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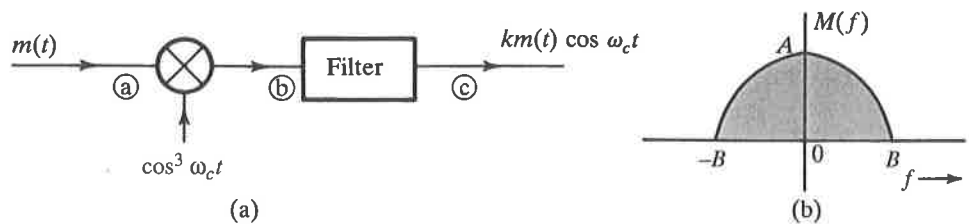
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## PROBLEMS

- 4.2-1** For each of the baseband signals: (i)  $m(t) = \cos 1000\pi t$ ; (ii)  $m(t) = 2 \cos 1000\pi t + \sin 2000\pi t$ ; (iii)  $m(t) = \cos 1000\pi t \cos 3000\pi t$ , do the following.
- (a) Sketch the spectrum of  $m(t)$ .
  - (b) Sketch the spectrum of the DSB-SC signal  $m(t) \cos 10,000\pi t$ .
  - (c) Identify the upper sideband (USB) and the lower sideband (LSB) spectra.
  - (d) Identify the frequencies in the baseband, and the corresponding frequencies in the DSB-SC, USB, and LSB spectra. Explain the nature of frequency shifting in each case.
- 4.2-2** Repeat Prob. 4.2-1 [parts (a), (b), and (c) only] if: (i)  $m(t) = \text{sinc}(100t)$ ; (ii)  $m(t) = e^{-|t|}$ ; (iii)  $m(t) = e^{-|t-1|}$ . Observe that  $e^{-|t-1|}$  is  $e^{-|t|}$  delayed by 1 second. For the last case you need to consider both the amplitude and the phase spectra.
- 4.2-3** Repeat Prob. 4.2-1 [parts (a), (b), and (c) only] for  $m(t) = e^{-|t|}$  if the carrier is  $\cos(10,000t - \pi/4)$ .  
*Hint:* Use Eq. (3.37).
- 4.2-4** You are asked to design a DSB-SC modulator to generate a modulated signal  $km(t) \cos(\omega_c t + \theta)$ , where  $m(t)$  is a signal band-limited to  $B$  Hz. Figure P4.2-4 shows a DSB-SC modulator available in the stock room. The carrier generator available generates not  $\cos \omega_c t$ , but  $\cos^3 \omega_c t$ . Explain whether you would be able to generate the desired signal using only this equipment. You may use any kind of filter you like.
- (a) What kind of filter is required in Fig. P4.2-3?
  - (b) Determine the signal spectra at points  $b$  and  $c$ , and indicate the frequency bands occupied by these spectra.
  - (c) What is the minimum usable value of  $\omega_c$ ?
  - (d) Would this scheme work if the carrier generator output were  $\sin^3 \omega_c t$ ? Explain.
  - (f) Would this scheme work if the carrier generator output were  $\cos^n \omega_c t$  for any integer  $n \geq 2$ ?

**Figure P.4.2-4**



4.2-5 You are asked to design a DSB-SC modulator to generate a modulated signal  $km(t) \cos \omega_c t$  with the carrier frequency  $f_c = 300$  kHz ( $\omega_c = 2\pi \times 300,000$ ). The following equipment is available in the stock room: (i) a signal generator of frequency 100 kHz; (ii) a ring modulator; (iii) a bandpass filter tuned to 300 kHz.

- (a) Show how you can generate the desired signal.
- (b) If the output of the modulator is  $k \cdot m(t) \cos \omega_c t$ , find  $k$ .

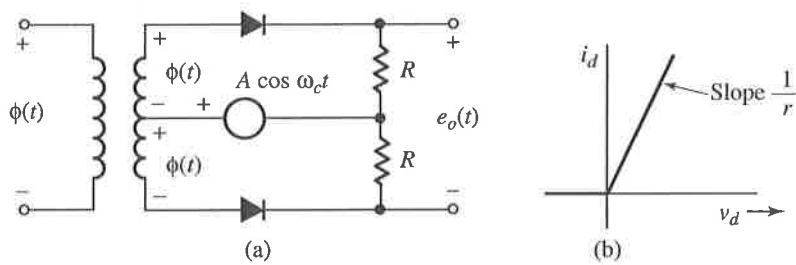
4.2-6 Amplitude modulators and demodulators can also be built without using multipliers. In Fig. P4.2-6, the input  $\phi(t) = m(t)$ , and the amplitude  $A \gg |\phi(t)|$ . The two diodes are identical, with a resistance of  $r$  ohms in the conducting mode and infinite resistance in the cutoff mode. Show that the output  $e_o(t)$  is given by

$$e_o(t) = \frac{2R}{R+r} w(t) m(t)$$

where  $w(t)$  is the switching periodic signal shown in Fig. 2.20a with period  $2\pi/\omega_c$  seconds.

