Digital Modulation Techniques

BPSK

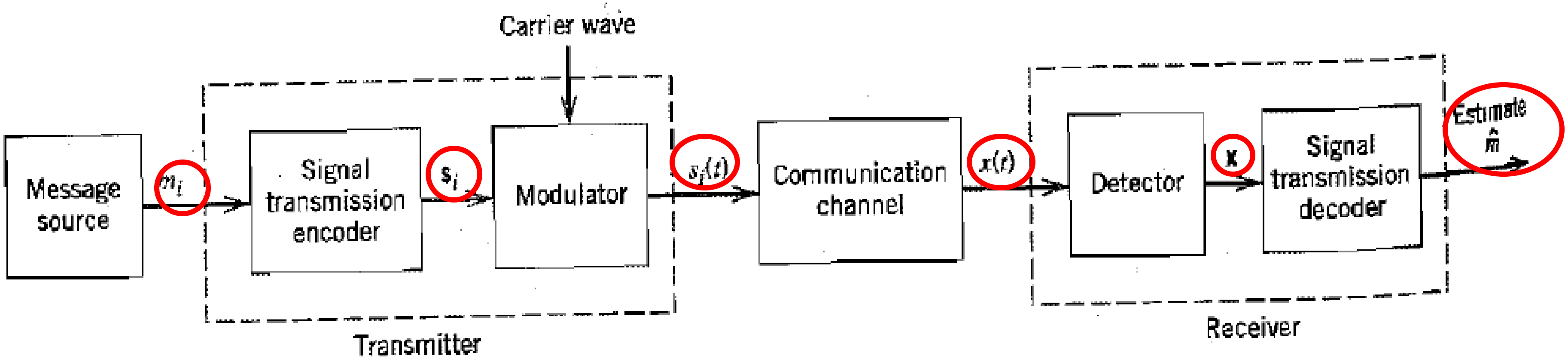


FIGURE 6.2 Functional model of passband data transmission system.

Passband Transmission Model

In a functional sense, we may model a passband data transmission system as shown in Figure 6.2. First, there is assumed to exist a *message source* that emits one *symbol* every T seconds, with the symbols belonging to an alphabet of M symbols, which we denote by $m_1, m_2, \ldots, m_M$. The *a priori probabilities* $P(m_1), P(m_2), \ldots, P(m_M)$ specify the message source output. When the M symbols of the alphabet are *equally likely*, we write

$$\begin{aligned} p_i &= P(m_i) \\ &= \frac{1}{M} \quad \text{for all } i \end{aligned} \tag{6.6}$$

BINARY PHASE-SHIFT KEYING

In a coherent binary PSK system, the pair of signals $s_1(t)$ and $s_2(t)$ used to represent binary symbols 1 and 0, respectively, is defined by

$$s_1(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t) \tag{6.8}$$

$$s_2(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t + \pi) = -\sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t) \tag{6.9}$$

where $0 \le t \le T_b$, and E_b is the *transmitted signal energy per bit*. To ensure that each transmitted bit contains an integral number of cycles of the carrier wave, the carrier frequency f_c is chosen equal to n_c/T_b for some fixed integer n_c. A pair of sinusoidal waves that differ only in a relative phase-shift of 180 degrees, as defined in Eqautions (6.8) and (6.9), are referred to as *antipodal signals*.

From this pair of equations it is clear that, in the case of binary PSK, there is only one basis function of unit energy, namely,

$$\phi_1(t) = \sqrt{\frac{2}{T_b}} \cos(2\pi f_c t), \qquad 0 \le t < T_b \tag{6.10}$$

Then we may express the transmitted signals $s_1(t)$ and $s_2(t)$ in terms of $\phi_1(t)$ as follows:

$$s_1(t) = \sqrt{E_b}\phi_1(t), \qquad 0 \le t < T_b \tag{6.11}$$

and

$$s_2(t) = -\sqrt{E_b}\phi_1(t), \qquad 0 \le t < T_b \tag{6.12}$$

$$s_{11} = \int_0^{T_b} s_1(t)\phi_1(t)\,dt = +\sqrt{E_b}$$

$$s_{21} = \int_0^{T_b} s_2(t)\phi_1(t)\,dt = -\sqrt{E_b}$$

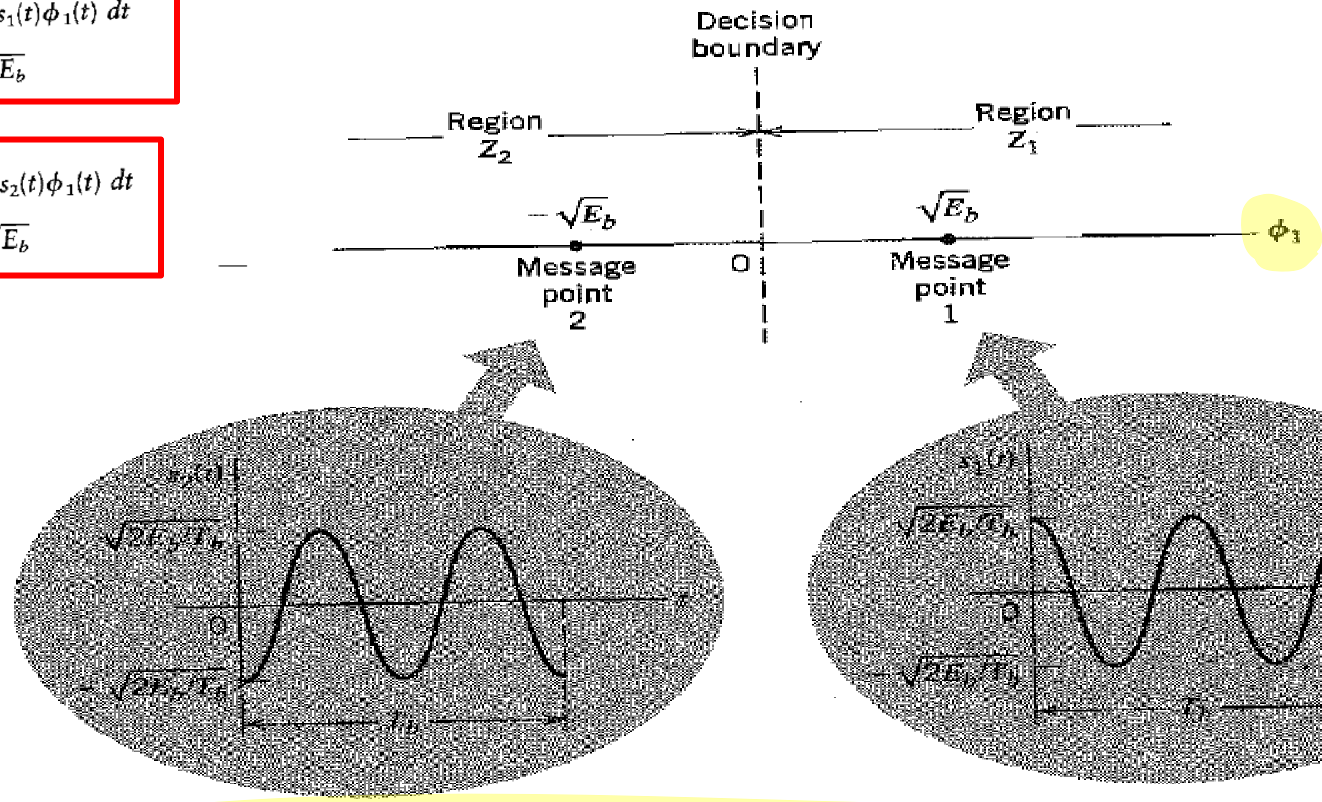


FIGURE 6.3 Signal-space diagram for coherent binary PSK system. The waveforms depicting the transmitted signals $s_1(t)$ and $s_2(t)$, displayed in the inserts, assume $n_c = 2$.

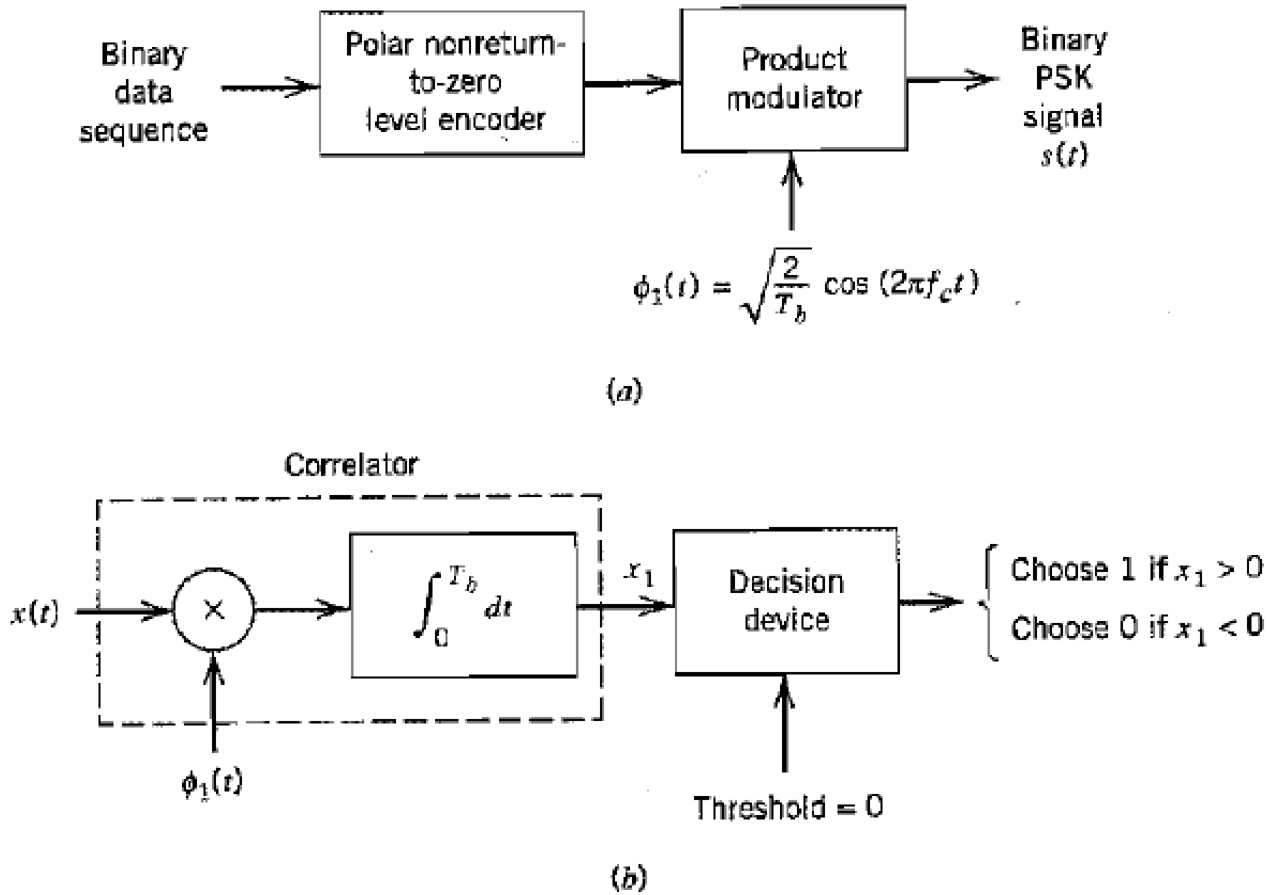


FIGURE 6.4 Block diagrams for (*a*) binary PSK transmitter and (*b*) coherent binary PSK receiver.

