table 4.1 Typical Measured Values of RMS Delay Spread

Environment	Frequency (MHz)	RMS Delay Spread (σ_τ)	Notes	Reference
Urban	910	1300 ns avg. 600 ns st. dev. 3500 ns max.	New York City	[Cox75]
Urban	892	10-25 μs	Worst case San Francisco	[Rap90]
Suburban	910	200-310 ns	Averaged typical case	[Cox72]
Suburban	910	1960-2110 ns	Averaged extreme case	[Cox72]
Indoor	1500	10-50 ns 25 ns median	Office building	[Sal87]
Indoor	850	270 ns max.	Office building	[Dev90a]
Indoor	1900	70-94 ns avg. 1470 ns max.	Three San Francisco buildings	[Sei92a]

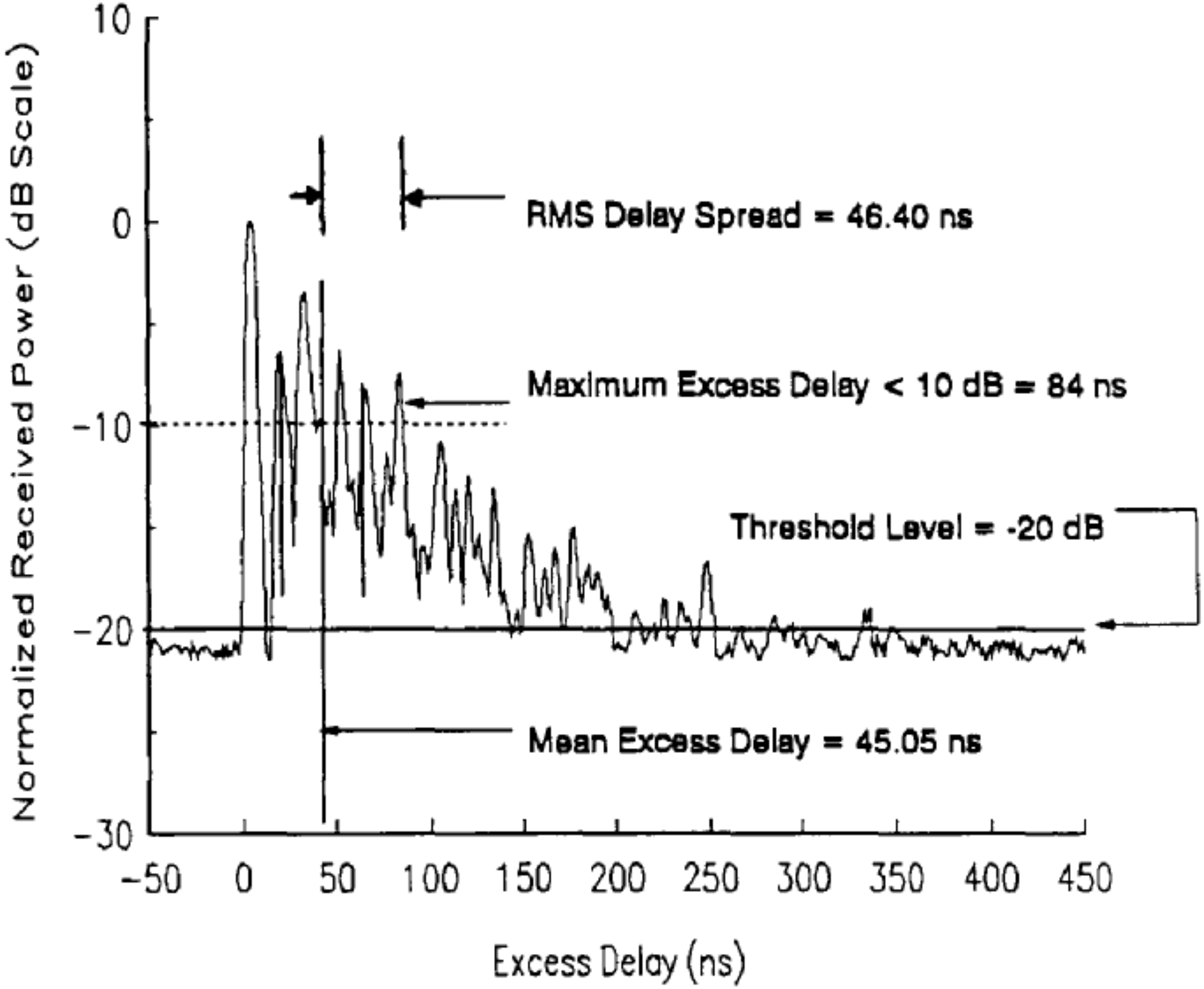
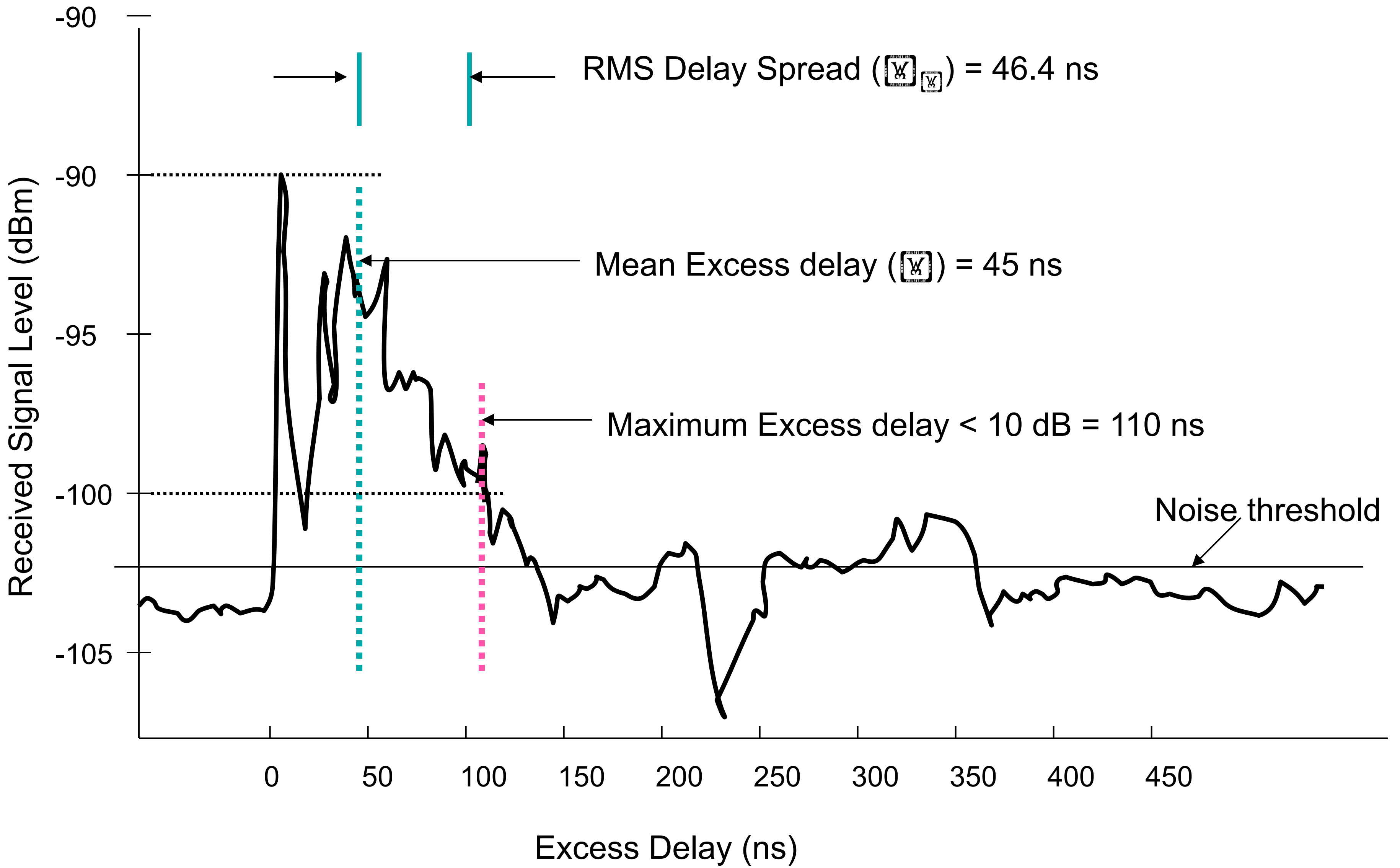


Figure 4.10

Example of an indoor power delay profile; rms delay spread, mean excess delay, maximum excess delay (10 dB), and threshold level are shown.