- (a) Hence, show that this circuit can be used as a DSB-SC modulator.
- (b) How would you use this circuit as a synchronous demodulator for DSB-SC signals.

Figure P.4.2-6



4.2-7 In Fig. P4.2-6, if  $\phi(t) = \sin(\omega_c t + \theta)$ , and the output  $e_o(t)$  is passed through a low-pass filter, then show that this circuit can be used as a phase detector, that is, a circuit that measures the phase difference between two sinusoids of the same frequency ( $\omega_c$ ).  
Hint: Show that the filter output is a dc signal proportional to  $\sin \theta$ .

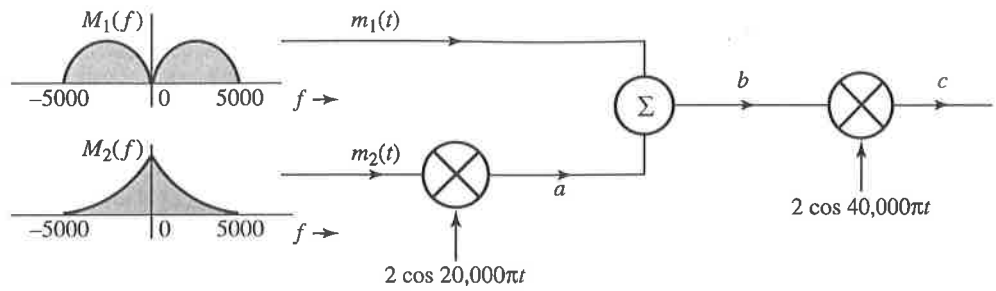
4.2-8 Two signals  $m_1(t)$  and  $m_2(t)$ , both band-limited to 5000 Hz, are to be transmitted simultaneously over a channel by the multiplexing scheme shown in Fig. P4.2-8. The signal at point  $b$  is the multiplexed signal, which now modulates a carrier of frequency 20,000 Hz. The modulated signal at point  $c$  is transmitted over a channel.

- (a) Sketch signal spectra at points  $a$ ,  $b$ , and  $c$ .
- (b) What must be the bandwidth of the channel?
- (c) Design a receiver to recover signals  $m_1(t)$  and  $m_2(t)$  from the modulated signal at point  $c$ .

4.2-9 The system shown in Fig. P4.2-9 is used for scrambling audio signals. The output  $y(t)$  is the scrambled version of the input  $m(t)$ .

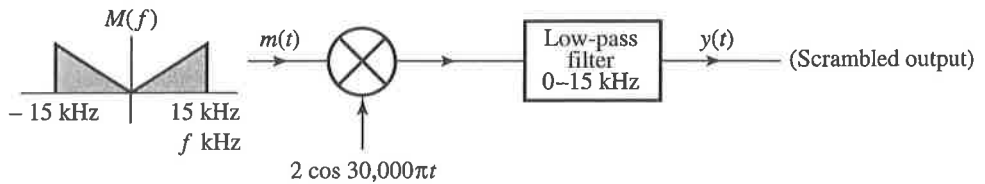
- (a) Find the spectrum of the scrambled signal  $y(t)$ .
- (b) Suggest a method of descrambling  $y(t)$  to obtain  $m(t)$ .

Figure P.4.2-8



A slightly modified version of this scrambler was first used commercially on the 25-mile radio-telephone circuit connecting Los Angeles and Santa Catalina island.

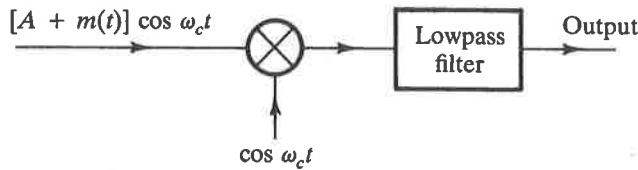
Figure P.4.2-9



4.2-10 A DSB-SC signal is given by  $m(t) \cos(2\pi)10^6t$ . The carrier frequency of this signal, 1 MHz, is to be changed to 400 kHz. The only equipment available consists of one ring modulator, a bandpass filter centered at the frequency of 400 kHz, and one sine wave generator whose frequency can be varied from 150 to 210 kHz. Show how you can obtain the desired signal  $cm(t) \cos(2\pi \times 400 \times 10^3t)$  from  $m(t) \cos(2\pi)10^6t$ . Determine the value of  $c$ .