QPSK

Quadriphase-Shift Keying

efficient utilization of channel bandwidth.

the phase of the carrier
$\pi/4$, $3\pi/4$, $5\pi/4$, and $7\pi/4$

values we may define the transmitted signal as

$$s_i(t) = \begin{cases} \sqrt{\dfrac{2E}{T}} \cos\left[2\pi f_c t + (2i - 1)\dfrac{\pi}{4}\right], & 0 \le t \le T \\ 0, & \text{elsewhere} \end{cases} \tag{6.23}$$

where $i = 1, 2, 3, 4$; E is the transmitted signal energy per symbol, and T is the symbol duration. The carrier frequency f_c equals n_c/T for some fixed integer n_c. Each possible value of the phase corresponds to a unique dibit. Thus, for example, we may choose the foregoing set of phase values to represent the *Gray-encoded* set of dibits: 10, 00, 01, and 11, where only a single bit is changed from one dibit to the next.

TABLE 6.1 *Signal-space characterization of QPSK*

Gray-encoded Input Dibit	*Phase of QPSK Signal (radians)*	*Coordinates of Message Points*	
		s_{i1}	s_{i2}
10	$\pi/4$	$+\sqrt{E/2}$	$-\sqrt{E/2}$
00	$3\pi/4$	$-\sqrt{E/2}$	$-\sqrt{E/2}$
01	$5\pi/4$	$-\sqrt{E/2}$	$+\sqrt{E/2}$
11	$7\pi/4$	$+\sqrt{E/2}$	$+\sqrt{E/2}$

Signal-Space Diagram of QPSK

Using a well-known trigonometric identity, we may use Equation (6.23) to redefine the transmitted signal $s_i(t)$ for the interval $0 \leq t \leq T$ in the equivalent form:

$$s_i(t) = \sqrt{\frac{2E}{T}} \cos\left[(2i - 1)\frac{\pi}{4}\right] \cos(2\pi f_c t) - \sqrt{\frac{2E}{T}} \sin\left[(2i - 1)\frac{\pi}{4}\right] \sin(2\pi f_c t) \quad (6.24)$$

where $i = 1, 2, 3, 4$. Based on this representation, we can make the following observations:

- There are two orthonormal basis functions, $\phi_1(t)$ and $\phi_2(t)$, contained in the expansion of $s_i(t)$. Specifically, $\phi_1(t)$ and $\phi_2(t)$ are defined by a pair of *quadrature carriers*:

$$\phi_1(t) = \sqrt{\frac{2}{T}} \cos(2\pi f_c t), \qquad 0 \leq t \leq T \quad (6.25)$$

$$\phi_2(t) = \sqrt{\frac{2}{T}} \sin(2\pi f_c t), \qquad 0 \leq t \leq T \quad (6.26)$$

- There are four message points, and the associated signal vectors are defined by

$$\mathbf{s}_i = \begin{bmatrix} \sqrt{E} \cos\left((2i - 1)\frac{\pi}{4}\right) \\ -\sqrt{E} \sin\left((2i - 1)\frac{\pi}{4}\right) \end{bmatrix}, \qquad i = 1, 2, 3, 4 \quad (6.27)$$

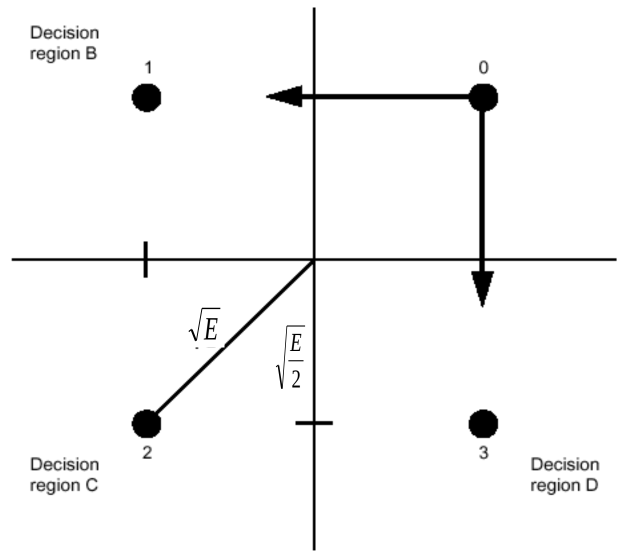


Figure 3.21 Necessary condition for an error to occur in a QPSK system.

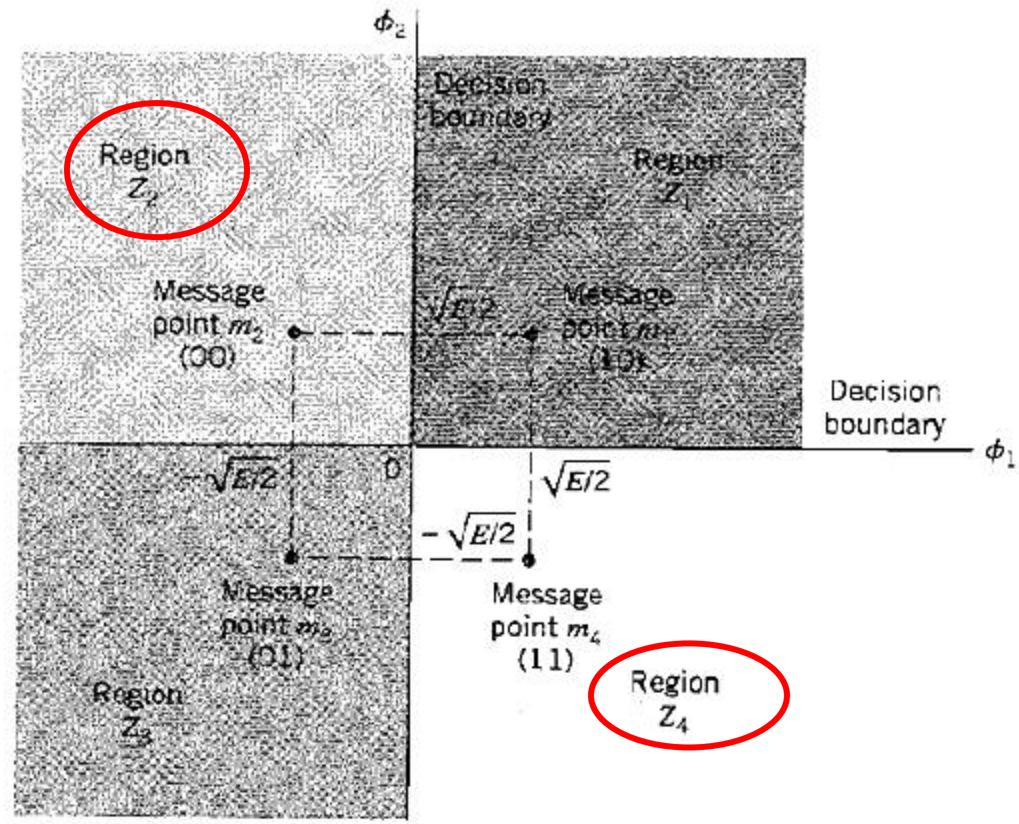


FIGURE 6.6 Signal-space diagram of coherent QPSK system.

TABLE 6.1 ***Signal-space characterization of QPSK***

Gray-encoded Input Dibit	*Phase of QPSK Signal (radians)*	*Coordinates of Message Points* s_{i1}	s_{i2}
10	$\pi/4$	$+\sqrt{E/2}$	$-\sqrt{E/2}$
00	$3\pi/4$	$-\sqrt{E/2}$	$-\sqrt{E/2}$
01	$5\pi/4$	$-\sqrt{E/2}$	$+\sqrt{E/2}$
11	$7\pi/4$	$+\sqrt{E/2}$	$+\sqrt{E/2}$