Power delay Profile



Example 4.4

Calculate the mean excess delay, rms delay spread, and the maximum excess delay (10 dB) for the multipath profile given in the figure below. Estimate the 50% coherence bandwidth of the channel. Would this channel be suitable for AMPS or GSM service without the use of an equalizer?

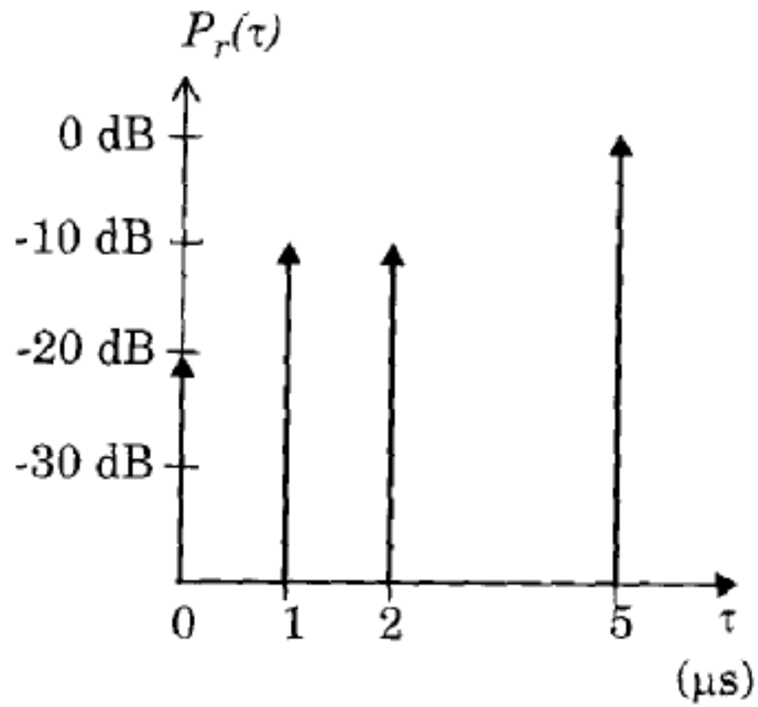


Figure E4.4

Solution to Example 4.4

The rms delay spread for the given multipath profile can be obtained using equations (4.35) — (4.37). The delays of each profile are measured relative to the first detectable signal. The mean excess delay for the given profile

$$\bar{\tau} = \frac{(1)(5) + (0.1)(1) + (0.1)(2) + (0.01)(0)}{[0.01 + 0.1 + 0.1 + 1]} = 4.38\mu s$$

The second moment for the given power delay profile can be calculated as

$$\overline{\tau^2} = \frac{(1)(5)^2 + (0.1)(1)^2 + (0.1)(2)^2 + (0.01)(0)}{1.21} = 21.07\mu s^2$$

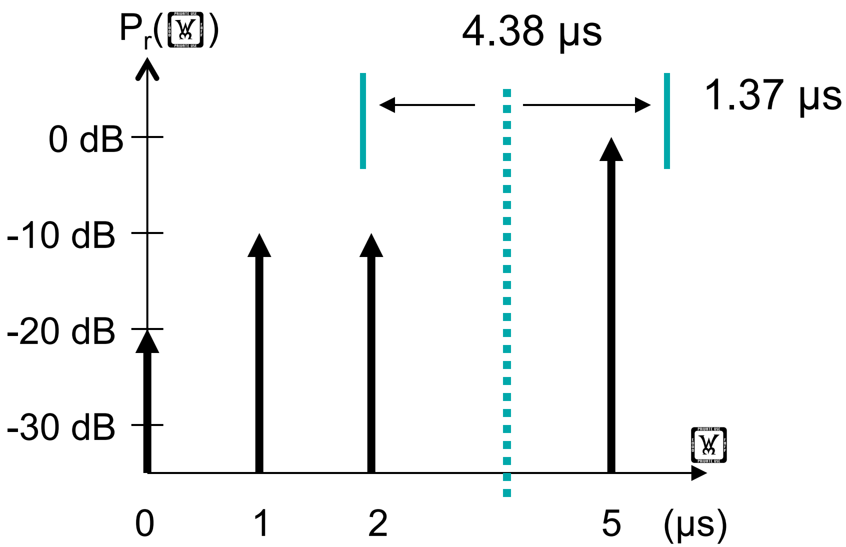
Therefore the rms delay spread, $\sigma_\tau = \sqrt{21.07 - (4.38)^2} = 1.37\mu s$

The coherence bandwidth is found from equation (4.39) to be

$$B_c \approx \frac{1}{5\sigma_\tau} = \frac{1}{5(1.37\mu s)} = 146 \text{ kHz}$$

Since B_c is greater than 30 kHz, AMPS will work without an equalizer. However, GSM requires 200 kHz bandwidth which exceeds B_c, thus an equalizer would be needed for this channel.

Example (Power delay profile)

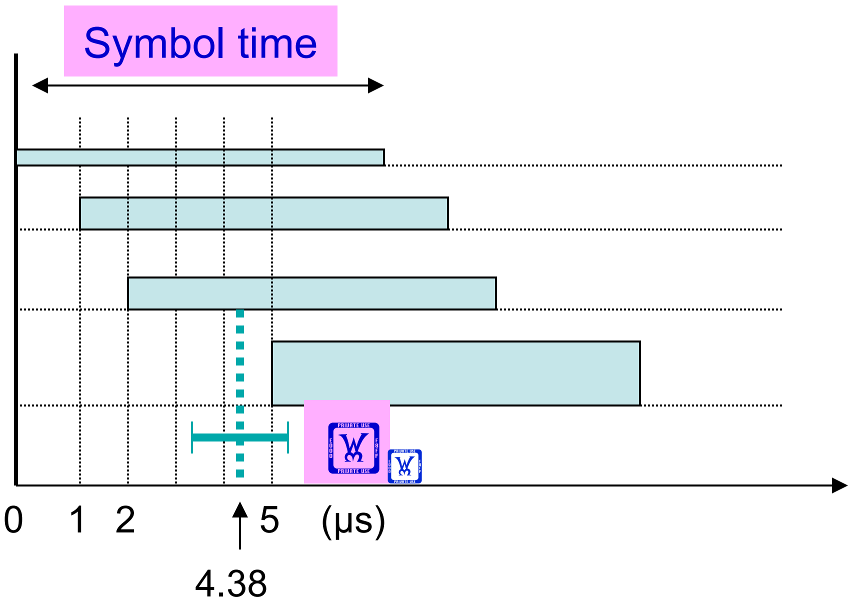


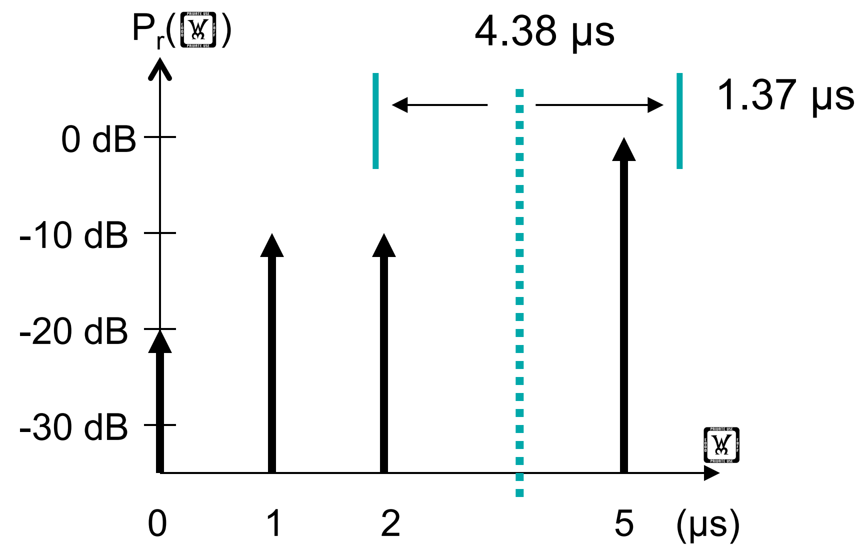
$$\bar{\tau} = \frac{(1)(5)+(0.1)(1)+(0.1)(2)+(0.01)(0)}{[0.01+0.1+0.1+1]} = 4.38\mu s$$

$$\overline{\tau^2} = \frac{(1)(5)^2+(0.1)(1)^2+(0.1)(2)^2+(0.01)(0)^2}{[0.01+0.1+0.1+1]} = 21.07\mu s^2$$

$$\sigma_\tau = \sqrt{21.07-(4.38)^2} = 1.37\mu s$$

Inter Symbol Interference





Symbol time > 10* --- No equalization required

Symbol time < 10* --- Equalization will be required to deal with ISI

In the above example, symbol time should be more than 14μs to avoid ISI.
This means that link speed must be less than 70Kbps (approx)