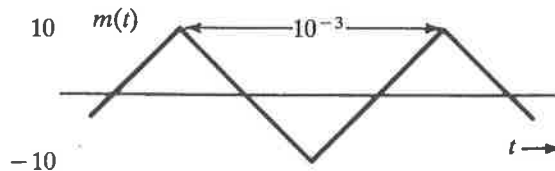
4.3-1 Figure P4.3-1 shows a scheme for coherent (synchronous) demodulation. Show that this scheme can demodulate the AM signal  $[A + m(t)] \cos(2\pi f_c t)$  regardless of the value of  $A$ .

Figure P.4.3-1



4.3-2 Sketch the AM signal  $[A + m(t)] \cos(2\pi f_c t)$  for the periodic triangle signal  $m(t)$  shown in Fig. P4.3-2 corresponding to the modulation indices (a)  $\mu = 0.5$ ; (b)  $\mu = 1$ ; (c)  $\mu = 2$ ; (d)  $\mu = \infty$ . How do you interpret the case of  $\mu = \infty$ ?

Figure P.4.3-2



4.3-3 For the AM signal with  $m(t)$  shown in Fig. P4.3-2 and  $\mu = 0.8$ :

- (a) Find the amplitude and power of the carrier.
- (b) Find the sideband power and the power efficiency  $\eta$ .

4.3-4 (a) Sketch the DSB-SC signal corresponding to the message signal  $m(t) = \cos 2\pi t$ .

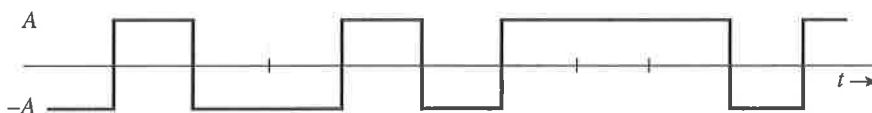
(b) The DSB-SC signal of part (a) is applied at the input of an envelope detector. Show that the output of the envelope detector is not  $m(t)$ , but  $|m(t)|$ . Show that, in general, if an AM signal  $[A + m(t)] \cos \omega_c t$  is envelope-detected, the output is  $|A + m(t)|$ . Hence, show that the condition for recovering  $m(t)$  from the envelope detector is  $A + m(t) > 0$  for all  $t$ .

4.3-5 Show that any scheme that can be used to generate DSB-SC can also generate AM. Is the converse true? Explain.

4.3-6 Show that any scheme that can be used to demodulate DSB-SC can also demodulate AM. Is the converse true? Explain.

4.3-7 In the text, the power efficiency of AM for a sinusoidal  $m(t)$  was found. Carry out a similar analysis when  $m(t)$  is a random binary signal as shown in Fig. P4.3-7 and  $\mu = 1$ . Sketch the AM signal with  $\mu = 1$ . Find the sideband's power and the total power (power of the AM signal) as well as their ratio (the power efficiency  $\eta$ ).

Figure P.4.3-7



4.3-8 In the early days of radio, AM signals were demodulated by a crystal detector followed by a low-pass filter and a dc blocker, as shown in Fig. P4.3-8. Assume a crystal detector to be basically a squaring device. Determine the signals at points  $a$ ,  $b$ ,  $c$ , and  $d$ . Point out the distortion term in the output  $y(t)$ . Show that if  $A \gg |m(t)|$ , the distortion is small.

Figure P.4.3-8



4.4-1 In a QAM system (Fig. 4.19), the locally generated carrier has a frequency error  $\Delta\omega$  and a phase error  $\delta$ ; that is, the receiver carrier is  $\cos [(\omega_c + \Delta\omega)t + \delta]$  or  $\sin [(\omega_c + \Delta\omega)t + \delta]$ . Show that the output of the upper receiver branch is

$$m_1(t) \cos [(\Delta\omega)t + \delta] - m_2(t) \sin [(\Delta\omega)t + \delta]$$

instead of  $m_1(t)$ , and the output of the lower receiver branch is

$$m_1(t) \sin [(\Delta\omega)t + \delta] + m_2(t) \cos [(\Delta\omega)t + \delta]$$

instead of  $m_2(t)$ .

4.4-2 A modulating signal  $m(t)$  is given by:

(a)  $m(t) = \cos 100\pi t + 2 \cos 300\pi t$

(b)  $m(t) = \sin 100\pi t \sin 500\pi t$

In each case:

(i) Sketch the spectrum of  $m(t)$ .