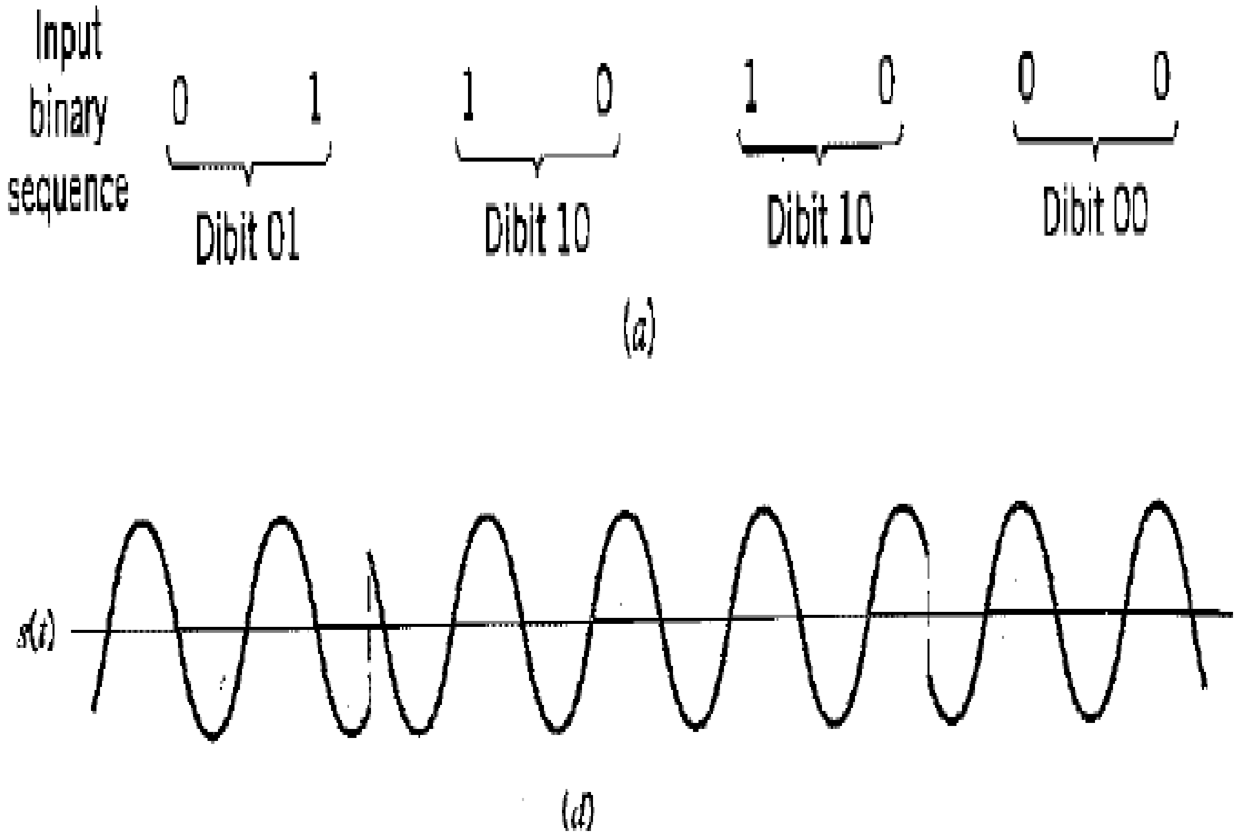
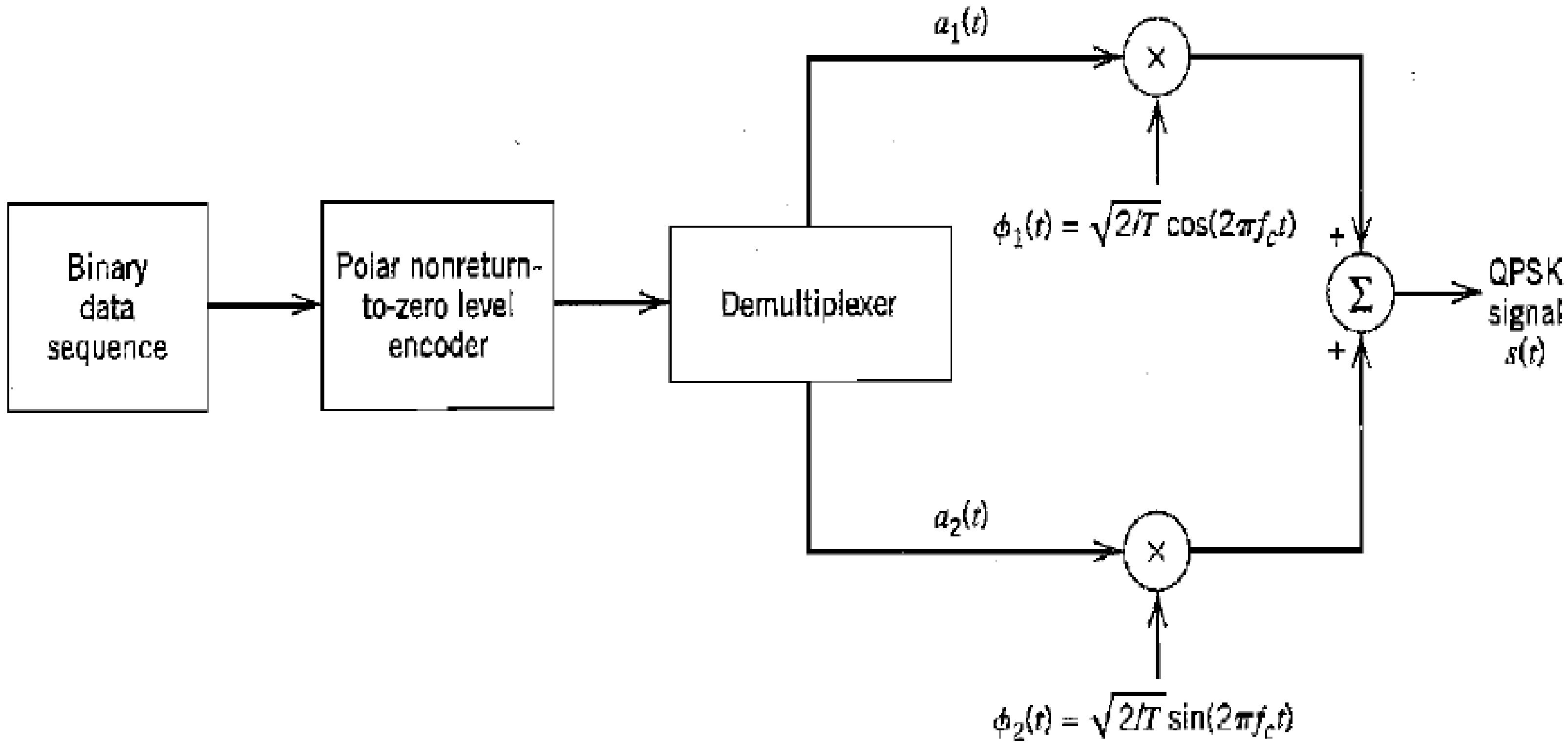
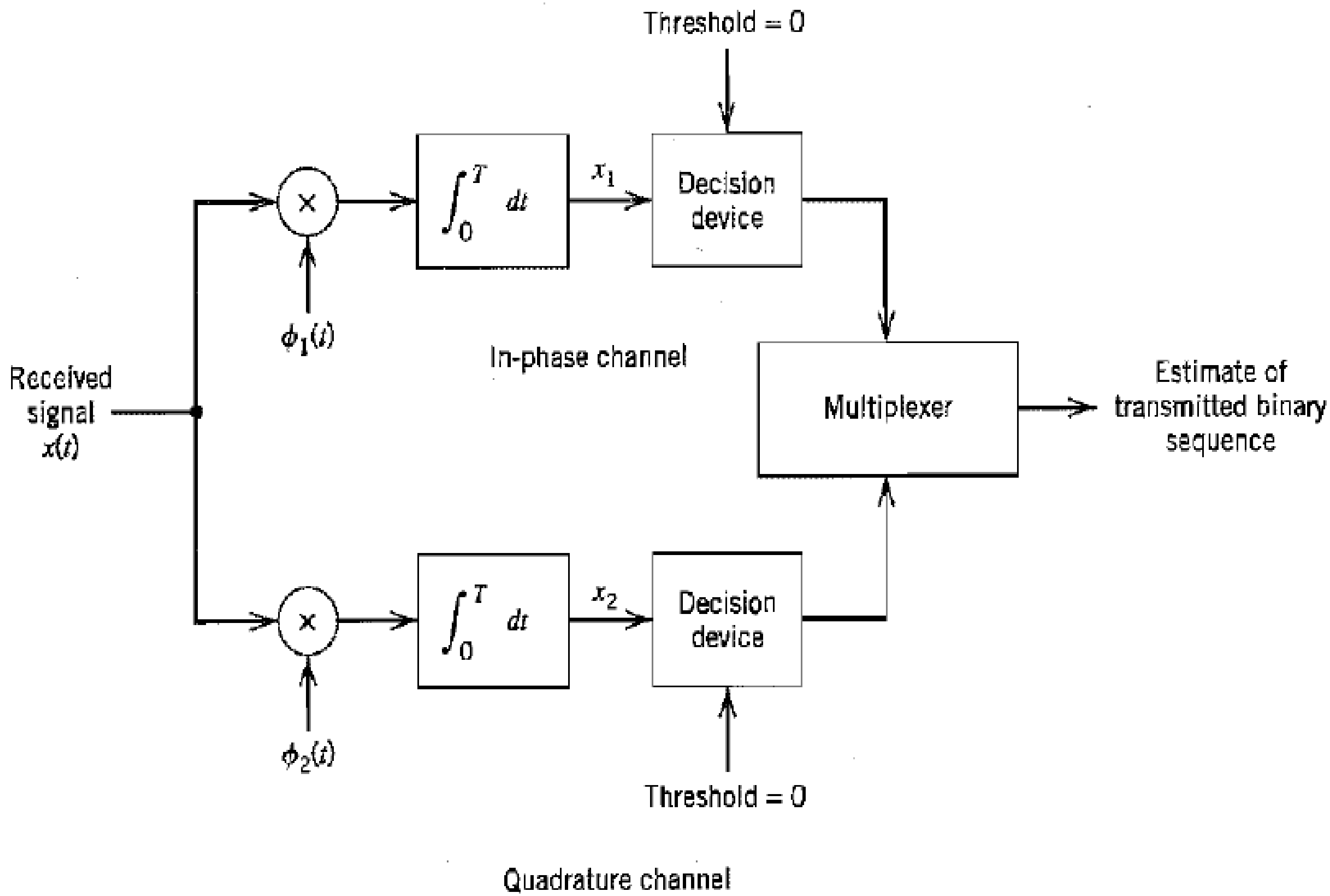


FIGURE 6.7 (a) Input binary sequence. (b) Odd-numbered bits of input sequence and associated binary PSK wave. (c) Even-numbered bits of input sequence and associated binary PSK wave. (d) QPSK waveform defined as $s(t) = s_{i1}\phi_1(t) + s_{i2}\phi_2(t)$.