2.5 Multipath Delay Spread

Multipath occurs when signals arrive at the receiver directly from the transmitter and, indirectly, due to transmission through objects or reflection. The amount of signal reflection depends on factors such as angle of arrival, carrier frequency, and polarization of incident wave. Because the path lengths are different between the direct path and the reflected path(s), different signal paths could arrive at the receiver at different times over different distances. Figure 2.3 illustrates the concept. An impulse is transmitted at time 0; assuming that there are a multitude of reflected paths present, a receiver approximately 1 km away should detect a series of pulses, or *delay spread*.

If the time difference Δt is significant compared to one symbol period, *intersymbol interference* (ISI) can occur. In other words, symbols arriving significantly earlier or later than their own symbol periods can corrupt preceding or trailing symbols. For a fixed-path difference and a given delay spread, a higher data rate system is more likely to suffer ISI due to delay spread. For a fixed data rate system, a propagation environment with longer path differences (and thus higher delay spread) is more likely to cause ISI.

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2. Coherence Bandwidth

- While the delay spread is a natural phenomenon caused by multipaths in the radio channel, the coherence bandwidth, **B**, is a defined relation derived from **the rms delay spread.**

- Coherence bandwidth is a statistical measure of the range of frequencies over which the channel can be considered "flat" (i.e., a channel which passes all spectral components with approximately equal gain and linear phase).

- **Two sinusoids with frequency separation greater than B are affected quite differently by the channel.**

If the coherence bandwidth is defined as the bandwidth over which the frequency correlation function is above 0.9, then the coherence bandwidth is approximately

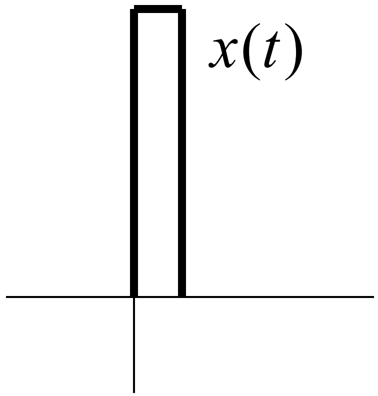
$$B_c \approx \frac{1}{50\sigma_\tau} \tag{4.38}$$

If the definition is relaxed so that the frequency correlation function is above 0.5, then the coherence bandwidth is approximately

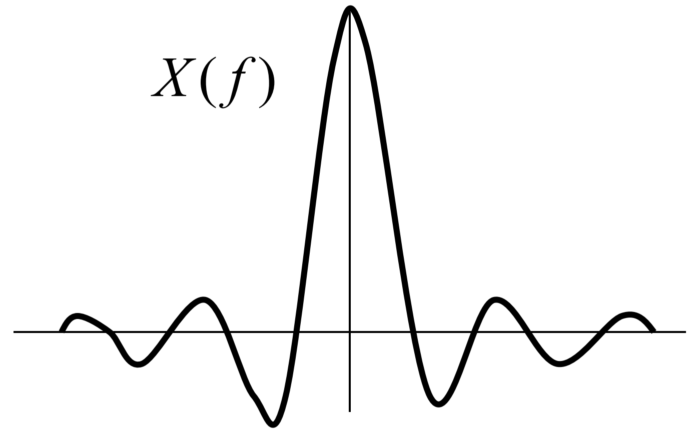
$$B_c \approx \frac{1}{5\sigma_\tau} \tag{4.39}$$