(ii) Find and sketch the spectrum of the DSB-SC signal  $2m(t) \cos 1000\pi t$ .

(iii) From the spectrum obtained in (ii), suppress the LSB spectrum to obtain the USB spectrum.

(iv) Knowing the USB spectrum in (ii), write the expression  $\varphi_{\text{USB}}(t)$  for the USB signal.

(v) Repeat (iii) and (iv) to obtain the LSB signal  $\varphi_{\text{LSB}}(t)$ .

4.4-3 For the signals in Prob. 4.4-2, use Eq. (4.20) to determine the time domain expressions  $\varphi_{\text{LSB}}(t)$  and  $\varphi_{\text{USB}}(t)$  if the carrier frequency  $\omega_c = 1000$ .

*Hint:* If  $m(t)$  is a sinusoid, its Hilbert transform  $m_h(t)$  is the sinusoid  $m(t)$  phase-delayed by  $\pi/2$  rad.

4.4-4 Find  $\varphi_{\text{LSB}}(t)$  and  $\varphi_{\text{USB}}(t)$  for the modulating signal  $m(t) = \pi B \text{sinc}^2(2\pi Bt)$  with  $B = 2000$  Hz and carrier frequency  $f_c = 10,000$  Hz. Follow these steps:

(a) Sketch spectra of  $m(t)$  and the corresponding DSB-SC signal  $2m(t) \cos \omega_c t$ .

(b) To find the LSB spectrum, suppress the USB in the DSB-SC spectrum found in part (a).

(c) Find the LSB signal  $\varphi_{\text{LSB}}(t)$ , which is the inverse Fourier transform of the LSB spectrum found in part (b). Follow a similar procedure to find  $\varphi_{\text{USB}}(t)$ .

4.4-5 If  $m_h(t)$  is the Hilbert transform of  $m(t)$ , then

(a) Show that the Hilbert transform of  $m_h(t)$  is  $-m(t)$ .

(b) Show also that the energies of  $m(t)$  and  $m_h(t)$  are identical.

4.4-6 An LSB signal is demodulated coherently, as shown in Fig. P4.4-6. Unfortunately, because of the transmission delay, the received signal carrier is not  $2 \cos \omega_c t$  as sent, but rather, is  $2 \cos [(\omega_c + \Delta\omega)t + \delta]$ . The local oscillator is still  $\cos \omega_c t$ . Show the following.

(a) When  $\delta = 0$ , the output  $y(t)$  is the signal  $m(t)$  with all its spectral components shifted (offset) by  $\Delta\omega$ .

*Hint:* Observe that the output  $y(t)$  is identical to the right-hand side of Eq. (4.20a) with  $\omega_c$  replaced with  $\Delta\omega$ .

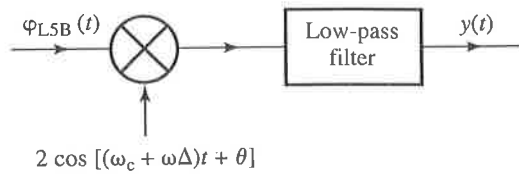
(b) When  $\Delta\omega = 0$ , the output is the signal  $m(t)$  with phases of all its spectral components shifted by  $\delta$ .

*Hint:* Show that the output spectrum  $Y(f) = M(f)e^{j\delta}$  for  $f \geq 0$ , and equal to  $M(f)e^{-j\delta}$  when  $f < 0$ .

(c) In each of these cases, explain the nature of distortion.

*Hint:* For part (a), demodulation consists of shifting an LSB spectrum to the left and right by  $\omega_c + \Delta\omega$  and low-pass-filtering the result. For part (b), use the expression (4.20b) for  $\varphi_{\text{LSB}}(t)$ , multiply it by the local carrier  $2 \cos(\omega_c t + \delta)$ , and low-pass-filter the result.

Figure P.4.4-6



4.4-7 A USB signal is generated by using the phase shift method (Fig. 4.17). If the input to this system is  $m_h(t)$  instead of  $m(t)$ , what will be the output? Is this signal still an SSB signal with bandwidth equal to that of  $m(t)$ ? Can this signal be demodulated [to get back  $m(t)$ ]? If so, how?

4.5-1 A vestigial filter  $H_i(f)$  shown in the transmitter of Fig. 4.21 has a transfer function as shown in Fig. P4.5-1. The carrier frequency is  $f_c = 10$  kHz and the baseband signal bandwidth is 4 kHz. Find the corresponding transfer function of the equalizer filter  $H_o(f)$  shown in the receiver of Fig. 4.21.  
*Hint:* Use Eq. (4.25).

Figure P.4.5-1