Binary data sequence
Polar nonreturn-to-zero level encoder
Demultiplexer
$a_1(t)$
$a_2(t)$
$\phi_1(t) = \sqrt{2/T}\cos(2\pi f_c t)$
$\phi_2(t) = \sqrt{2/T}\sin(2\pi f_c t)$
QPSK signal $s(t)$

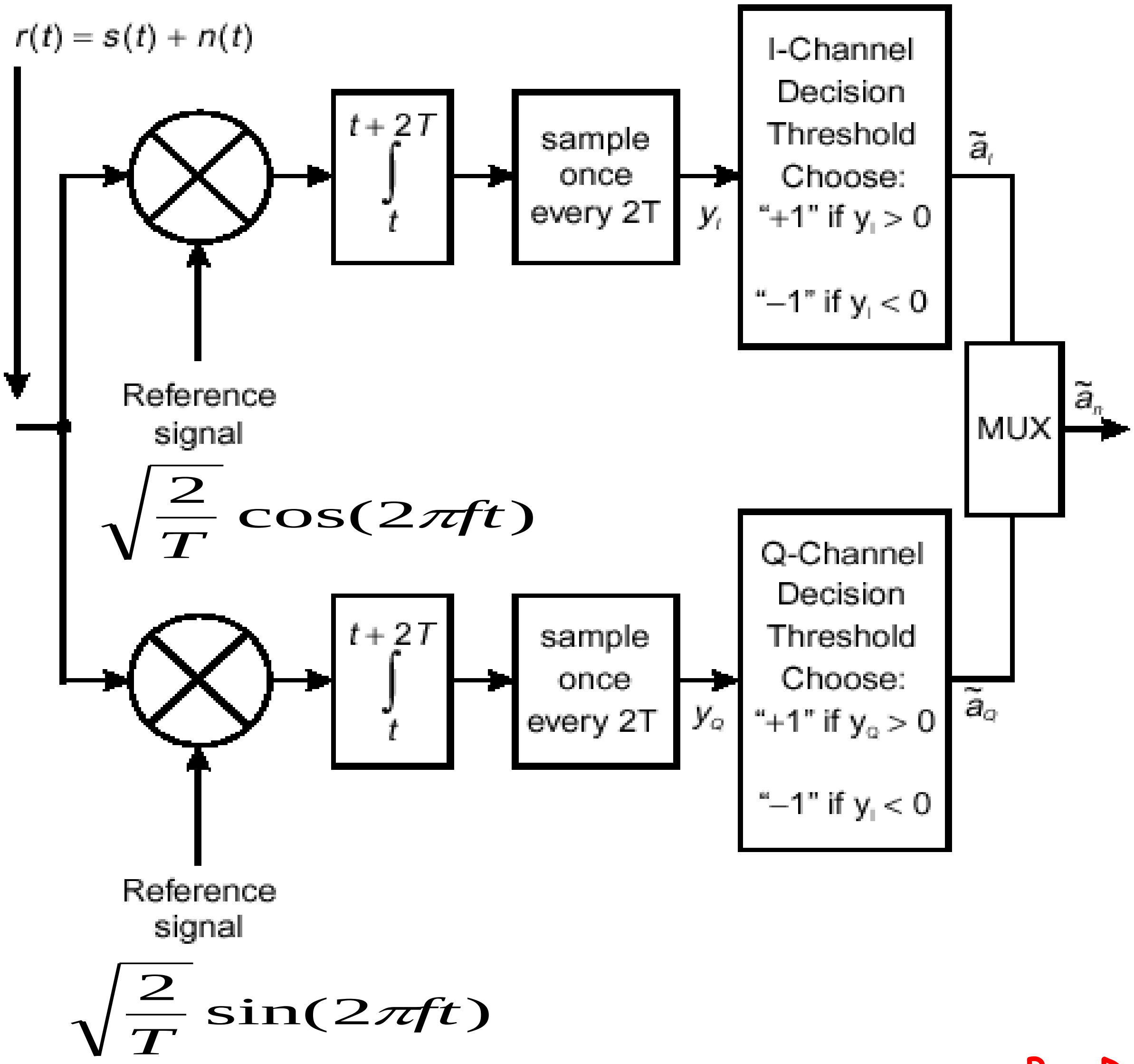
(a)

(a)



(b)

FIGURE 6.8 Block diagrams of (*a*) QPSK transmitter and (*b*) coherent QPSK receiver.



By: Dr.Mohab Mangoud

M ary - PSK

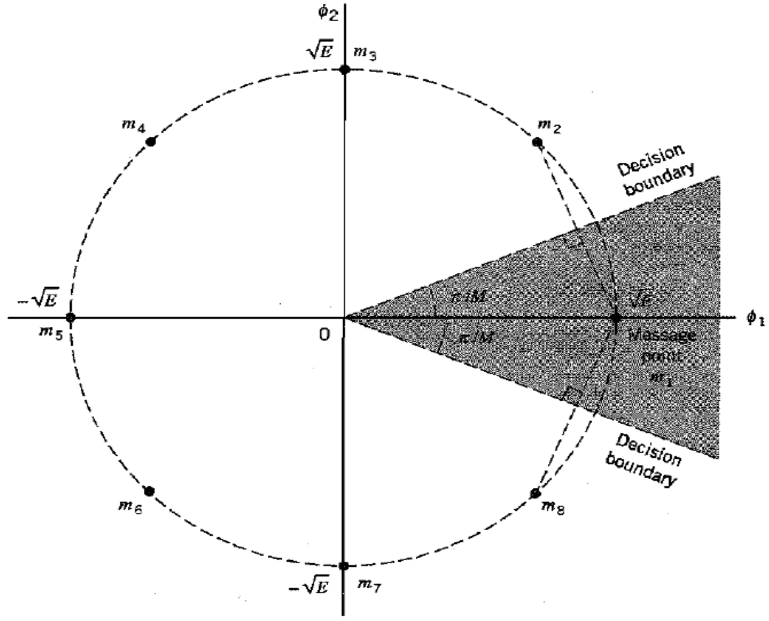
M-ARY PSK

QPSK is a special case of *M-ary PSK*, where the phase of the carrier takes on one of M possible values, namely, $\theta_i = 2(i-1)\pi/M$, where $i = 1, 2, \ldots, M$. Accordingly, during each signaling interval of duration T, one of the M possible signals

$$s_i(t) = \sqrt{\frac{2E}{T}} \cos\left(2\pi f_c t + \frac{2\pi}{M}(i-1)\right), \qquad i = 1, 2, \ldots, M \tag{6.46}$$

is sent, where E is the signal energy per symbol. The carrier frequency $f_c = n_c/T$ for some fixed integer n_c.

Each $s_i(t)$ may be expanded in terms of the same two basis functions $\phi_1(t)$ and $\phi_2(t)$ defined in Equations (6.25) and (6.26), respectively. The signal constellation of M-ary PSK is therefore two-dimensional. The M message points are equally spaced on a circle of radius $\sqrt{E}$ and center at the origin, as illustrated in Figure 6.15*a*, for the case of *octaphase-shift-keying* (i.e., $M = 8$).



(b)

FIGURE 6.15 (a) Signal-space diagram for octaphase-shift keying (i.e., $M = 8$). The decision boundaries are shown as dashed lines. (b) Signal-space diagram illustrating the application of the

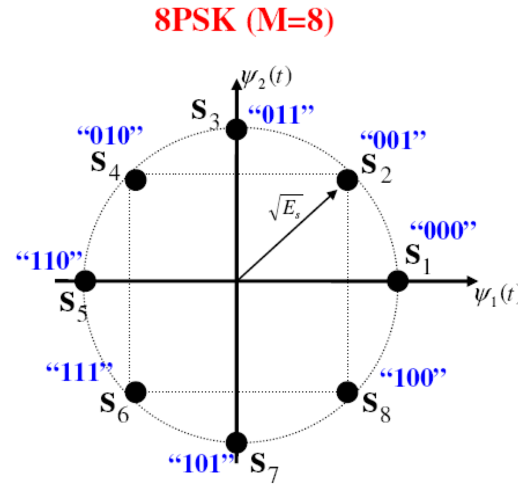
$$d_{12} = d_{18} = 2\sqrt{E}\sin\left(\frac{\pi}{M}\right)$$

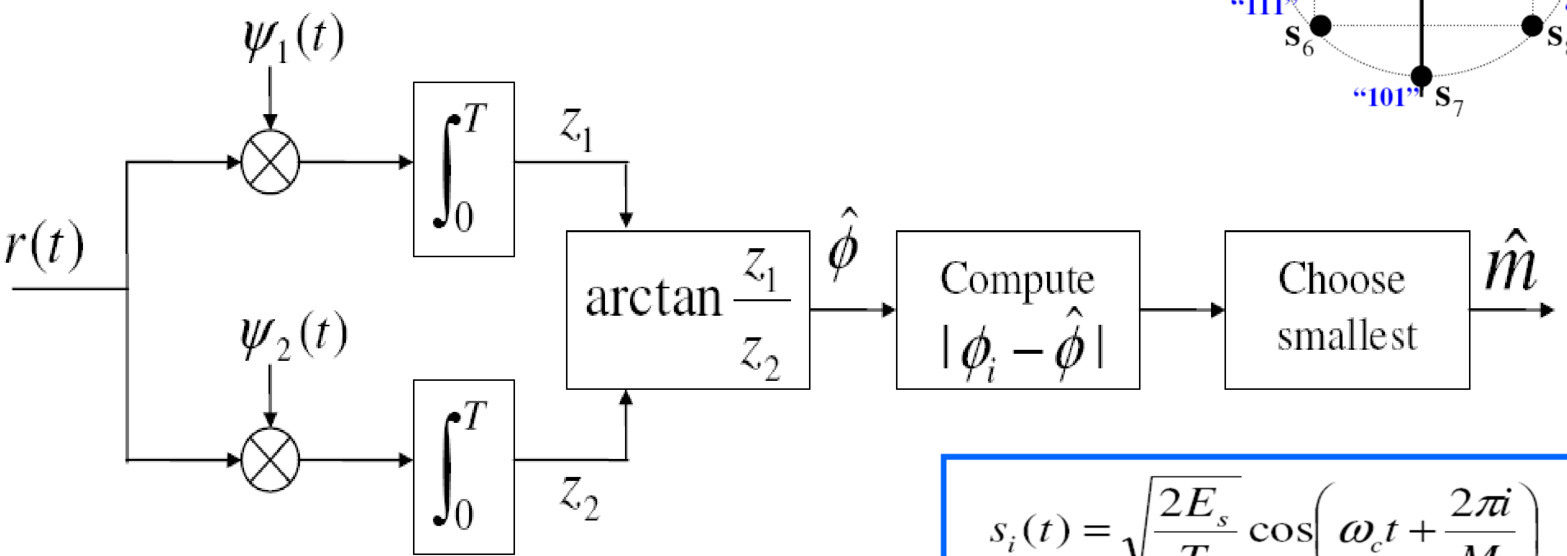
Hence, the use of Equation (5.92) of Chapter 5 yields the average probability of symbol error for coherent M-ary PSK as

$$P_e \simeq \operatorname{erfc}\left(\sqrt{\frac{E}{N_0}}\sin\left(\frac{\pi}{M}\right)\right) \tag{6.47}$$

where it is assumed that $M \geq 4$. The approximation becomes extremely tight, for fixed M, as E/N_0 is increased. For $M = 4$, Equation (6.47) reduces to the same form given in Equation (6.34) for QPSK.