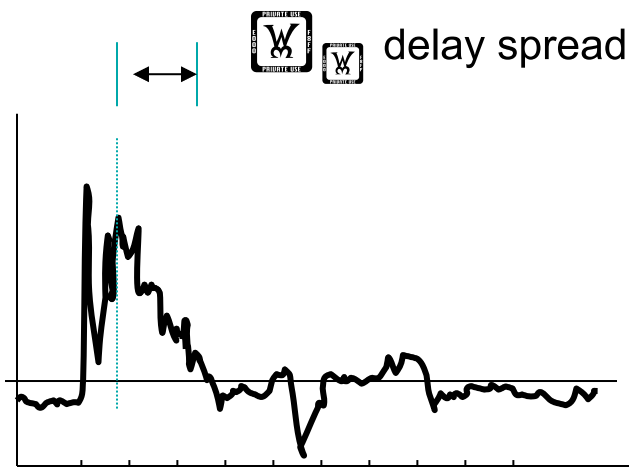
Coherence Bandwidth

Time domain view

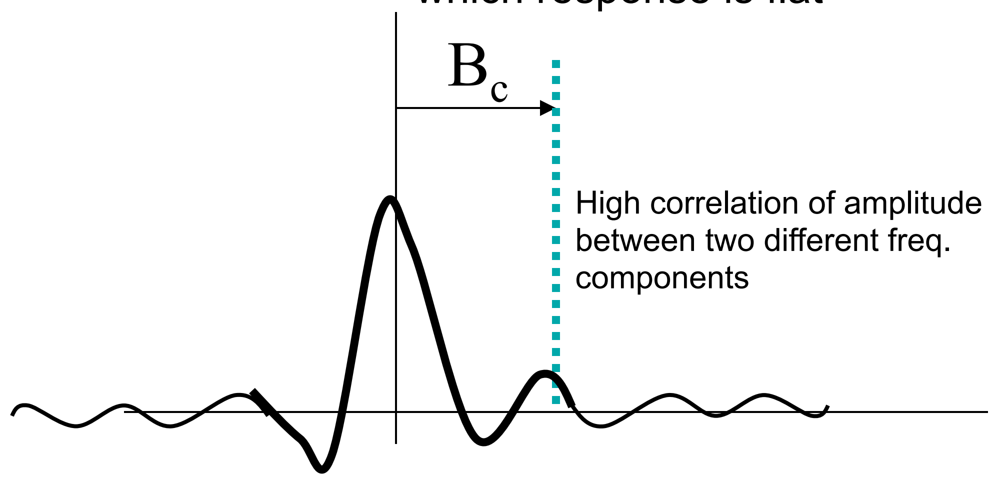


Freq. domain view





Range of freq over which response is flat



RMS delay spread and coherence BW

$$B_c \alpha \frac{1}{\sigma_\tau}$$

- RMS delay spread and coherence b/w (B_c) are inversely proportional

$$B_c \approx \frac{1}{50.\sigma_\tau}$$

For 0.9 correlation

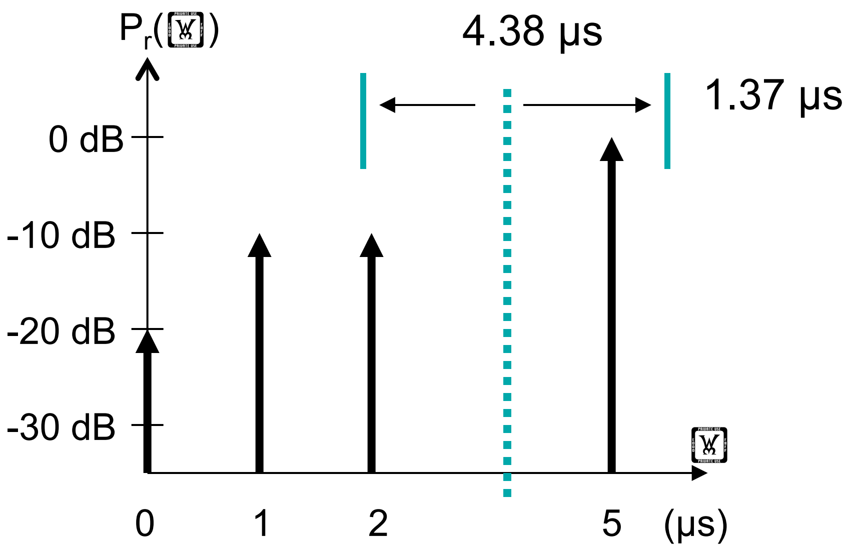
$$B_c \approx \frac{1}{5.\sigma_\tau}$$

For 0.5 correlation

Coherence Bandwidth

- Example:
  - For a multipath channel, s is given as 1.37ms.
  - The 50% coherence bandwidth is given as: 1/5s = 146kHz.
    - This means that, for a good transmission from a transmitter to a receiver, the range of transmission frequency (channel bandwidth) should not exceed 146kHz, so that all frequencies in this band experience the same channel characteristics.
    - Equalizers are needed in order to use transmission frequencies that are separated larger than this value.
    - This coherence bandwidth is enough for an AMPS channel (30kHz band needed for a channel), but is not enough for a GSM channel (200kHz needed per channel).

Revisit Example (Power delay profile)



$$\bar{\tau} = 4.38 \mu s$$

$$\sigma_\tau = 1.37 \mu s$$

$$\bar{\tau}^2 = 21.07 \mu s^2$$

$$(50\% - coherence) B_c \approx \frac{1}{5.\sigma_\tau} = 146 kHz$$

Signal bandwidth for Analog Cellular = 30 KHz
Signal bandwidth for GSM = 200 KHz

Doppler spread and coherence time

- Delay spread and Coherence bandwidth describe **the time dispersive nature** of the channel in a local area.
  - They don't offer information about the time varying nature of the channel caused by relative motion of transmitter and receiver or the movement of objects in the channel.

- Doppler Spread and Coherence time are parameters which describe **the time varying nature** of the channel in a small-scale region.

Doppler spread and coherence time

Coherence time definition implies that two signals arriving with a time separation greater than T_C are affected differently by the channel.

- Doppler spread and coherence time (T_c) are inversely proportional

$$T_c \alpha \frac{1}{f_m}$$

f_m is the max doppler shift

$$T_c \approx \frac{9}{16\pi f_m}$$

For 0.5 correlation

$$T_c \approx \frac{0.423}{f_m}$$

Rule of thumb

Doppler Spread

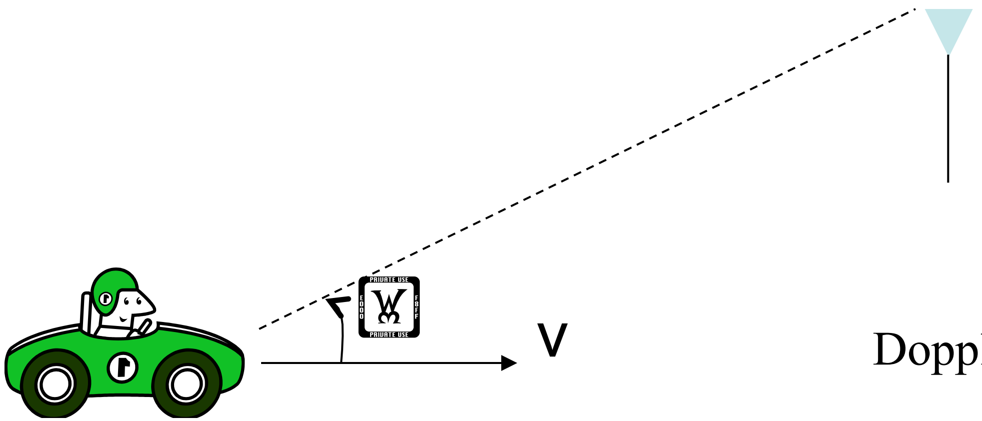
- Measure of spectral broadening caused by motion

- We know how to compute Doppler shift: f_d

- Doppler spread: **B_D**, is defined as the maximum Doppler shift: **f_m = v/l**

- If the baseband signal bandwidth (B_s) is much greater than (B_D) then effect of Doppler spread is negligible at the receiver.

Doppler Shift



Doppler shift $$\Delta f = \frac{v\cos\theta}{\lambda}$$

Example

- Carrier frequency f_c = 1850 MHz (i.e. [illegible] = 16.2 cm)
- Vehicle speed v = 60 mph = 26.82 m/s

- If the vehicle is moving directly towards the transmitter

$$\Delta f = \frac{26.82}{0.162} = 165Hz$$

- If the vehicle is moving perpendicular to the angle of arrival of the transmitted signal

$$\Delta f = 0$$

Types of Small-Scale Fading

The type of fading experienced by a signal propagating through a mobile radio channel depends on:

the nature of the transmitted signal with respect to the characteristics of the channel.

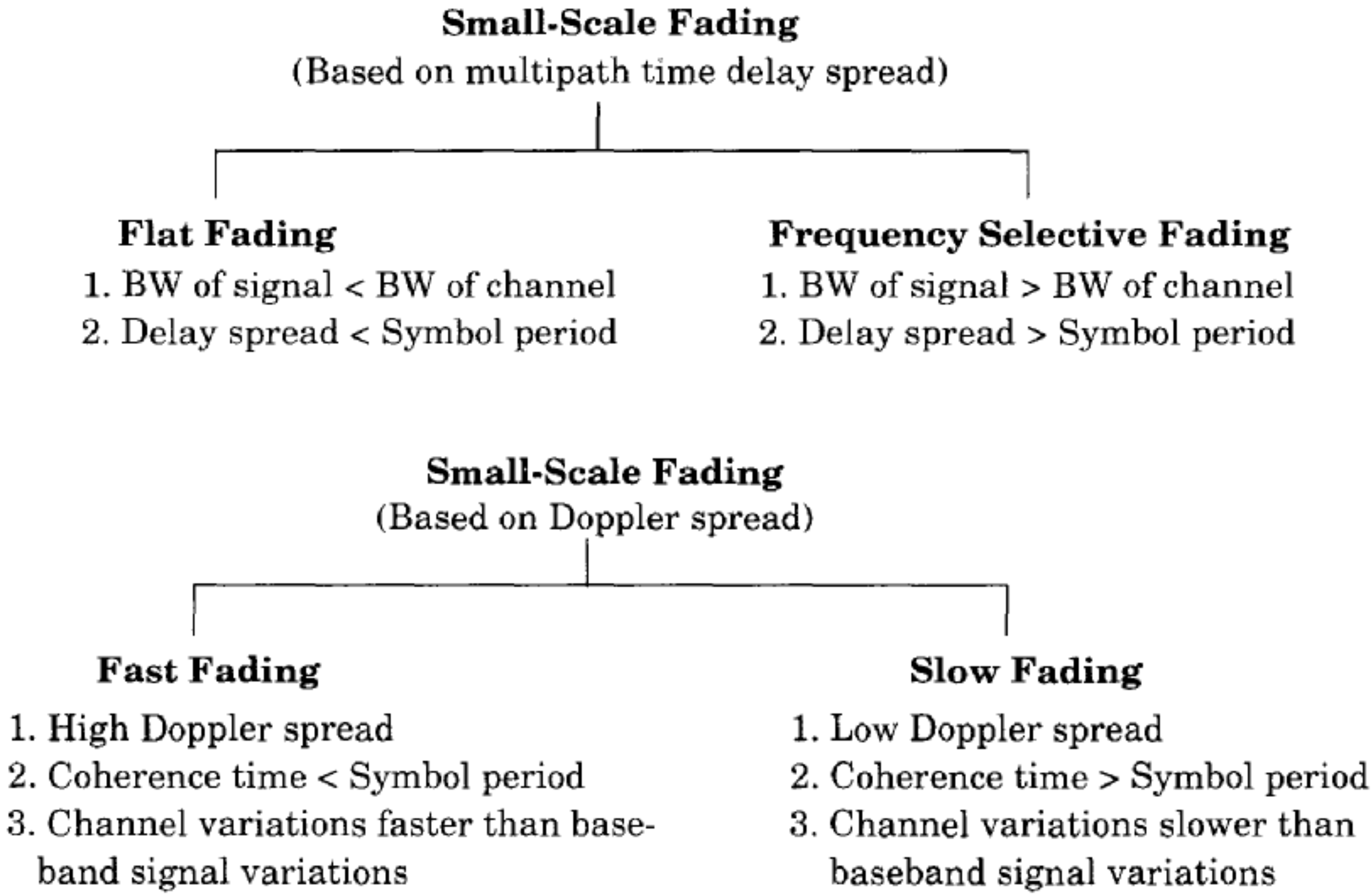


Figure 4.11
Types of small-scale fading.