- Coherent detection of MPSK





$$s_i(t) = \sqrt{\frac{2E_s}{T}} \cos\left(\omega_c t + \frac{2\pi i}{M}\right)$$

$$s_i(t) = a_{i1}\psi_1(t) + a_{i2}\psi_2(t) \quad i = 1, \ldots, M$$

$$\psi_1(t) = \sqrt{\frac{2}{T}} \cos(\omega_c t) \quad \psi_2(t) = -\sqrt{\frac{2}{T}} \sin(\omega_c t)$$

$$a_{i1} = \sqrt{E_s} \cos\left(\frac{2\pi i}{M}\right) \quad a_{i2} = \sqrt{E_s} \sin\left(\frac{2\pi i}{M}\right)$$

$$E_s = E_i = \|\mathbf{s}_i\|^2$$

Power Spectra of M-ary PSK Signals

The symbol duration of M-ary PSK is defined by

$$T = T_b \log_2 M \tag{6.48}$$

where T_b is the bit duration. Proceeding in a manner similar to that described for a QPSK signal, we may show that the baseband power spectral density of an M-ary PSK signal is given by

$$\begin{aligned} S_B(f) &= 2E \operatorname{sinc}^2(Tf) \\ &= 2E_b \log_2 M \operatorname{sinc}^2(T_b f \log_2 M) \end{aligned} \tag{6.49}$$

In Figure 6.16, we show the normalized power spectral density $S_B(f)/2E_b$ plotted versus the normalized frequency fT_b for three different values of M, namely, $M = 2, 4, 8$.

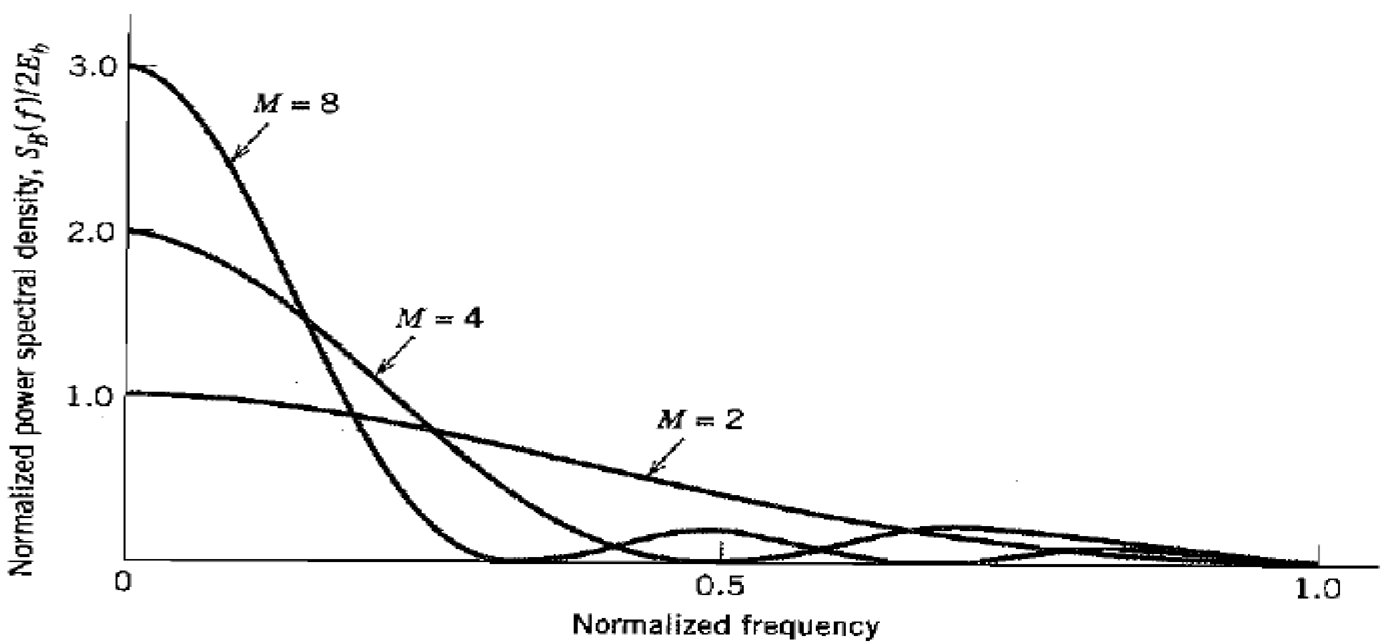


FIGURE 6.16 Power spectra of M-ary PSK signals for $M = 2, 4, 8$.

For the passband basis functions defined in Equations (6.25) and (6.26), the channel bandwidth required to pass M-ary PSK signals (more precisely, the main spectral lobe of M-ary signals) is given by

$$B = \frac{2}{T} \tag{6.50}$$

where T is the symbol duration. But the symbol duration T is related to the bit duration T_b by Equation (6.48). Moreover, the bit rate $R_b = 1/T_b$. Hence, we may redefine the channel bandwidth of Equation (6.50) in terms of the bit rate R_b as

$$B = \frac{2R_b}{\log_2 M} \tag{6.51}$$

Based on this formula, the bandwidth efficiency of M-ary PSK signals is given by

$$\begin{aligned} \rho &= \frac{R_b}{B} \\ &= \frac{\log_2 M}{2} \end{aligned} \tag{6.52}$$

TABLE 6.4 ***Bandwidth efficiency of M-ary PSK signals***

M	2	4	8	16	32	64
ρ (bits/s/Hz)	0.5	1	1.5	2	2.5	3

6.4 Hybrid Amplitude/Phase Modulation Schemes

M-QAM

M-ary Quadrature Amplitude Modulation

In Chapters 4 and 5, we studied *M*-ary pulse amplitude modulation (PAM), which is one-dimensional. *M*-ary QAM is a two-dimensional generalization of *M*-ary PAM in that its formulation involves two orthogonal passband basis functions, as shown by

$$\phi_1(t) = \sqrt{\frac{2}{T}}\cos(2\pi f_c t), \qquad 0 \le t \le T \tag{6.53}$$

$$\phi_2(t) = \sqrt{\frac{2}{T}}\sin(2\pi f_c t), \qquad 0 \le t \le T \tag{6.54}$$

Let the *i*th message point s_i in the (ϕ_1, ϕ_2) plane be denoted by $(a_i d_{\min}/2, b_i d_{\min}/2)$, where $d_{\min}$ is the minimum distance between any two message points in the constellation, a_i and b_i are integers, and $i = 1, 2, \ldots, M$. Let $(d_{\min}/2) = \sqrt{E_0}$, where E_0 is the energy of the signal with the lowest amplitude. The transmitted *M*-ary QAM signal for symbol *k*, say, is then defined by

$$s_k(t) = \sqrt{\frac{2E_0}{T}}\, a_k \cos(2\pi f_c t) - \sqrt{\frac{2E_0}{T}}\, b_k \sin(2\pi f_c t), \qquad \begin{array}{l} 0 \le t \le T \\ k = 0, \pm 1, \pm 2, \ldots \end{array} \tag{6.55}$$

The signal $s_k(t)$ consists of two phase-quadrature carriers with each one being modulated by a set of discrete amplitudes, hence the name *quadrature amplitude modulation*.

Two dimensional mod.,… (M-QAM)

- M-ary Quadrature Amplitude Mod. (M-QAM)

$$s_i(t) = \sqrt{\frac{2E_i}{T}} \cos(\omega_c t + \varphi_i)$$

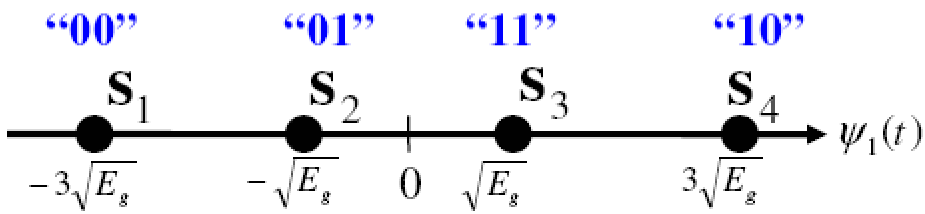
$$s_i(t) = a_{i1}\psi_1(t) + a_{i2}\psi_2(t) \quad i = 1, \dots, M$$

$$\psi_1(t) = \sqrt{\frac{2}{T}} \cos(\omega_c t) \quad \psi_2(t) = \sqrt{\frac{2}{T}} \sin(\omega_c t)$$

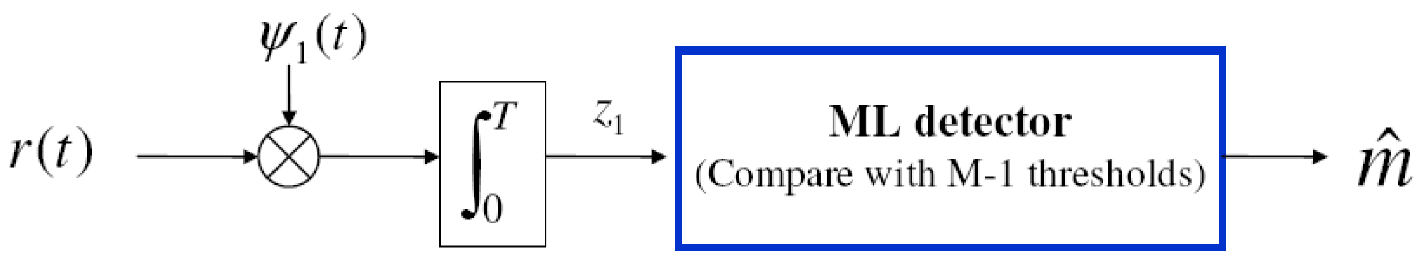
where a_{i1} and a_{i2} are PAM symbols and $E_s = \dfrac{2(M-1)}{3}$

$$(a_{i1}, a_{i2}) = \begin{bmatrix} (-\sqrt{M}+1, \sqrt{M}-1) & (-\sqrt{M}+3, \sqrt{M}-1) & \cdots & (\sqrt{M}-1, \sqrt{M}-1) \\ (-\sqrt{M}+1, \sqrt{M}-3) & (-\sqrt{M}+3, \sqrt{M}-3) & \cdots & (\sqrt{M}-1, \sqrt{M}-3) \\ \vdots & \vdots & \vdots & \vdots \\ (-\sqrt{M}+1, -\sqrt{M}+1) & (-\sqrt{M}+3, -\sqrt{M}+1) & \cdots & (\sqrt{M}-1, -\sqrt{M}+1) \end{bmatrix}$$

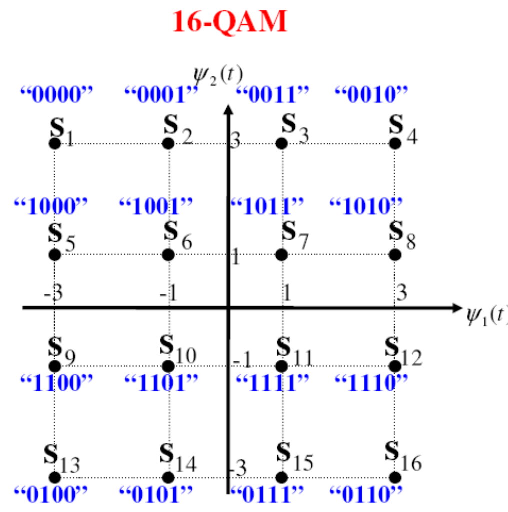
4-PAM:

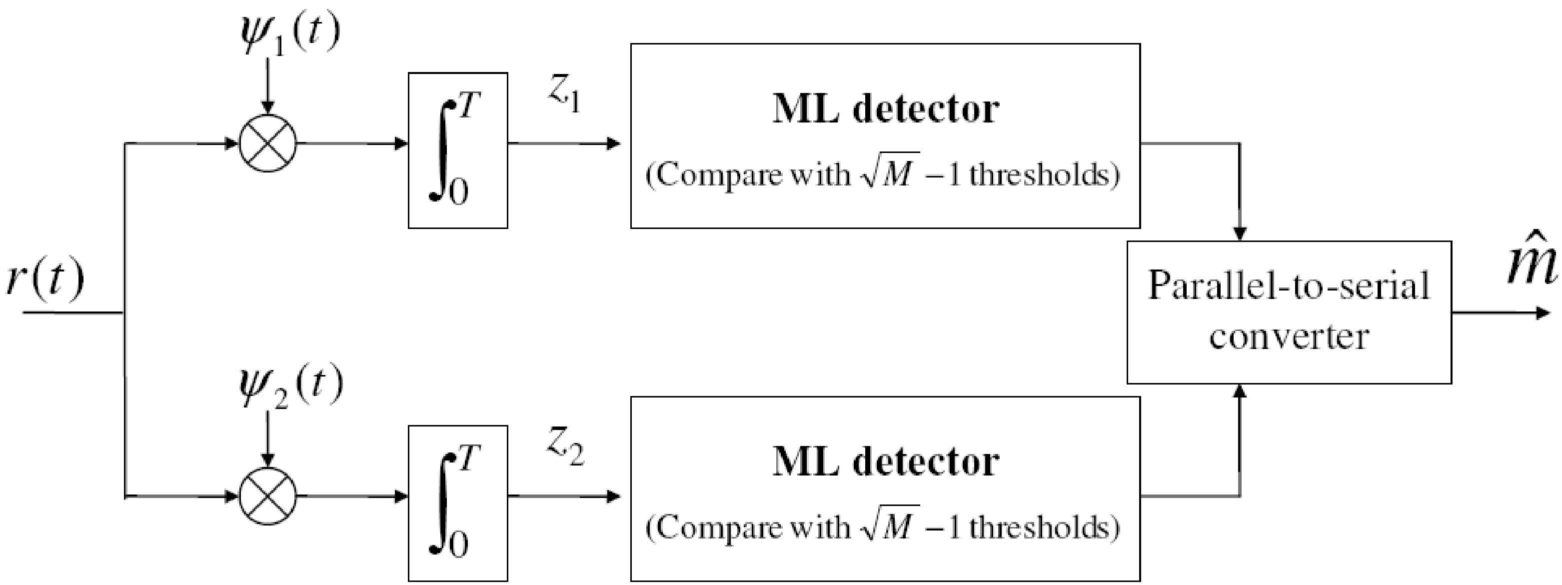


- Coherent detection of M-PAM



- Coherent detection of M-QAM





FSK

5.8.1 Binary Frequency Shift Keying

In binary frequency shift keying (BFSK), the frequency of a constant amplitude carrier signal is switched between two values according to the two possible message states (called *high* and *low* tones), corresponding to a binary 1 or 0. Depending on how the frequency variations are imparted into the transmitted waveform, the FSK signal will have either a discontinuous phase or continuous phase between bits. In general, an FSK signal may be represented as

$$s_{FSK}(t) = v_H(t) = \sqrt{\frac{2E_b}{T_b}}\cos(2\pi f_c + 2\pi\Delta f)t \quad 0 \le t \le T_b \text{ (binary 1)} \tag{5.95.a}$$

$$s_{FSK}(t) = v_L(t) = \sqrt{\frac{2E_b}{T_b}}\cos(2\pi f_c - 2\pi\Delta f)t \quad 0 \le t \le T_b \text{ (binary 0)} \tag{5.95.b}$$

where $2\pi\Delta f$ is a constant offset from the nominal carrier frequency.

One obvious way to generate an FSK signal is to switch between two independent oscillators according to whether the data bit is a 0 or a 1. Normally, this form of FSK generation results in a waveform that is discontinuous at the switching times, and for this reason this type of FSK is called *discontinuous FSK*. A discontinuous FSK signal is represented as

$$s_{FSK}(t) = v_H(t) = \sqrt{\frac{2E_b}{T_b}}\cos(2\pi f_H t + \theta_1) \quad 0 \le t \le T_b \text{ (binary 1)} \tag{5.96.a}$$

$$s_{FSK}(t) = v_L(t) = \sqrt{\frac{2E_b}{T_b}}\cos(2\pi f_L t + \theta_2) \quad 0 \le t \le T_b \text{ (binary 0)} \tag{5.96.b}$$

Since the phase discontinuities pose several problems, such as spectral spreading and spurious transmissions, this type of FSK is generally not used in highly regulated wireless systems.

Binary FSK

In a *binary FSK system*, symbols 1 and 0 are distinguished from each other by transmitting one of two sinusoidal waves that differ in frequency by a fixed amount. A typical pair of sinusoidal waves is described by

$$s_i(t) = \begin{cases} \sqrt{\dfrac{2E_b}{T_b}} \cos(2\pi f_i t), & 0 \le t \le T_b \\ 0, & \text{elsewhere} \end{cases} \tag{6.86}$$

where $i = 1, 2$, and E_b is the transmitted signal energy per bit; the transmitted frequency is

$$f_i = \frac{n_c + i}{T_b} \quad \text{for some fixed integer } n_c \text{ and } i = 1, 2 \tag{6.87}$$

Thus symbol 1 is represented by $s_1(t)$, and symbol 0 by $s_2(t)$. The FSK signal described here is known as *Sunde's FSK*. It is a *continuous-phase signal* in the sense that phase continuity is always maintained, including the inter-bit switching times. This form of digital modulation is an example of *continuous-phase frequency-shift keying* (CPFSK), on which we have more to say later on in the section.

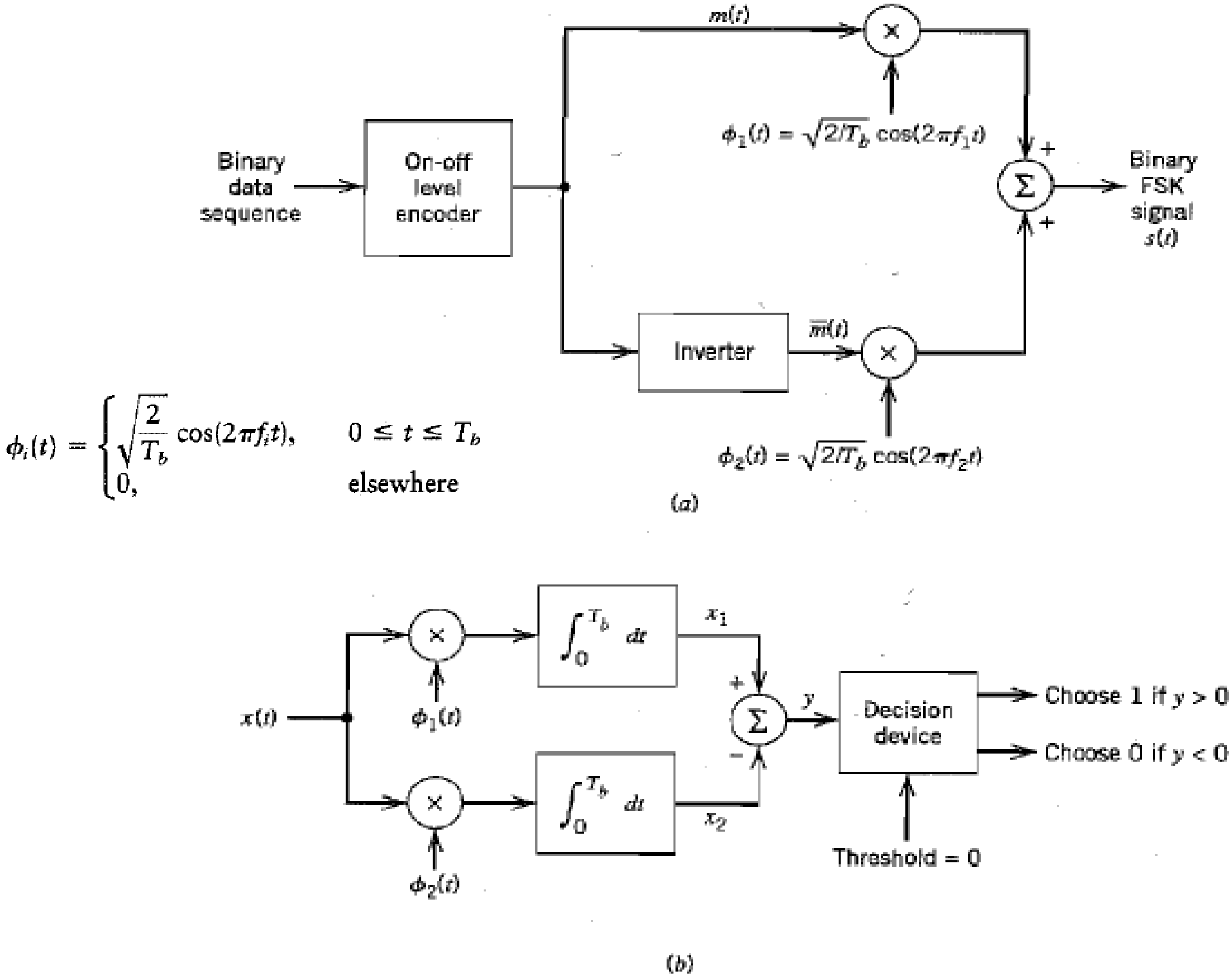
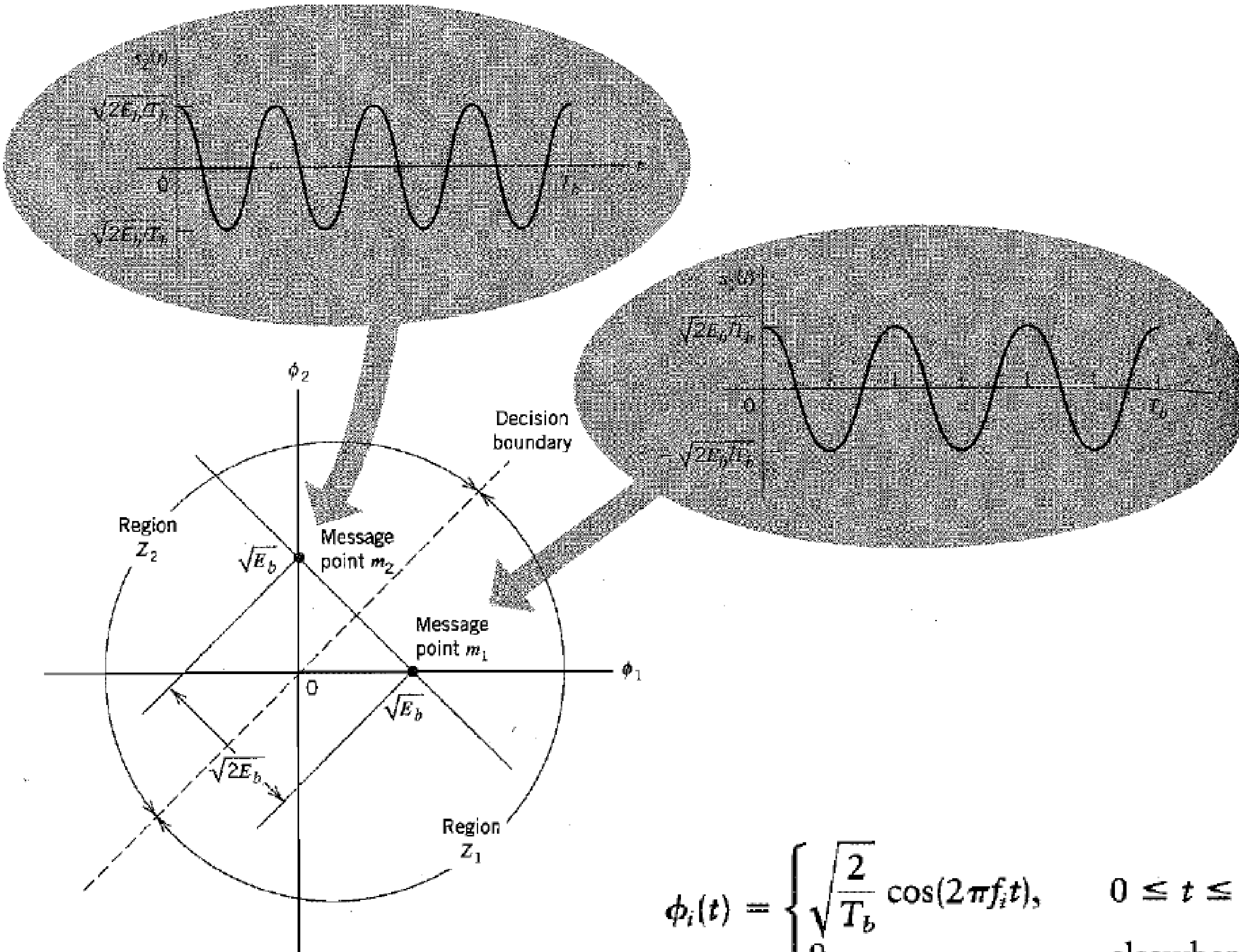


FIGURE 6.26 Block diagrams for (*a*) binary FSK transmitter and (*b*) coherent binary FSK receiver.

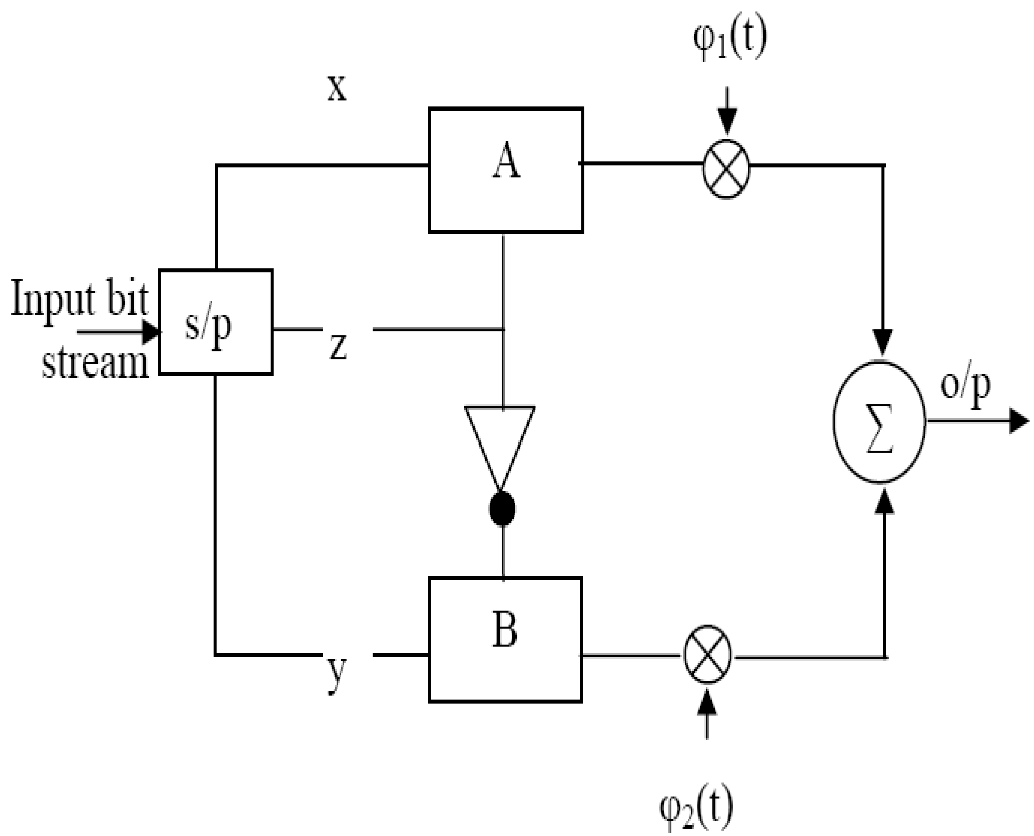


$$\phi_i(t) = \begin{cases} \sqrt{\dfrac{2}{T_b}} \cos(2\pi f_i t), & 0 \leq t \leq T_b \\ 0, & \text{elsewhere} \end{cases}$$

Examples

Q1. In the shown transmitter the amplifiers A& B are controlled by a control bit "z". If "z" is '1' the amplification ratio (for A and B) is 2:1 and if "z" is '-1' the ratio (for A and B)is 1:1. The input stream is divided into symbols each of 3 bits designated as xyz in order. The bits are represented using polar NRZ format with =5v and –A=-5v.

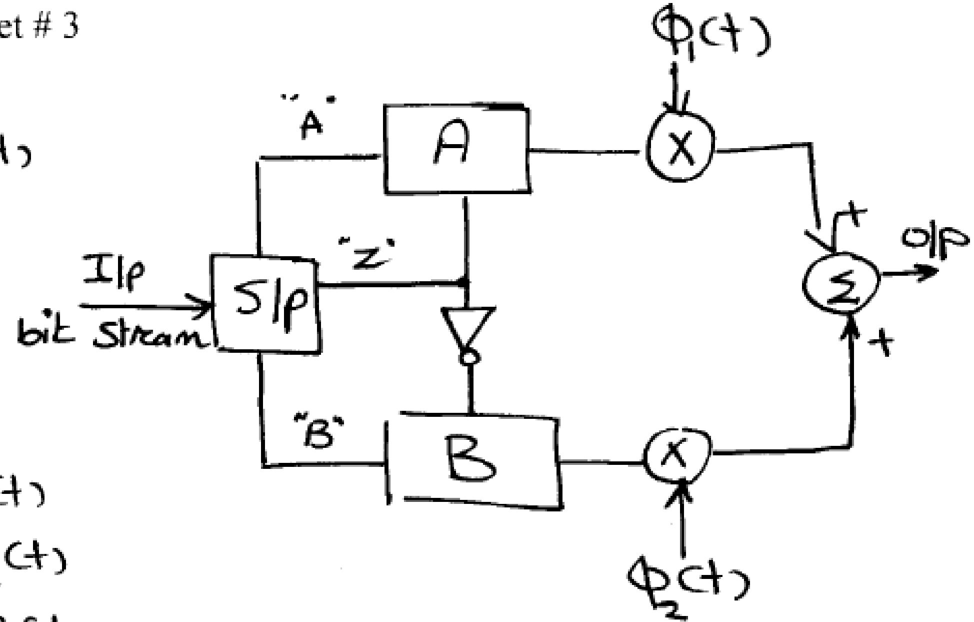
1. Find all possible outputs of the transmitter in terms of $\phi 1$ and $\phi 2$.
2. Sketch to scale the signals in Signal space. and define the Decision Regions (DR) and the Decision Boundaries(DB).



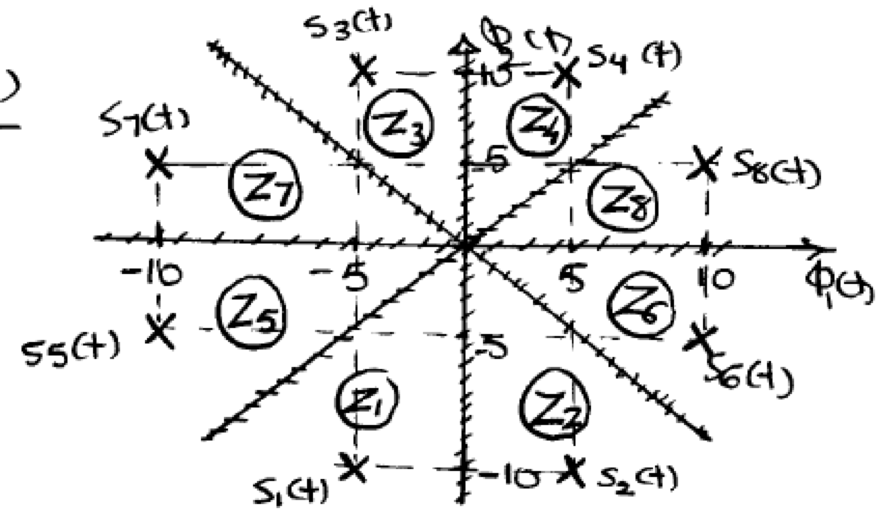
Sheet # 3

Q.1 1) Possible o/ps

"Z"	"B"	"A"	o/p
0	0	0	$s_1(t) = -5\phi_1(t) - 10\phi_2(t)$
0	0	1	$s_2(t) = 5\phi_1(t) - 10\phi_2(t)$
0	1	0	$s_3(t) = -5\phi_1(t) + 10\phi_2(t)$
0	1	1	$s_4(t) = 5\phi_1(t) + 10\phi_2(t)$
1	0	0	$s_5(t) = -10\phi_1(t) - 5\phi_2(t)$
1	0	1	$s_6(t) = 10\phi_1(t) - 5\phi_2(t)$
1	1	0	$s_7(t) = -10\phi_1(t) + 5\phi_2(t)$
1	1	1	$s_8(t) = 10\phi_1(t) + 5\phi_2(t)$



2)



Q2. A communication system uses a signal $s_1(t) = 3\cos(200\pi t) \quad 0 \le t \le 2\sec$ to represent the digit '1'.