Types of Small-scale Fading

Small-scale Fading
(Based on Multipath Tİme Delay Spread)

Flat Fading

1. BW Signal < BW of Channel
2. Delay Spread < Symbol Period

Frequency Selective Fading

1. BW Signal > Bw of Channel
2. Delay Spread > Symbol Period

Small-scale Fading
(Based on Doppler Spread)

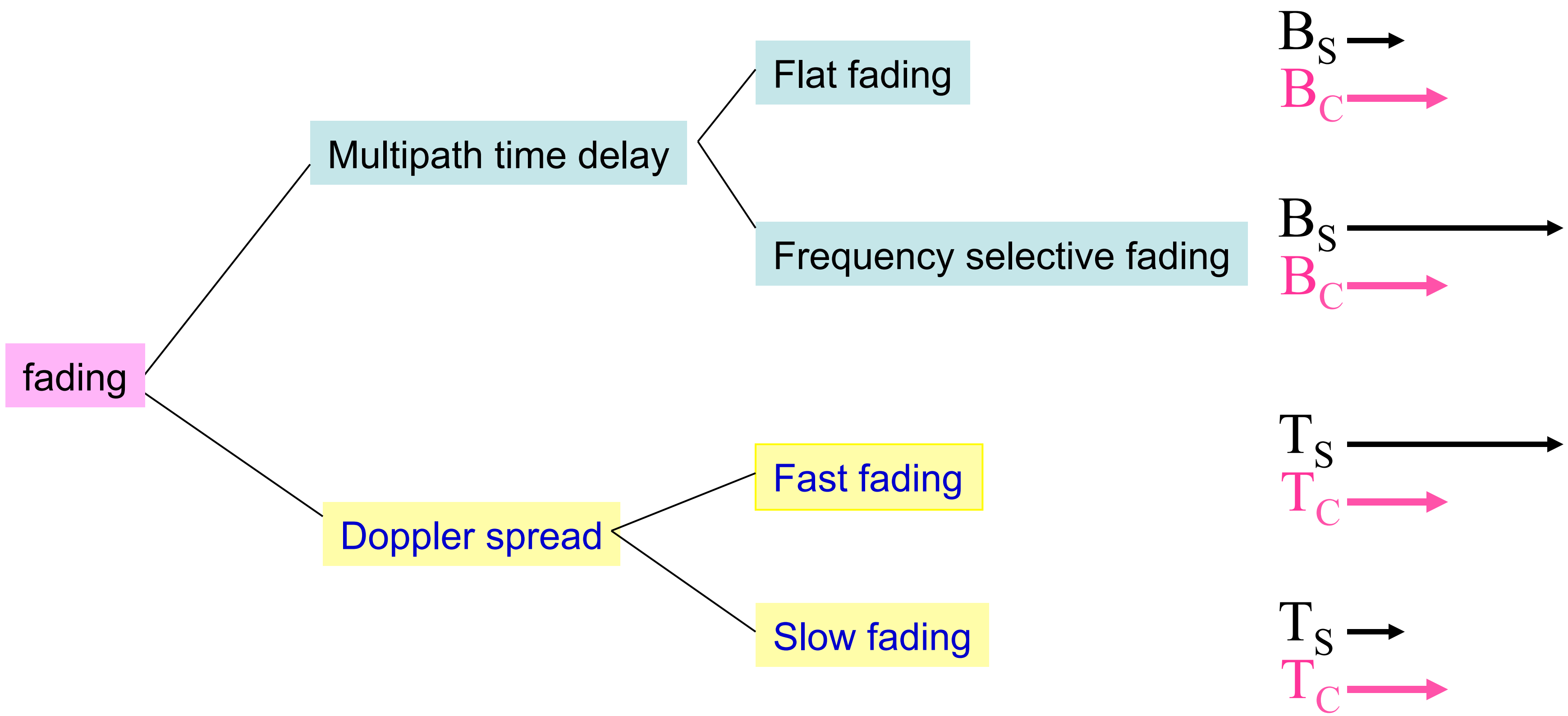
Fast Fading

1. High Doppler Spread
2. Coherence Time < Symbol Period
3. Channel variations faster than baseband signal variations

Slow Fading

1. Low Doppler Spread
2. Coherence Time > Symbol Period
3. Channel variations smaller than baseband signal variations

Small scale fading



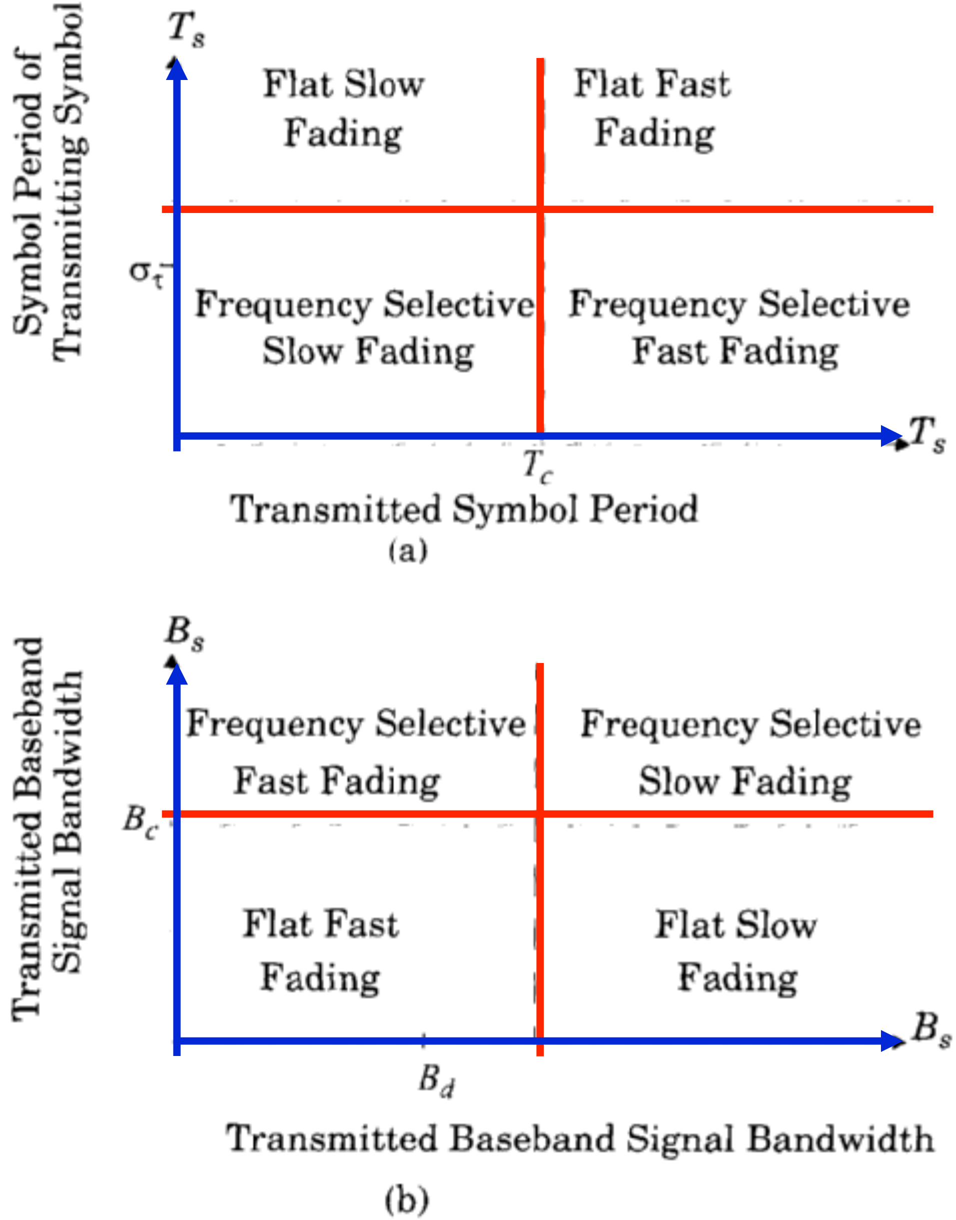


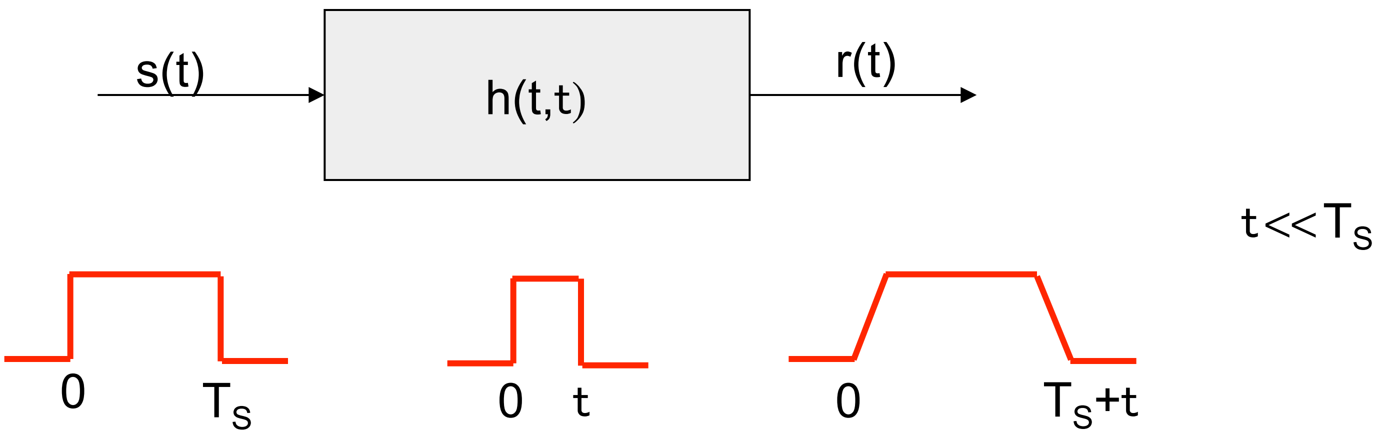
Figure 4.14

Matrix illustrating type of fading experienced by a signal as a function of
(a) symbol period
(b) baseband signal bandwidth.

Flat Fading

- Occurs when the amplitude of the received signal changes with time
    - For example according to Rayleigh Distribution
- Occurs when symbol period of the transmitted signal is much larger than the Delay Spread of the channel
    - Bandwidth of the applied signal is narrow.
- May cause deep fades.
    - Increase the transmit power to combat this situation.

Flat Fading



Occurs when:

$B_S << B_C$

and

$T_S >> s_t$

B_C: Coherence bandwidth
B_S: Signal bandwidth
T_S: Symbol period
s_t: Delay Spread

Flat Fading

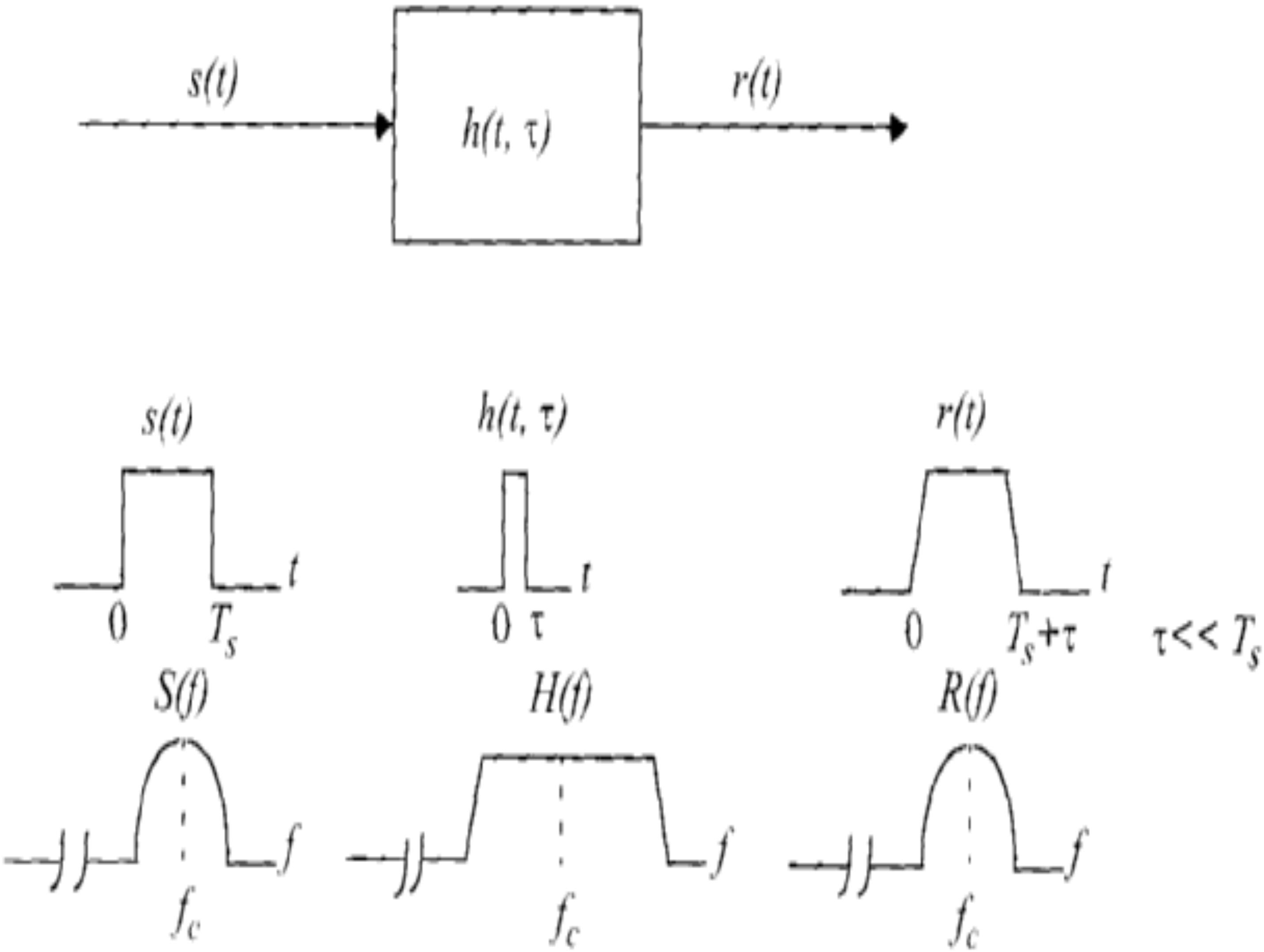
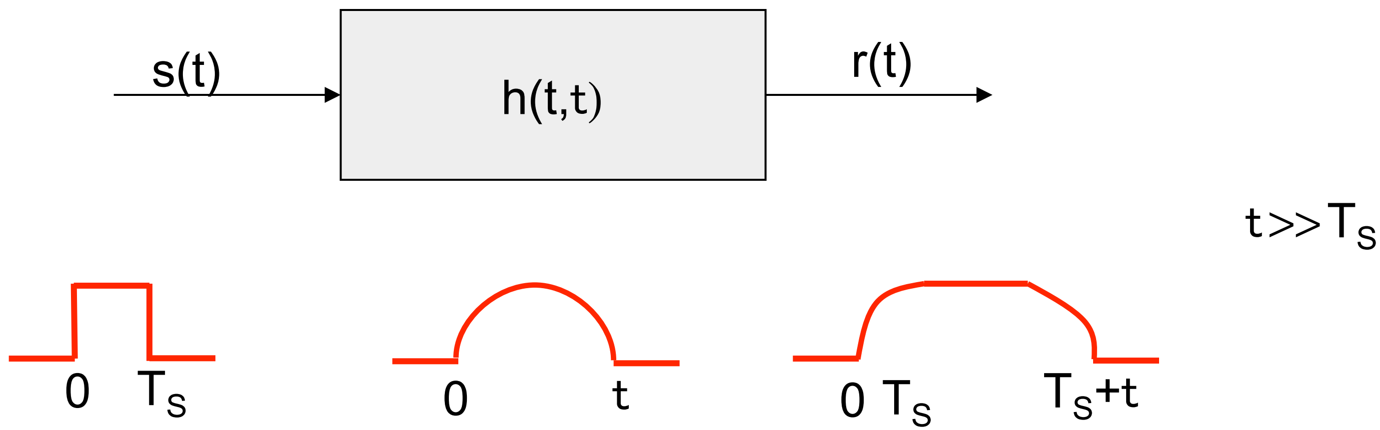


Figure 4.12
Flat fading channel characteristics.