To present the digit '0' either $s_2(t)$ or $s_2'(t)$ is available, where

$s_2(t) = -4\cos(200\pi t) \qquad s_2'(t) = 4\cos(400\pi t) \quad 0 \le t \le 2\sec$.

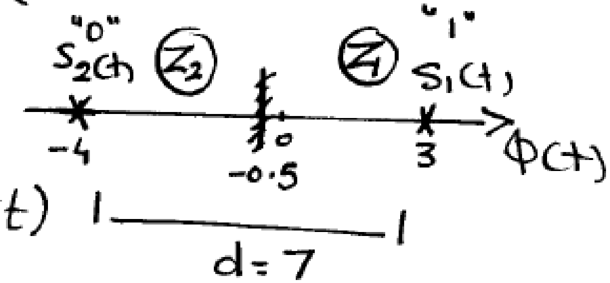
The noise is assumed to be AWGN with two-sided PSD= $\frac{N_0}{2}$ =2watt/Hz.

1. Sketch to scale the two cases in S.S. showing the DRs and the DBs.
2. Calculate the minimum average probability of error.
3. Show that the receiver in both cases can be implemented using a single arm receiver and define each part of the receiver.

Q2.

Case I 1) $s_1(t) = 3\cos(200\pi t)$ $\quad 0 \le t \le 2\,\text{sec}$

$s_2(t) = -4\cos(200\pi t)$

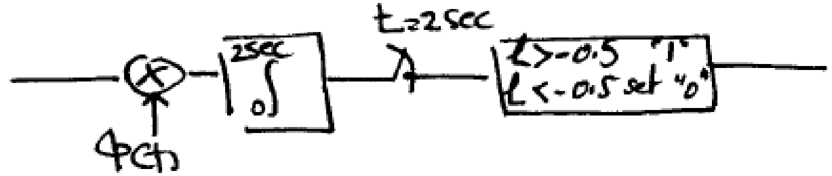


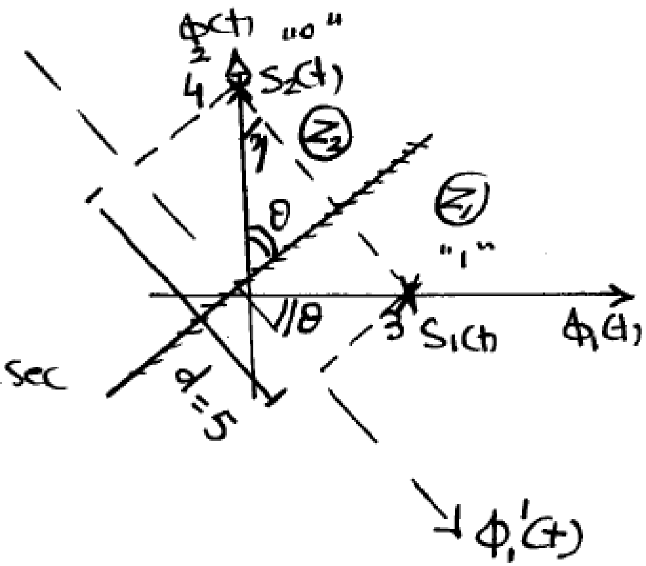
$f_{c_1} = f_{c_2} = f_c = 100\,\text{Hz}$

$\phi(t) = \sqrt{\frac{2}{T}}\cos(200\pi t) = \cos(200\pi t)$

2) $P_e[\varepsilon] = P_e = Q\left[\frac{d}{\sqrt{2N_0}}\right] = Q\left[\frac{7}{\sqrt{8}}\right]$

3)





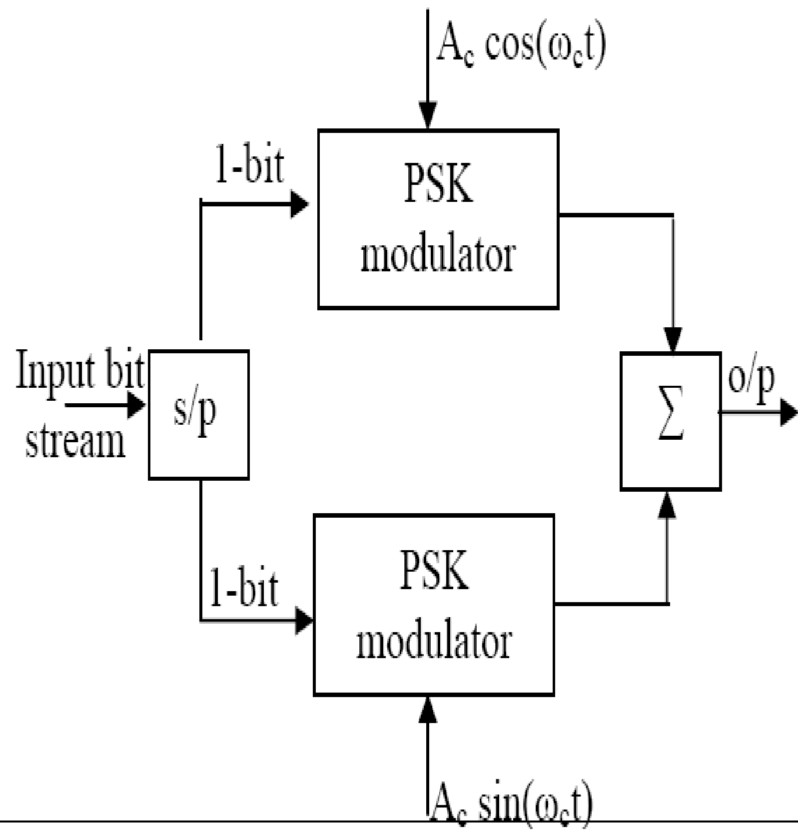
Case II

1) $s_1(t) = 3\cos(200\pi t)$ $\quad 0 \le t \le 2\,\text{sec}$

$s_2'(t) = 4\cos(400\pi t)$

Q3. The below digital modulator scheme produces 4 equally likely messages.

1. Sketch the output possible signals in SS.
2. Draw the DRs and DBs.
3. Calculate the average energy.
4. Calculate the minimum average probability of error if the noise is assumed to be AWGN of zero mean and PSD= $\frac{N_0}{2}$

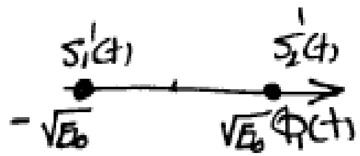


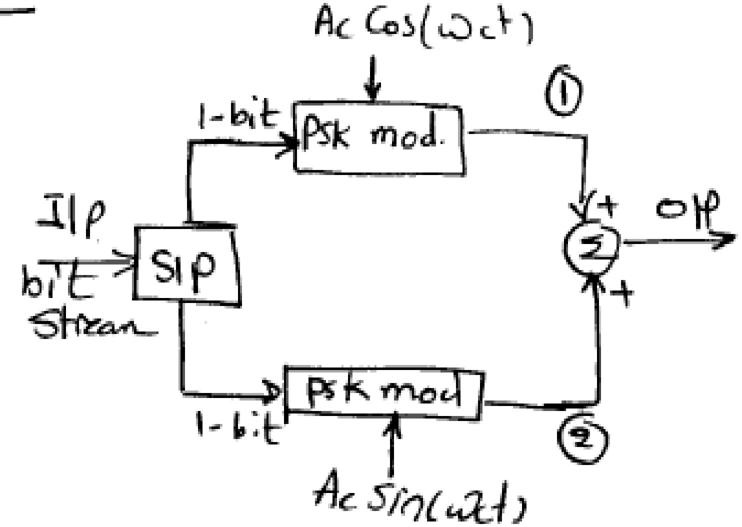
Q4

O/P at pt. ①

$s_1'(t) = -\sqrt{E_b}\,\phi_1(t)$

$s_2'(t) = \sqrt{E_b}\,\phi_1(t)$

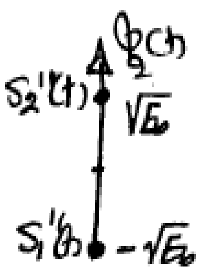




O/P at Pt. ②

$s_1''(t) = -\sqrt{E_b}\,\phi_2(t)$

$s_2''(t) = \sqrt{E_b}\,\phi_2(t)$



Tx. O/P

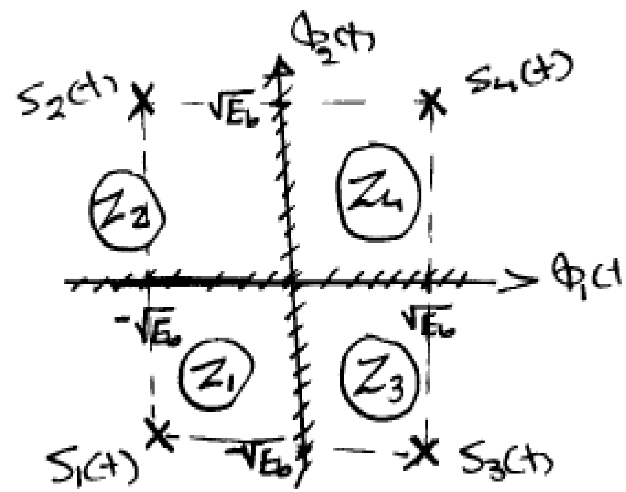
$s_1(t) = -\sqrt{E_b}\,\phi_1(t) - \sqrt{E_b}\,\phi_2(t)$

$s_2(t) = -\sqrt{E_b}\,\phi_1(t) + \sqrt{E_b}\,\phi_2(t)$

$s_3(t) = \sqrt{E_b}\,\phi_1(t) - \sqrt{E_b}\,\phi_2(t)$

$s_4(t) = \sqrt{E_b}\,\phi_1(t) + \sqrt{E_b}\,\phi_2(t)$

2)



3) $E_{av} = \dfrac{4\,(E_b + E_b)}{4} = 2E_b$ Joule