Frequency Selective Fading

- Occurs when channel multipath delay spread is greater than the symbol period.
  - Symbols face time dispersion
  - Channel induces Intersymbol Interference (ISI)
- Bandwidth of the signal s(t) is wider than the channel impulse response.

Frequency Selective Fading



Causes distortion of the received baseband signal

Causes Inter-Symbol Interference (ISI)

Occurs when:
$B_S > B_C$
and
$T_S < s_t$

As a rule of thumb: $T_S < s_t$

Frequency Selective Fading

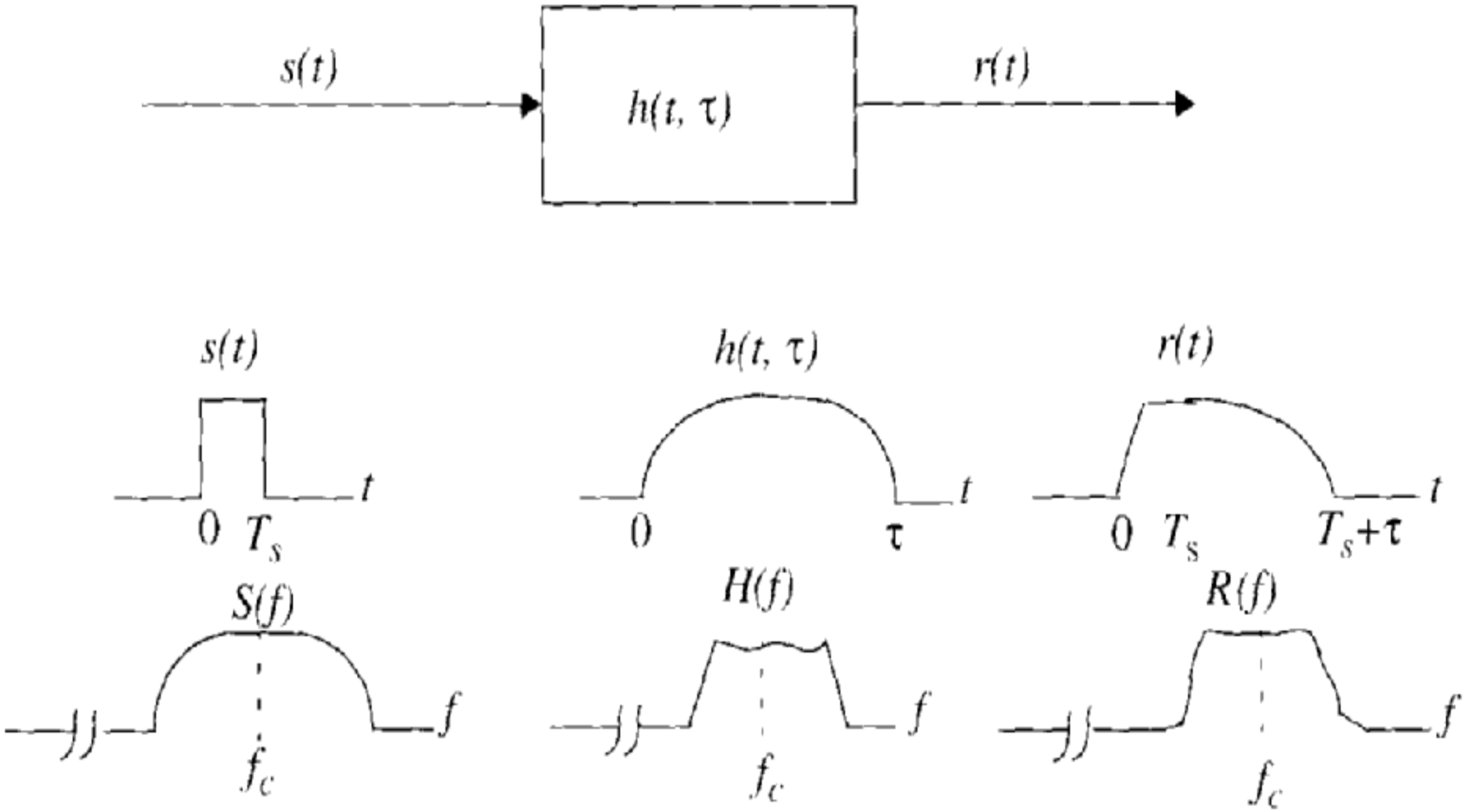


Figure 4.13
Frequency selective fading channel characteristics.

Fast Fading

- Due to Doppler Spread
  - Rate of change of the channel characteristics
    is **larger** than the
    Rate of change of the transmitted signal
  - The channel changes during a symbol period.
  - The channel changes because of receiver motion.
  - Coherence time of the channel is smaller than the symbol period of the transmitter signal

Occurs when:
$B_S < B_D$
and
$T_S > T_C$

B_S: Bandwidth of the signal
B_D: Doppler Spread
T_S: Symbol Period
T_C: Coherence Bandwidth

Slow Fading

- Due to Doppler Spread
  - Rate of change of the channel characteristics is **much smaller** than the Rate of change of the transmitted signal

Occurs when:
$B_S >> B_D$
and
$T_S << T_C$

B_S: Bandwidth of the signal
B_D: Doppler Spread
T_S: Symbol Period
T_C: Coherence Bandwidth

Fading Distributions

- Describes how the received signal amplitude changes with time.
  - Remember that the received signal is combination of multiple signals arriving from different directions, phases and amplitudes.
  - With the received signal we mean the baseband signal, namely the envelope of the received signal (i.e. r(t)).
- Its is a statistical characterization of the multipath fading.
- Two distributions
  - Rayleigh Fading
  - Ricean Fading

Rayleigh and Ricean Distributions

- Describes the received signal envelope distribution for channels, where all the components are non-LOS:
  - i.e. there is no line-of–sight (LOS) component.
- Describes the received signal envelope distribution for channels where one of the multipath components is LOS component.
  - i.e. there is one LOS component.

Rayleigh

Rayleigh distribution has the probability density function (PDF) given by:

$$p(r) = \begin{cases} \dfrac{r}{\sigma^2} e^{\left(-\frac{r^2}{2\sigma^2}\right)} & (0 \le r \le \infty) \\ 0 & (r < 0) \end{cases}$$

s^2 is the time average power of the received signal before envelope detection.
s is the rms value of the received voltage signal before envelope detection

Remember: $\overline{P}$ (average power) $\propto V_{rms}^2$ (see end of slides 5)

Rayleigh

The probability that the envelope of the received signal does not exceed a specified value of R is given by the CDF:

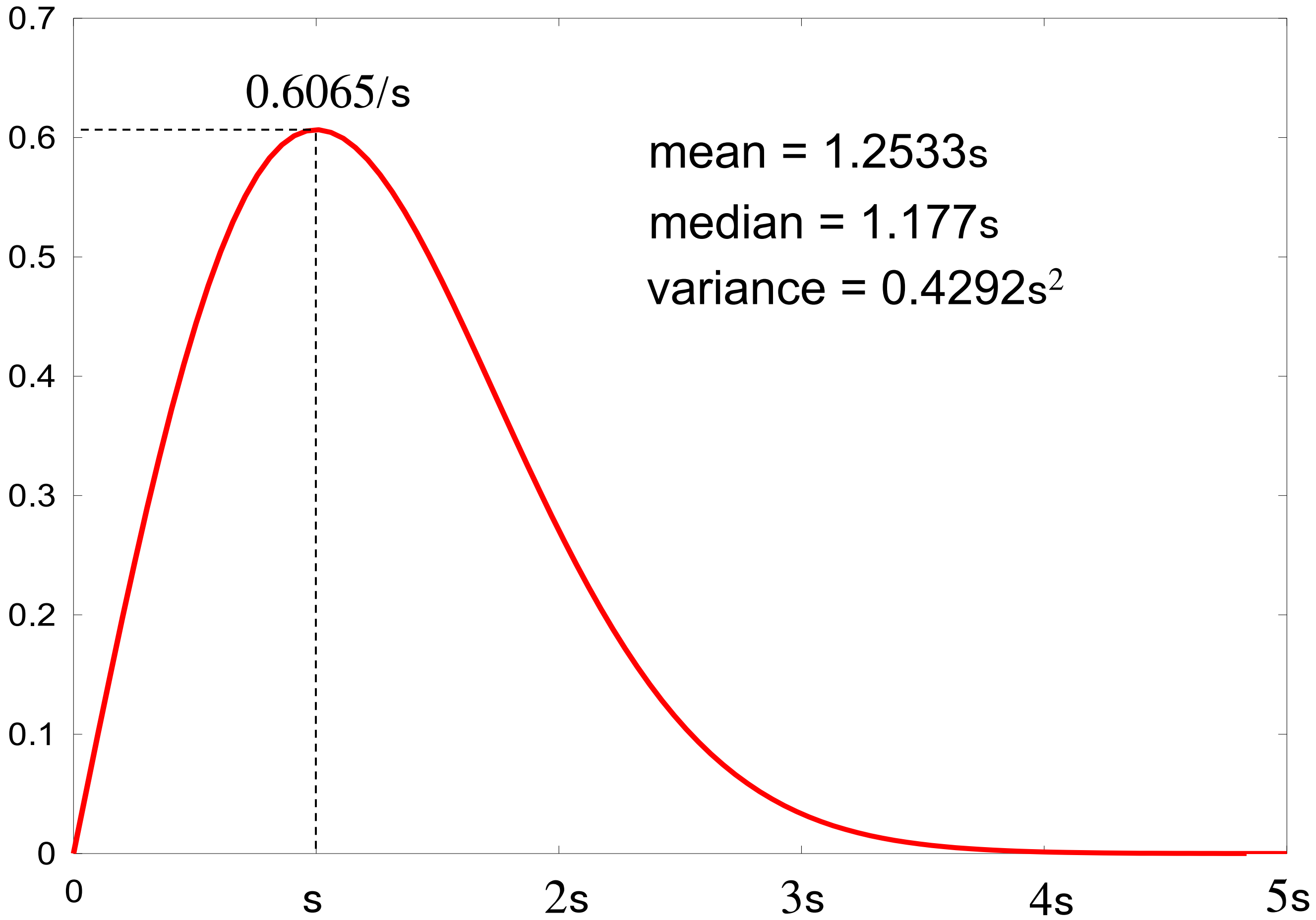
$$P(R) = P_r(r \le R) = \int_0^R p(r)dr = 1 - e^{-\frac{R^2}{2\sigma^2}}$$

$$r_{mean} = E[r] = \int_0^\infty rp(r)dr = \sigma\sqrt{\frac{\pi}{2}} = 1.2533\sigma$$

$r_{median} = 1.177\sigma$ found by solving $\frac{1}{2} = \int_0^{r_{median}} p(r)dr$

$$r_{rms} = \sqrt{2}\sigma$$

Rayleigh PDF



Ricean Distribution

- When there is a stationary (non-fading) LOS signal present, then the envelope distribution is Ricean.
- The Ricean distribution degenerates to Rayleigh when the dominant component fades away.

How do systems handle fading problem?

Analog

- Narrowband transmission

GSM

- Adaptive channel equalization
- Channel estimation training sequence

DECT

- Use the handset only in small cells with small delay spreads
- Diversity and channel selection can help a little bit (pick a channel where late reflections are in a fade)

IS95 Cellular CDMA

- Rake receiver separately recovers signals over paths with excessive delays

Digital Audio Broacasting

- OFDM multi-carrier modulation: The radio channel is split into many narrowband (ISI- free) subchannels

4.2 If a particular modulation provides suitable BER performance whenever $\sigma/T_s \leq 0.1$, determine the smallest symbol period T_s (and thus the greatest symbol rate) that may be sent through RF channels shown in Figure P4.2, without using an equalizer.

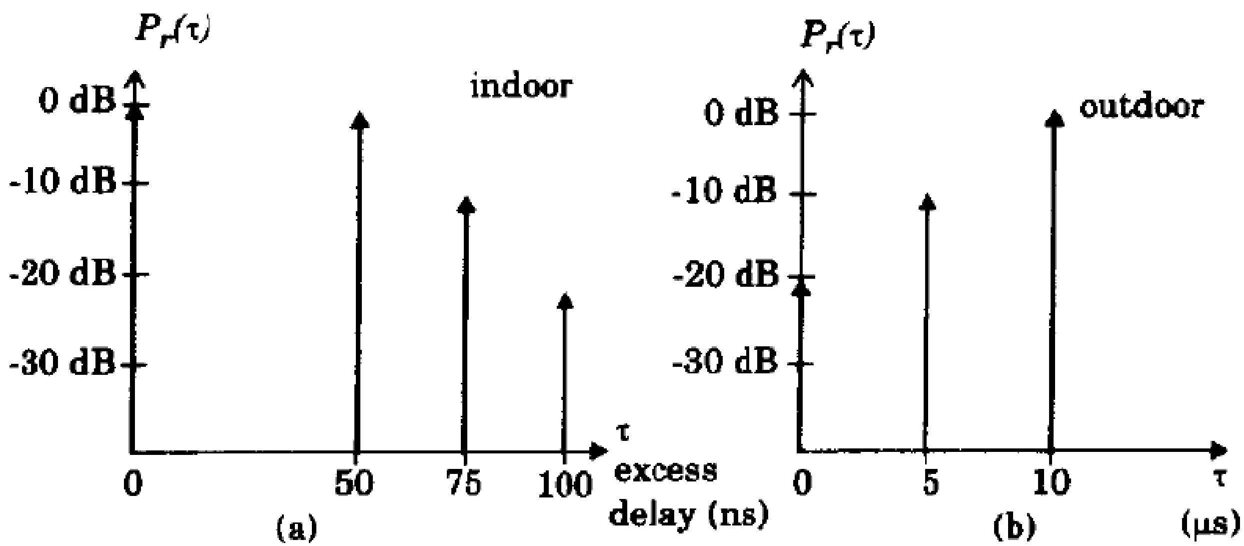


Figure P4.2: Two channel responses for Problem 4.2

5.6 For (a), $\bar{\tau} = \frac{1\times0+1\times50+0.1\times75+0.01\times100}{1+1+0.1+0.01} \doteq 27.725\,(ns)$

$$\overline{\tau^2} = \frac{1\times0+1\times50^2+0.1\times75^2+0.01\times100^2}{1+1+0.1+0.01} \doteq 1498.8\,(ns^2)$$

$\Rightarrow$ the rms delay spread $\sigma_\tau = \sqrt{1498.8-27.725^2} \doteq 27\,(ns)$

Since $\frac{\sigma_\tau}{T_s} \le 0.1$, $T_s \ge 10\sigma_\tau = 270\,ns$

$\Rightarrow$ Smallest symbol period $T_{s\,min} = 270ns$

greatest data rate $R_{max} = \frac{1}{T_{s\,min}} \doteq 3.7\,Mbps$

For (b), $\bar{\tau} = \frac{0.01\times0+0.1\times5+1\times10}{0.01+0.1+1} \doteq 9.46\,(\mu s)$

$$\overline{\tau^2} = \frac{0.01\times0+0.1\times5^2+1\times10^2}{0.01+0.1+1} = 92.34\,(\mu s^2)$$

$$\sigma_\tau = \sqrt{\overline{\tau^2}-(\bar{\tau})^2} = \sqrt{92.34-(9.46)^2} \doteq 1.688\,(\mu s)$$

$T_{s\,min} = 10\,\sigma_\tau = 16.88\,(\mu s)$ $\quad R_{max} = \frac{1}{T_{s\,min}} \doteq 59.25\,kbps$